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Justification by Coherence from Scratch*

Can coherence among independent pieces of evidence make them credible even if each piece has no individual credibility? It is generally agreed among recent epistemologists that coherence of independent pieces of evidence can make them *more* credible if each piece already has some credibility of its own. Take a group of somewhat questionable witnesses. Even if each witness's report has only a modest amount of credibility, a close match among many independently produced reports makes people quite confident that they are true. Acknowledging this effect, even the opponents of coherentism in epistemology usually agree that coherence of independent pieces of evidence *enhances* their existing credibility.[1] Our question, however, is whether coherence among independent pieces of evidence with no individual credibility at all can make them credible. We will call this type of evidential support "justification by coherence from scratch" (JCS for short).

JCS has some initial appeal. To continue the example of testimonial evidence, even if we have no trust at all in what each witness says individually, a surprising match among their reports will prompt people to reconsider their assessment, provided it is clear that these reports are produced independently of each other. The reason is that it is highly unlikely—or so it seems intuitively—that unreliable witnesses testifying independently of each other happen to produce matching reports. However, intuition can be misleading especially in matters of probability (Kahneman, *et al*. 1982). This is not merely a general note of caution in the case of JCS. There have been formal arguments against JCS by the use of the probability calculus (Huemer 1997; Olsson 2002). If JCS is impossible by the rules of the probability calculus, we should regard its intuitive appeal as another instance of bias in our pre-theoretical intuitions.

Against this background we aim at three goals in this paper. First, we will confirm formally that coherence does not make independent pieces of evidence credible if each piece has no individual credibility.[2] There are some limitations in the recent formal

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[1] Two of the most prominent foundationalists of the last century, C. I. Lewis and Roderick Chisholm, both stress this role of coherence in epistemic justification. See Lewis (1946, Ch.11) on 'congruence' and Chisholm (1989, Ch.7) on 'concurrence'. For a more recent analysis of the justification-enhancing role of coherence in the foundationalist setting, see Shogenji (2001a, Sec.2).

[2] By the term "credible" we do not mean here *credible enough for acceptance*. As we use the term in this paper, a piece of evidence is "credible" (has "credibility") with regard to a proposition provided it has *some* positive impact on the probability of the proposition. The first claim of our paper is therefore that coherence of independent pieces of evidence has *no positive impact at all* on the probabilities of

arguments against JCS—viz., Olsson (2002) argues against JCS in cases involving *two* independent pieces of evidence, while Huemer (1997) examines cases of *complete agreement* among independent pieces of evidence. We want to show more generally with no restrictions on the number of pieces of evidence or the type of relation that holds among them that JCS is impossible.

Our second goal is to explain the false intuitive appeal of JCS. We will analyze an intuitively plausible informal reasoning for JCS and uncover an ambiguity in the notion of "individual credibility" used in the reasoning.

These findings are discouraging for supporters of JCS, but our analysis of JCS also points to a way of circumventing the formal result against it. We will see that coherence, *when it is recurrent*, can make independent sources of evidence with no individual credibility credible. For example, if the same group of independent witnesses with no individual credibility repeatedly produce reports that are in agreement with each other, the witnesses—and hence their reports—can become credible. In other words, *recurrent* coherence makes justification from scratch possible although coherence *per se* does not. Our third goal in this paper is to establish justification by recurrent coherence from scratch (JRCS for short) by the formal probability calculus.

1. Formal Argument against JCS

In this section we will show formally and generally that JCS is impossible. An obstacle to making the argument against JCS general is disagreements over the proper probabilistic formulation of coherence (Akiba 2000; Shogenji 1999; 2001b; Olsson 2001; Fitelson 2003). Faced with the uncertainty, some people may consider refuting JCS with regard to all known formalizations of coherence, but that would still leave the possibility open that some hitherto unknown formalization of coherence may make JCS possible. Another concern from a broader perspective on epistemology is that if some relation among independent pieces of evidence makes justification from scratch possible, whether this relation is something we ordinarily call "coherence" is probably of little importance. For, arguably the most interesting aspect of JCS from the epistemological standpoint is that pieces of evidence with no individual credibility become credible simply because of their *relation* in contents—i.e., because of the relation among propositions they support. From this standpoint, whether or not the relation in question is coherence-like is much less important than the existence of a relation with such power. The following remark by BonJour (1985, p.88) is indicative of this attitude:

> [A system of empirical beliefs] can only be justified from within, by virtue of the relations of its component beliefs to each other—if, that is, it is justified at all. And the idea of coherence should for the moment be taken merely to indicate whatever property (or complex set of properties) is requisite for the justification of such a system of beliefs

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the propositions they purport to support if each piece of evidence individually has no positive impact on the probabilities of these propositions.

Given this understanding of the epistemological significance of JCS, we will construct a general argument against justification by *relation* from scratch. In other words, we are going to establish formally that no matter what relation one takes coherence to be, coherence does not make independent pieces of evidence with no individual credibility credible.

