Infinitism Regained

Jeanne Peijnenburg

Consider the following process of epistemic justification: proposition E_0 is made probable by E_1, which in turn is made probable by E_2, which is made probable by E_3, and so on. Can this process go on indefinitely? Foundationalists, coherentists, and sceptics claim that it cannot. I argue that it can: there are many infinite regresses of probabilistic reasoning that can be completed. This leads to a new form of epistemic infinitism.

Today epistemologists are usually probabilists: they hold that epistemic justification is mostly probabilistic in nature. If a person S is epistemically (rather than prudentially) justified in believing a proposition E_0, and if this justification is inferential (rather than noninferential or 'immediate'), then typically S believes a proposition E_1 which makes E_0 probable.[1]

How to justify E_1 epistemically? Again, if the justification is inferential, then there is a proposition E_2 that makes E_1 probable. Imagine that E_2 is in turn made probable by E_3, and that E_3 is made probable by E_4, and so on, *ad infinitum*. Is such a process possible? Does the 'ad infinitum' makes sense? The question is known as the Regress Problem and the reactions to it are fourfold. Sceptics have hailed it as another indication of the fact that we can never be justified in believing anything. Foundationalists famously argued that the process must come to an end in a proposition that is itself noninferentially justified. Coherentists, too, maintain that the infinite regress can be blocked, but unlike foundationalists they hold that the inferential justification need not be linear and may not terminate at a unique stopping point. Finally, infinitists have claimed that there is nothing troublesome about infinite regresses, the reason being that an infinite chain of reasoning need not be *actually* completed for a proposition or belief to be justified. One of the leading infinitists, Peter Klein, has even stated that the requirement

[1] This relation of making probable might be conceived externalistically or internalistically. In the first case the emphasis will be on an objective interpretation of probability, in the second case a subjective interpretation seems more appropriate. For the present argument, it does not matter which stance one takes.

Mind, Vol. 116 . 463 . July 2007
doi:10.1093/mind/fzm597

that an infinite chain must be completed 'would be tantamount to rejecting infinitism' (Klein 1998, p. 920).

In this paper I will defend a view that is different from all four. Against sceptics, foundationalists, and coherentists I will show that an infinite regress can make sense; against infinitists I will show that beliefs may be justified by an infinite chain of reasons that *can* be actually completed.

Suppose that E_0 is made probable by E_1. The probability of E_0, $P(E_0)$, can be calculated by means of the rule of total probability:

$$(1)\quad P(E_0) = P(E_0|E_1)\,P(E_1) + P(E_0|\neg E_1)\,P(\neg E_1)$$

where $P(E_0|E_1)$ is the probability of E_0 given E_1 and $P(E_0|\neg E_1)$ is the probability of E_0 given not-E_1. Since E_0 is made probable by E_1, E_0 is more probable if E_1 is true than if E_1 is false, so we have

$$P(E_0|E_1) > P(E_0|\neg E_1)$$

If E_1 is in turn made probable by E_2, we must of course repeat the rule:

$$(2)\quad P(E_1) = P(E_1|E_2)\,P(E_2) + P(E_1|\neg E_2)\,P(\neg E_2)$$

where again it is assumed that $P(E_1|E_2) > P(E_1|\neg E_2)$.

Can we continue this repetition, thus allowing for propositions made probable by other propositions, made probable by still other propositions, and so on, *ad infinitum*? Supporting foundationalism, Richard Fumerton has claimed that we cannot, since 'finite minds cannot complete an infinitely long chain of reasoning and so, if all justification were inferential, no-one would be justified in believing anything at all to any extent whatsoever' (Fumerton 2006a, p. 40; Fumerton 2006b, p. 2; Fumerton 2004, p. 150).

Fumerton does not say *why* he believes that finite minds cannot complete an infinitely long chain of reasoning. Presumably he thinks that such a task would be infinitely complicated, or would take an infinite time to finish. Such worries are understandable. If $P(E_0)$ is the outcome of an infinite regression, the calculation of $P(E_0)$ seems at first sight too lengthy and too complex for us to complete. After all, insertion of Equation (2), together with

$$(3)\quad P(\neg E_1) = P(\neg E_1|E_2)\,P(E_2) + P(\neg E_1|\neg E_2)\,P(\neg E_2)$$

into the right-hand side of Equation (1) leads to an expression with four terms, namely

$$(4)\quad P(E_0) = P(E_0|E_1)\,P(E_1|E_2)\,P(E_2) + P(E_0|\neg E_1)\,P(\neg E_1|E_2)\,P(E_2) + P(E_0|E_1)\,P(E_1|\neg E_2)\,P(\neg E_2) + P(E_0|\neg E_1)\,P(\neg E_1|\neg E_2)\,P(\neg E_2)$$

A repetition of this manoeuvre to express $P(E_2)$ and $P(\neg E_2)$ in terms of $P(E_3)$ and $P(\neg E_3)$ would produce no less than eight terms. After $n+1$ steps, the number of terms is 2^{n+1}, which yields an ungainly expression that seems hard to calculate in a simple, closed form.