The following is then the initial statement of our goal in this section. Let $E_1, \ldots, Ew$ be pieces of evidence in support, respectively, of propositions, $A_1, \ldots, Aw$. We will establish that if $E_1, \ldots, Ew$ are independent of each other and have no individual credibility, then $P(Ai|E_1, \ldots, Ew) = P(Ai)$ for any $i = 1, \ldots, w$. In other words, if the pieces of evidence are independent of each other and have no individual credibility, then they have no impact on the probability of any propositions they (purport to) support. There is no reference in this statement to any relation that may hold among $A_1, \ldots, Aw$—the claim is true regardless of their relation. This means that JCS is impossible not only in any probabilistic interpretation of coherence but in any non-probabilistic interpretation of it as well, e.g., in terms of explanatory relations (Thagard 1989; 1992). Probabilistic or otherwise, the relation among the propositions has no impact on their posterior probabilities provided the pieces of evidence are independent of each other and have no individual credibility.

Our next task is to express the two key concepts in the statement in formal terms of the probability calculus—viz., the concept of independence among pieces of evidence and the concept of individual credibility. We start with the former. The concept of independence among pieces of evidence (evidential independence) needs to be distinguished from probabilistic independence. Probabilistic independence is a relation among propositions such that truth of one does not affect the probability of any other. For example, two propositions A_1 and A_2 are probabilistically independent if and only if $P(A_1|A_2) = P(A_1)$, or equivalently $P(A_1 \wedge A_2) = P(A_1) \times P(A_2)$, assuming that $P(A_2) \neq 0$. Evidential independence, on the other hand, is a relation among pieces of evidence such that the existence of one piece does not influence the existence of another. For example, in order for two reports to be evidentially independent, they must be produced by different witnesses who have had no communication with each other.

There is a general consensus among probability theorists on how to formalize the condition that two pieces of evidence E_1 and E_2 are independent of each other with respect to proposition A. The standard formalization is that $P(E_1|E_2, A) = P(E_1|A)$ and $P(E_1| E_2, \neg A) = P(E_1|\neg A)$, or equivalently $P(E_1 \wedge E_2|A) = P(E_1|A) \times P(E_2|A)$ and $P(E_1 \wedge E_2|\neg A) = P(E_1|\neg A) \times P(E_2|\neg A)$, assuming that $P(E_2 \wedge A) \neq 0$ and $P(E_2 \wedge \neg A) \neq 0$. The idea is that if two pieces of evidence are independent of each other with respect to a proposition, then they affect each other probabilistically only through the truth or falsity of the proposition. As a result, once the truth or falsity of the proposition is given, the two pieces are probabilistically independent of each other. We can extend this formalization naturally to cases involving three or more pieces of evidence. Namely, pieces of evidence are independent of each other with respect to a proposition if and only if, given the truth or falsity of the proposition, any of these pieces of evidence are probabilistically independent of each other. More precisely, given the truth or falsity of the proposition, the members of any subset (except the empty set and singletons) of the set of these pieces of evidence are probabilistically independent of each other. For example, three pieces of evidence E_1, E_2 and E_3 are independent of each other with respect to proposition A if and only if:

$$P(E_1 \wedge E_2|A) = P(E_1|A) \times P(E_2|A) \qquad P(E_1 \wedge E_2|\neg A) = P(E_1|\neg A) \times P(E_2|\neg A)$$
$$P(E_2 \wedge E_3|A) = P(E_2|A) \times P(E_3|A) \qquad P(E_2 \wedge E_3|\neg A) = P(E_2|\neg A) \times P(E_3|\neg A)$$
$$P(E_3 \wedge E_1|A) = P(E_3|A) \times P(E_1|A) \qquad P(E_3 \wedge E_1|\neg A) = P(E_3|\neg A) \times P(E_1|\neg A)$$
$$P(E_1 \wedge E_2 \wedge E_3|A) = P(E_1|A) \times P(E_2|A) \times P(E_3|A)$$
$$P(E_1 \wedge E_2 \wedge E_3|\neg A) = P(E_1|\neg A) \times P(E_2|\neg A) \times P(E_3|\neg A).$$

It follows immediately from this definition that if pieces of evidence $E_1, \ldots, Ew$ are independent with respect to proposition A, then $P(E_1 \wedge \ldots \wedge Ew|A) = P(E_1|A) \times \ldots \times P(Ew|A)$ and $P(E_1 \wedge \ldots \wedge Ew|\neg A) = P(E_1|\neg A) \times \ldots \times P(Ew|\neg A)$.

We also need to formalize the concept of individual credibility. More specifically, we want to express formally the condition that pieces of evidence $E_1, \ldots, Ew$ have no individual credibility with respect to propositions $A_1, \ldots, Aw$ they purport to support. It is clear that the formal condition should include $P(Ai|Ei) = P(Ai)$ for $i = 1, \ldots, w$—e.g., in the absence of individual credibility, evidence E_1 on its own should not affect the probability of A_1. But this is not enough. For, this would still allow Ei to be credible evidence for Aj for some $j \neq i$. If, for example, E_1 can be credible evidence for A_2, it is no surprise that the combined evidence $E_1 \wedge E_2$ can affect the probability of A_1. For this reason, the condition of no individual credibility should be understood more broadly that for any $i = 1, \ldots, w$ and any $j = 1, \ldots, w$, $P(Ai|Ej) = P(Ai)$, or equivalently $P(Ej|Ai) = P(Ej)$, assuming that $P(Ej) \neq 0$ and $P(Aj) \neq 0$.

We are now ready to show formally and generally that JCS is impossible.

Claim: Suppose the following conditions hold:
(i) Pieces of evidence $E_1, \ldots, Ew$ are independent of each other with respect to propositions $A_1, \ldots, Aw$.
(ii) Pieces of evidence $E_1, \ldots, Ew$ have no individual credibility with respect to propositions $A_1, \ldots, Aw$.
(iii) Propositions $A_1, \ldots, Aw$, their negations $\neg A_1, \ldots, \neg Aw$, and the conjunction $E_1 \wedge \ldots \wedge Ew$ of the pieces of evidence have nonzero probabilities.
Then, for any $i = 1, \ldots, w$, $P(Ai|E_1, \ldots, Ew) = P(Ai)$.

The substantive conditions are (i) and (ii), with condition (iii) excluding trivial cases where conditional probabilities are undefined. The proof of the claim is straightforward.

Proof: By condition (i) of independence among the pieces of evidence,

$$P(E_1 \wedge \ldots \wedge Ew|Ai) = P(E_1|Ai) \times \ldots \times P(Ew|Ai). \qquad (1)$$

By condition (ii) of no individual credibility,

$$P(Ej|Ai) = P(Ej). \qquad (2)$$

From (1) and (2) it follows that

$$P(E_1 \wedge \ldots \wedge Ew|Ai) = P(E_1) \times \ldots \times P(Ew). \qquad (3)$$

By similar reasoning,

$$P(E_1 \wedge \ldots \wedge Ew | \neg Ai) = P(E_1) \times \ldots \times P(Ew). \quad (4)$$

From (3) and (4) it follows that

$$P(E_1 \wedge \ldots \wedge Ew | Ai) = P(E_1 \wedge \ldots \wedge Ew | \neg Ai). \quad (5)$$

And hence,

$$P(E_1 \wedge \ldots \wedge Ew | Ai) = P(E_1 \wedge \ldots \wedge Ew). \quad (6)$$

From (6) it follows by Bayes' Theorem that

$$P(Ai | E_1, \ldots, Ew) = P(Ai).$$

2. Intuitive Appeal of JCS

We have now established formally that coherence—no matter what relation we take coherence to be—does not make independent pieces of evidence with no individual credibility credible. But there is a seemingly plausible informal reasoning in support of JCS. This section describes the reasoning and examines it critically in light of our formal result.

The most prominent advocate of JSC in the recent literature is Laurence BonJour (1985). Referring to C. I. Lewis's (1946, p. 346) example in which "congruence" of witness reports *enhances* their existing credibility, BonJour (1985, p.148) makes the following remark:

> What Lewis does not see, however, is that his own example shows quite convincingly that no antecedent degree of warrant or credibility is required. For as long as we are confident that the reports of various witnesses are genuinely independent of each other, a high enough degree of coherence among them will eventually dictate the hypothesis of truth telling as the only available explanation of their agreement—even, indeed, if those individual reports initially have a high degree of negative credibility, that is, are much more likely to be false than true (for example in the case where all the witnesses are known to be habitual liars).

The basic idea here seems to be that if they are produced independently, it is extremely unlikely that reports by unreliable or lying witnesses happen to be in agreement with each other, and therefore independent reports that are in agreement with each other are very likely to be true.

Before examining this reasoning, we want to note that the kind of reasoning envisioned here does not by itself support coherentism in epistemology, which we take to be the view that coherence among beliefs is a *source* of epistemic justification. For, the proposed justification treats the existence of relevant pieces of evidence as *given*, which is in line with foundationalism in epistemology. In the case of witness reports, we are only evaluating the credibility of the reports in light of their coherence without questioning the existence of these reports themselves. Foundationalists may suggest that the beliefs about the existence of relevant pieces of evidence are *basic beliefs* and that JCS is only a claim about the justification of *derived beliefs*.[3] Indeed this seems to be BonJour's (1999) more recent view, where beliefs about one's own sensory experiences have independent justification prior to the consideration of coherence. Under this construal JCS is primarily an attempt to meet the challenge of justifying our beliefs about the external world within the traditional framework of foundationalism. In this paper we do not address the issue of how JCS is to be incorporated into the general theory of epistemic justification. We only examine whether JCS is possible.

Let us take a closer look at BonJour's reasoning. Witnesses to whom we initially assign no credibility may turn out to be truthful, but they may also turn out to be unreliable witnesses whose reports are sometimes true and sometimes false, or even liars who deliberately make false statements. But the latter two hypotheses do not adequately explain surprising agreement among independent reports. Let us assume here for the sake of simplicity that the reports are in complete agreement—for example, every witness states that Jones is the murderer. Clearly the witnesses cannot include both a truth teller and a liar since their reports are in agreement. Also, it is unlikely that they include many unreliable witnesses since chances are slim that many unreliable witnesses happen to produce the same report independently. Consequently, if independently produced reports turn out to be in agreement, it is likely that either most of the witnesses are truth tellers or most of the witnesses are liars. Further, where there are many ways of telling a lie while there is only one way of telling the truth, as is commonly the case, we can disregard the mostly-liar hypothesis because it is very unlikely that many liars producing their reports independently happen to tell the same lie. Thus, the only plausible explanation of surprising agreement among independently produced reports is that most of the witnesses are truth tellers, and hence it is very likely that their reports are true.

We want to note first that this reasoning depends on some additional conditions beyond agreement of independent pieces of evidence. First, it starts with the assumption that the witnesses may turn out to be truth tellers though they may also turn out to be unreliable witnesses or liars. This assumption is important because if it is already certain that the witnesses are unreliable, then no amount of agreement among their reports makes them credible. Plausible or not, fortuitous coincidence is the only available explanation of the agreement.[4] Supporters of the informal reasoning for JCS, therefore, must assume that the truth teller hypothesis has a positive prior probability. Some people may worry that this assumption violates the condition of no individual credibility—i.e., if a witness is possibly a truth teller, isn't her report individually a weak but still positive piece of evidence for the truth of its content? That, however, does not follow because a positive

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[3] Shogenji (2001a) describes a way coherence may *channel* justification from basic meta-beliefs to derived first-order beliefs.

[4] This is essentially what Huemer (1997) shows.

prior probability for the truth teller hypothesis may be offset by a positive prior probability for the liar hypothesis. If the witness is possibly a truth teller but she is also possibly a liar, there may be no reason to regard her report as a positive piece of evidence. One way of interpreting the informal reasoning for JCS is that the initial balance between the truth teller and the liar hypotheses is tipped by the coherence of reports in favor of the truth teller hypothesis.

The informal reasoning for JCS also requires that there are many ways of telling a lie while there is only one way of telling the truth. This condition is not always satisfied. For example, if the witnesses are reporting on the result of a coin toss, there is only one way they can tell a lie. It is therefore just as likely that the agreement of reports result from the witnesses being all liars as it is from their being all truth tellers. There is no reason in such a case to favor the truth teller hypothesis over the liar hypothesis because of the reports' agreement. It is critical to the success of the informal reasoning for JCS that there are many ways of telling a lie.

The informal reasoning for JCS turns out to depend on two additional conditions, but we do not think this is a defect in the reasoning. These are conditions that hold commonly and it is unreasonable in our view to disallow the use of common conditions in the epistemic evaluation of evidence. In fact it is quite clear that we cannot defend the universal claim that under any conditions coherence makes independent reports with no individual credibility credible. A more interesting issue is whether there are some (fairly common) conditions under which JCS is possible. The claim at issue is, therefore, existential. We also want to note that these additional assumptions do not make the informal reasoning for JCS consistent with the formal result against it in the preceding section. For, the formal result states that no pieces of evidence alter the probability of the propositions they purport to support provided two conditions hold—viz., the pieces of evidence are independently produced and they have no individual credibility. The formal result should hold even if there is the possibility initially that the witnesses are truthful and that there are many ways the witness may tell a lie.

We have, in other words, yet to resolve the conflict between the informal reasoning for JCS and the formal argument against it. We believe the key to resolving the conflict is an ambiguity in the notion of individual credibility. In our understanding evidence E_1 has no individual credibility with respect to proposition A_1 if and only if the existence of E_1 does not alter the probability of A_1—i.e., $P(A_1|E_1) = P(A_1)$. E_1 has positive (or negative) credibility with respect to A_1 if and only if the existence of E_1 raises (or lowers) the probability of A_1. However, this is not the way BonJour understands the notion of individual credibility in the passage quoted above. Note in particular his characterization of individual reports with "a high degree of negative credibility" as those which "are much more likely to be false than true." This indicates that individual reports that are more likely to be false than true have negative credibility, while those that are more likely to be true than false have positive credibility. In other words, in BonJour's understanding evidence E_1 has no individual credibility (positive or negative) with regard to proposition A_1 if and only if given E_1, A_1 is as likely to be true as it is false—i.e., $P(A_1|E_1) = P(\neg A_1|E_1)$, or equivalently $P(A_1|E_1) = ½$.

We now have two ways of formalizing the condition of no individual credibility—$P(A_1|E_1) = P(A_1)$ and $P(A_1|E_1) = ½$. The difference is inconsequential in some cases, e.g., if there are only two equally probable propositions, A_1 and $\neg A_1$, to choose from. For, in that case $P(A_1) = ½$, and therefore $P(A_1|E_1) = P(A_1)$ if and only if $P(A_1|E_1) = ½$. However, where there are many propositions to choose from, as the informal reasoning for JCS requires so that there are many ways of telling a lie, we must decide which formalization is appropriate. Let us see this in a concrete example. Suppose there are ten suspects in a murder case who are initially equally likely to be the murderer so that $P(A_1) = \ldots = P(A_{10}) = .1$. Report E_1 in support of A_1 has no credibility in BonJour's sense just in case $P(A_1|E_1) = .5$—i.e., just in case there are equal chances that the report is true or false. Note, however, that the existence of such evidence makes A_1 five times more likely to be true than it is without the evidence—i.e., before obtaining E_1, the probability of A_1 is .1, but given the evidence it is .5. E_1 is therefore a powerful piece of positive evidence for A_1 on its own. It is no surprise that an increasing number of matching reports with "no individual credibility" in BonJour's sense makes A_1 more and more probable. This understanding of "no individual credibility" is inappropriate in the context of evaluating justification *from scratch*. If we are allowed to use pieces of evidence each of which increases the probability of the proposition five-fold on its own, then a further rise in probability due to their coherence is not justification from scratch.

The condition of no individual credibility used in our formal argument against JCS requires that in the murder case above if E_1 has no individual credibility, then $P(A_1|E_1) = P(A_1) = .1$. This means that the report must be nine times more likely to be false than it is true. Once we realize this implication of the condition of no individual credibility, it is no longer clear intuitively that coherence of independent pieces of evidence with no individual credibility makes them credible. For example, if two independent witnesses who are individually nine times more likely to be lying than truthful identify the same person out of ten equally likely suspects as the murderer, is it reasonable to conclude that the chances of the person being the real murderer are higher than one out of ten? We should note that as the number of matching reports increases, it becomes more and more unlikely that the witnesses happen to be lying in the same way. However, this improbability is offset by the rapid decrease in the probability that all witnesses are truthful since each witness with no individual credibility is already unlikely to be truthful. For example, the probability of all three witnesses being truthful is only 9^{-3} of the probability that all of them are lying. This surely makes us much less confident that in the absence of individual credibility coherence of evidence has a positive impact on the probability of the proposition, and the formal result in the preceding section bears out this doubt.

3. Justification by Recurrent Coherence from Scratch

We have discovered that the seemingly plausible informal reasoning for JCS is deficient. The reasoning is based on the plausible idea that where there are many ways of lying, it is unlikely that independently produced lies agree with each other. The problem is that where there are many ways of lying, a witness with no individual credibility in the sense appropriate for JCS must be much more likely to be lying than truthful. We grant that as the number of agreeing witnesses increases, it becomes more and more unlikely that many liars happen to produce matching reports. However, this rapid decrease in the probability of the liar hypothesis is offset by the rapidly decreasing probability that the witnesses with no individual credibility are all truth tellers.

In order to overcome this difficulty, supporters of justification from scratch must find a scenario where there is a decrease in the probability that lying witnesses are fortuitously producing matching reports but there is no comparable decrease in the probability that the witnesses are all truth tellers. This is possible if we can increase the number of matching reports (thus decreasing the probability that liars happen to be telling the same lie) without increasing the number of witnesses. We find this situation in the case of recurrent coherence, e.g., when the same group of independent witnesses repeatedly produces agreeing reports. We are going to show formally that recurrent coherence of independent pieces of evidence with no individual credibility can make them credible. We want to note that our claim of justification by recurrent coherence from scratch (JRCS) is existential—i.e., we claim that there are certain (fairly common) conditions under which there is such justification. Therefore, it suffices to present a clear case in which no conditions are extraordinary and there is justification by recurrent coherence from scratch. We now describe such a case.

Colored marbles are drawn randomly from an urn. Each draw has a name: *a*, *b*, … The marble drawn has one of *n* different colors with equal probabilities. We assume that $n > 2$ so that the witness can tell a lie in more than one way. We let predicates '*Bx*', '*Gx*', '*Rx*' mean that *x* is blue, green, red, respectively. Variables *p*, *q*, … range over propositions, such as *Ba* and *Gb*. When a marble is drawn, the subject receives reports from witnesses 1, 2, … about its color. Variables *i*, *j*, … range over the witnesses. A *report statement* consists of a proposition and a witness subscript. For example, 'Gc_2' means that draw *c* is green according to witness 2; $\ulcorner p_j \urcorner$ means that it is the case that *p* according to witness *j*. The subject judges the color of the marble based on report statements.[5] We distinguish three types of witnesses—viz., witness *i* may be a truth teller (*Ti*), a randomizer (*Ri*), or a liar (*Li*). A truth teller invariably produces true reports and hence her report is reliable; a randomizer's report is probabilistically independent of its truth and hence her report is unreliable; and finally a liar invariably produces false reports and hence her report is anti-reliable.[6] We also assume that a liar is as likely to produce one false report as any other. We express these properties formally, as follows:

For any *p* and *i*, $P(p_i|p, Ti) = 1$ and $P(p_i|\neg p, Ti) = 0$.
For any *p* and *i*, $P(p_i|p, Ri) = P(p_i|\neg p, Ri) = P(p_i)$.
For any *p* and *i*, $P(p_i|p, Li) = 0$ and $P(p_i|\neg p, Li) = 1/(n-1)$.

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[5] When the witness *i* states that *p*, *p* is a *report* while p_i is a *report statement*. Accordingly, the conjunction *X* of *report statements* has the form: p_i & q_j & … Meanwhile, a relation among *reports* such as coherence is a relation that holds among *p*, *q*, …, and not among p_i, q_j, …

[6] The characterization of a liar as a producer of invariably false reports only applies to atomic propositions. In producing her reports a liar assigns the opposite truth values to all atomic propositions. The liar then assigns truth values to non-atomic propositions in the standard fashion based on the atomic assignment. This proviso is necessary to keep the liar's identity in the dark in the face of some trick questions—say, the question whether the drawn marble is *both blue and not blue*, to which a simple-minded liar would answer 'Yes' thereby revealing her mendacious nature immediately. Our atomic liar avoids detection by answering 'No' to this question regardless of the marble's true color. Propositional and predicate letters used in this paper should be understood to be atomic unless noted otherwise.

For the sake of simplicity, we assume that every witness is either a truth teller, a randomizer, or a liar; and thus no witness has a mixed nature—e.g., no witness is more reliable than a randomizer but less reliable than a truth teller. To express this formally, for any i, $P(Ti) + P(Ri) + P(Li) = 1$. We also assume that a witness retains the same nature in all her reports—e.g., no witness is a truth teller in one report but a liar in another. In other words, for any i and p, $P(Ti|p_i) = P(Ti)$, $P(Ri|p_i) = P(Ri)$, and $P(Li|p_i) = P(Li)$.

Earlier we pointed out informally that where there are many ways of telling a lie, a witness with no individual credibility must be more likely to be a liar than she is a truth teller. We now prove formally that $P(p|\, p_i) = P(p)$ requires that $P(Li) = (n - 1)P(Ti)$ and hence $P(Li) > P(Ti)$ for $n > 2$. We need to consider three possibilities. First, if witness i is a truth teller, i tells that p if and only if it is actually the case that p. Second, if i is a randomizer, i tells that p one out of n times no matter what is actually the case. Finally, if i is a liar, i tells that p only if it is not actually the case that p; and further, if it is not actually the case that p, then i tells that p one out of $n - 1$ times since there are $n - 1$ ways of telling a lie. To put these observations together, the probability that i tells that p is:

$$
\begin{aligned}
P(p_i) &= P(Ti) \times P(p) + P(Ri) \times 1/n + P(Li) \times P(\neg p) \times 1/(n - 1) \\
&= P(Ti) \times 1/n + P(Ri) \times 1/n + P(Li) \times (n - 1)/n \times 1/(n - 1) \\
&= P(Ti) \times 1/n + P(Ri) \times 1/n + P(Li) \times 1/n \\
&= [P(Ti) + P(Ri) + P(Li)] \times 1/n \\
&= 1/n.
\end{aligned}
$$

Meanwhile the probability that i tells that p given that it is actually the case that p is:

$$P(p_i|p) = P(Ti) + P(Ri) \times 1/n.$$

It follows from these that if $P(p|p_i) = P(p)$, or equivalently if $P(p_i|p) = P(p_i)$, then:

$$
\begin{aligned}
&P(Ti) + P(Ri) \times 1/n = 1/n. \\
\therefore\ &P(Ri) = 1 - P(Ti) \times n.
\end{aligned}
$$

However, $P(Li) = 1 - P(Ti) - P(Ri)$, and hence:

$$
\begin{aligned}
P(Li) &= 1 - P(Ti) - (1 - P(Ti) \times n) \\
&= (n - 1)P(Ti).
\end{aligned}
$$

It follows from this further that $P(Li) > P(Ti)$ for $n > 2$.

We now turn to our main task of showing that under the condition of no individual credibility, recurrent coherence among independent pieces of evidence can raise the probability that the propositions they support are true. We use the concrete example that witnesses 1 and 2, each of whom has no individual credibility and is therefore $n - 1$ times more likely to be a liar than she is a truth teller, agree that draw a is blue and further that draw b is green. We will show that these reports raise the probability that a is blue—i.e., $P(Ba|Ba_1, Ba_2, Gb_1, Gb_2) > P(Ba)$.[7] Our strategy is to calculate $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|Ba)$ and $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|\neg Ba)$ to show that the former is greater than the latter. It follows from this that $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|Ba) > P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2)$, and further by Bayes' Theorem that $P(Ba|Ba_1, Ba_2, Gb_1, Gb_2) > P(Ba)$, which is our claim.

We start with $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|Ba)$.[8] Given that a is actually blue, no liar reports that a is blue. Therefore, given that a is actually blue, there are three possible cases in which 1 and 2 agree that a is blue and b is green. First, both witnesses may be truth tellers, in which case b must be actually green. Second, only one witness may be a truth teller while the other is a randomizer, in which case b must be actually green. Third, both witnesses may be randomizers, in which case b may or may not be green. We also note that where there is a randomizer(s), each randomizer tells that a is blue one out of n times, and that b is green one out of n times. So, each randomizer tells that a is blue *and* b is green one out of n^2 times. We combine these points to calculate the value of $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|Ba)$ as follows, where $\alpha = P(T_1) = P(T_2)$ and $\beta = P(R_1) = P(R_2)$, assuming that the two witnesses have the same initial probabilities with regard to their reliable, unreliable, or anti-reliable nature:

$$\begin{aligned} P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|Ba) &= \alpha^2 \times P(Gb) + 2 \times \alpha\beta \times P(Gb) \times 1/n^2 + \beta^2 \times 1/n^2 \times 1/n^2 \\ &= \alpha^2 \times 1/n + 2 \times \alpha\beta \times 1/n \times 1/n^2 + \beta^2 \times 1/n^2 \times 1/n^2 \\ &= \alpha^2/n + 2\alpha\beta/n^3 + \beta^2/n^4. \end{aligned}$$

Next we calculate $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|\neg Ba)$.[9] Given that a is not actually blue, no truth teller reports that a is blue. Therefore, given that a is not actually blue, there are three possible cases in which 1 and 2 agree that a is blue and b is green. First, both witnesses may be liars, in which case b is not actually green. Second, only one witness may be a liar while the other is a randomizer, in which case b is not actually green. Third, both witnesses may be randomizers, in which case b may or may not be green. As noted above, each randomizer tells that a is blue and b is green one out of n^2 times. Meanwhile each liar chooses the particular lie that a is blue one out of $n - 1$ times, and another lie that b is green one out of $n - 1$ times since there are $n - 1$ ways of telling a lie. So, each liar tells the particular lie that a is blue *and* b is green one out of $(n - 1)^2$ times. We combine these observations to calculate the value of $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|\neg Ba)$ as follows, where $\gamma = P(L_1) = P(L_2)$ and $\beta = P(R_1) = P(R_2)$:

$$\begin{aligned} P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2|\neg Ba) &= \gamma^2 \times P(\neg Gb) \times 1/(n-1)^2 \times 1/(n-1)^2 \\ &\quad + 2 \times \gamma\beta \times P(\neg Gb) \times 1/(\mathrm{n}-1)^2 \times 1/n^2 + \beta^2 \times 1/n^2 \times 1/n^2 \\ &= \gamma^2 \times (n-1)/n \times 1/(n-1)^2 \times 1/(n-1)^2 \\ &\quad + 2 \times \gamma\beta \times (n-1)/n \times 1/(\mathrm{n}-1)^2 \times 1/n^2 + \beta^2 \times 1/n^2 \times 1/n^2 \\ &= \gamma^2/n(n-1)^3 + 2\gamma\beta/n^3(n-1) + \beta^2/n^4 \end{aligned}$$

[7] Similarly, $P(Gb|Ba_1, Ba_2, Gb_1, Gb_2) > P(Gb)$.
[8] Compare Table 1 in the appendix to keep track of different cases and sub-cases in the explanations and calculations that follow.
[9] Compare Table 2 in the appendix to keep track of different cases and sub-cases in the explanations and calculations that follow.

It follows from the two calculations that:

$$\begin{aligned}&P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | Ba) - P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | \neg Ba)\\ &\quad = [\alpha^2/n + 2\alpha\beta/n^3 + \beta^2/n^4] - [\gamma^2/n(n-1)^3 + 2\gamma\beta/n^3(n-1) + \beta^2/n^4]\\ &\quad = [\alpha^2/n - \gamma^2/n(n-1)^3] + [2\alpha\beta/n^3 - 2\gamma\beta/n^3(n-1)]\end{aligned}$$

But, as we saw above, the condition of no individual credibility entails that $\gamma = (n-1)\alpha$, and therefore:

$$\begin{aligned}&P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | Ba) - P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | \neg Ba)\\ &\quad = [\alpha^2/n - (n-1)^2\alpha^2/n(n-1)^3] + [2\alpha\beta/n^3 - 2(n-1)\alpha\beta/n^3(n-1)]\\ &\quad = \alpha^2/n - \alpha^2/n(n-1)\\ &\quad = \alpha^2/n \times (1 - 1/(n-1))\\ &\quad = \alpha^2(n-2)/(n-1)n\end{aligned}$$

Since $\alpha > 0$ and $n > 2$, it follows that:

$$\begin{aligned}&P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | Ba) - P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | \neg Ba) > 0.\\ \therefore\ &P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | Ba) > P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | \neg Ba).\\ \therefore\ &P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | Ba) > P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2).\end{aligned}$$

We then use Bayes' Theorem to conclude that:

$$P(Ba | Ba_1, Ba_2, Gb_1, Gb_2) > P(Ba).$$

The conclusion means that repeated agreement by the two witnesses that *a* is blue and that *b* is green raises the probability that *a* is indeed blue even if each report has no individual credibility.

One final remark is in order. Some people may wonder how the formal result against JCS obtained in Section 1 meshes with the new result in support of JRCS. The result in Section 1 states that no relation among pieces of evidence alters the probability of the propositions they support if these pieces are produced independently and have no individual credibility. Since recurrent coherence of independent pieces of evidence alters the probability of the propositions they support, one of the conditions for the result against JCS must be absent, and the absent condition is evidential independence of the reports. JRCS still assumes that the *witnesses* are independent—e.g., the two witnesses 1 and 2 produce their reports independently of each other, so that Ba_1 and Ba_2 are evidentially independent of each other with respect to *Ba*. The next pair of reports Gb_1 and Gb_1 is also evidentially independent of each other with respect to both *Ba* and *Gb*. However, the four reports, Ba_1, Ba_2, Gb_1 and Gb_2, are not evidentially independent of each other with respect to *Ba*. To state the reason for this informally, given that Ba, the reports Ba_1 and Ba_2 eliminate the possibility that the witnesses are liars. This makes their next reports more likely to agree than otherwise. Thus, given that *Ba*, the three reports Ba_1, Ba_2 and Gb_1 make Gb_2 more likely than otherwise—i.e., $P(Gb_2 | Ba_1, Ba_2, Gb_1, Ba) >$

$P(Gb_2|Ba)$, which means that the four reports are not independent of each other with respect to *Ba*.

4. Summary

We made three points in this paper. First, we argued formally and generally that when pieces of evidence are produced independently of each other and have no individual credibility, their coherence does not make them credible no matter what relation we take coherence to be. JSC is impossible. Second, we argued that BonJour's intuitively plausible informal reasoning for JCS is deficient since it relies on an understanding of individual credibility that is not appropriate for justification from scratch. Third, we argued for justification by recurrent coherence from scratch. We showed specifically that under certain conditions that are not extraordinary, repeated agreements between reports produced by two independent witnesses with no individual credibility raises the probability that their reports are true. This third point restores the hope, which the first two results seem to have eliminated, that the probability of our beliefs about the external world might be raised by repeated agreements among pieces of evidence from independent sources of information—e.g., visual and tactile sensory experiences if they are indeed evidentially independent[10]—even if each piece of evidence has no individual credibility.

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[10] See Shogenji (2002) for an exploration of the methods of confirming evidential independence of informational sources from internally available evidence.

Appendix

We let $\alpha = P(T_1) = P(T_2)$, $\beta = P(R_1) = P(R_2)$ and $\gamma = P(L_1) = P(L_2)$. In Table 1 below it is assumed that *Ba*. The value at the top of each cell (in square brackets) in Table 1 is the conditional probability of $Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2$ given the row header, the column header, and *Ba*. For example, the value at the top of the cell in the second row of the first column is $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | T_1, R_2, Gb, Ba)$. The value at the bottom of each cell in Table 1 is the conditional probability of the conjunction of $Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2$, the row header, and the column header, given *Ba*. This value is calculated by multiplying the value below the row header (in square brackets), the value below the column header (in square brackets), and the value at the top of the cell (in square brackets). The sum total of all the values at the bottom of the cells in Table 1 is $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | Ba)$.

	Gb $[1/n]$	$\neg Gb$ $[(n-1)/n]$
T_1 & T_2 $[\alpha^2]$	$[1 \times 1]$ α^2/n	[0] 0
T_1 & R_2 $[\alpha\beta]$	$[1 \times 1/n^2]$ $\alpha\beta/n^3$	[0] 0
T_1 & L_2 $[\alpha\gamma]$	[0] 0	[0] 0
R_1 & T_2 $[\beta\alpha]$	$[1/n^2 \times 1]$ $\alpha\beta/n^3$	[0] 0
R_1 & R_2 $[\beta^2]$	$[1/n^2 \times 1/n^2]$ β^2/n^5	$[1/n^2 \times 1/n^2]$ $\beta^2(n-1)/n^5$
R_1 & L_2 $[\beta\gamma]$	[0] 0	[0] 0
L_1 & T_2 $[\gamma\alpha]$	[0] 0	[0] 0
L_1 & R_2 $[\gamma\beta)]$	[0] 0	[0] 0
L_1 & L_2 $[\gamma^2]$	[0] 0	[0] 0

Table 1

In Table 2 below it is assumed that ¬*Ba*. The value at the top of each cell (in square brackets) in Table 2 is the conditional probability of $Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2$ given the row header, the column header, and ¬*Ba*. For example, the value at the top of the cell in the fifth row of the second column is $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | R_1, R_2, \neg Gb, \neg Ba)$. The value at the bottom of each cell in Table 1 is the conditional probability of the conjunction of $Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2$, the row header, and the column header, given ¬*Ba*. This value is calculated by multiplying the value below the row header (in square brackets), the value below the column header (in square brackets), and the value at the top of the cell (in square brackets). The sum total of all the values at the bottom of the cells in this table is $P(Ba_1 \wedge Ba_2 \wedge Gb_1 \wedge Gb_2 | \neg Ba)$.

	Gb $[1/n]$	$\neg Gb$ $[(n-1)/n]$
T_1 & T_2 $[\alpha^2]$	[0] 0	[0] 0
T_1 & R_2 $[\alpha\beta]$	[0] 0	[0] 0
T_1 & L_2 $[\alpha\gamma]$	[0] 0	[0] 0
R_1 & T_2 $[\beta\alpha]$	[0] 0	[0] 0
R_1 & R_2 $[\beta^2]$	$[1/n^2 \times 1/n^2]$ β^2/n^5	$[1/n^2 \times 1/n^2]$ $\beta^2(n-1)/n^5$
R_1 & L_2 $[\beta\gamma]$	[0] 0	$[1/n^2 \times 1/(n-1)^2]$ $\beta\gamma/n^3(n-1)$
L_1 & T_2 $[\gamma\alpha]$	[0] 0	[0] 0
L_1 & R_2 $[\gamma\beta)]$	[0] 0	$[1/(n\text{-}1)^2 \times 1/n^2]$ $\beta\gamma/n^3(n-1)$
L_1 & L_2 $[\gamma^2]$	[0] 0	$[1/(n\text{-}1)^2 \times 1/(n\text{-}1)^2]$ $\gamma^2/n(n-1)^3$

Table 2

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