There is however an easy way to reduce this complication of the rapidly increasing number of terms. Replace $P(\neg E_1)$ in Equation (1) by $1 - P(E_1)$, and then rewrite this equation as

$$(5)\quad P(E_0) = P(E_0|\neg E_1) + [\,P(E_0|E_1) - P(E_0|\neg E_1)\,]\,P(E_1)$$

A similar treatment can be applied to Equation (2), which then becomes

$$(6)\quad P(E_1) = P(E_1|\neg E_2) + [\,P(E_1|E_2) - P(E_1|\neg E_2)\,]\,P(E_2)$$

and so on. Although these changes may seem minimal, their advantages are significant. For they enable us to obtain a closed and completable expression for $P(E_0)$, not only when the number of steps is finite, but also when it is infinite. This can be further explained as follows.

Clearly we can only use Equation (5) to compute the value of $P(E_0)$ if we know the value of $P(E_1)$. Similarly, we can only use (6) to compute $P(E_1)$ if we know $P(E_2)$. So knowing $P(E_1)$ is necessary for knowing $P(E_0)$, knowing $P(E_2)$ is necessary for knowing $P(E_1)$, knowing $P(E_3)$ is necessary for knowing $P(E_2)$, and so on. If we generalize (5)–(6) to

$$(7)\quad P(E_m) = P(E_m|\neg E_{m+1}) + [P(E_m|E_{m+1}) - P(E_m|\neg E_{m+1})]\,P(E_{m+1})$$

which gives the probability of E_m, the conclusion remains unaltered: we need to know the value of $P(E_{m+1})$ in order to be able to compute the value of $P(E_m)$. Now let us call the probability of E_m α if E_{m+1} is true, and β if E_{m+1} is false:

$$P(E_m|E_{m+1}) = \alpha \qquad \text{and} \qquad P(E_m|\neg E_{m+1}) = \beta$$

For simplicity we assume that neither of these two conditional probabilities depends on m, in other words, α and β are the same for any m. For example, we might have $\alpha = 0.9$ and $\beta = 0.3$, or any other pair of numbers that satisfies $1 > \alpha > \beta > 0$. With α and β in place, Equation (7) becomes

(8) $P(E_m) = \beta + (\alpha - \beta)\, P(E_{m+1})$

Being special cases of (7), Equations (5) and (6) can also be written

(9) $P(E_0) = \beta + (\alpha - \beta)\, P(E_1)$

(10) $P(E_1) = \beta + (\alpha - \beta)\, P(E_2)$

Let us now apply the rule expressed in these equations to the finite case, beginning with $m = 0, 1, 2$. This move gives us a finite series, consisting of two steps:

(11) $$\begin{aligned} P(E_0) &= \beta + (\alpha - \beta)\, P(E_1) \\ &= \beta + (\alpha - \beta)\,[\beta + (\alpha - \beta)\, P(E_2)] \end{aligned}$$

We can continue this process for any finite $m = 0, 1, 2, 3, \ldots, n$. The result is still a finite series, and moreover one that can be summed explicitly, yielding

(12) $$\begin{aligned} P(E_0) &= \beta + (\alpha - \beta)\,[\beta + (\alpha - \beta)\,[\beta + (\alpha - \beta)\,[\ldots P(E_{n+1})\,]]\ldots]] \\ &= \beta[1+(\alpha-\beta) + (\alpha-\beta)^2 + \ldots (\alpha-\beta)^n] + (\alpha-\beta)^{n+1} P(E_{n+1}) \\ &= \frac{\beta}{1-\alpha+\beta} + (\alpha-\beta)^{n+1}\left[P(E_{n+1}) - \frac{\beta}{1-\alpha+\beta}\right] \end{aligned}$$

Here the value of $P(E_0)$ is ultimately derived from one single term, the remainder term $(\alpha - \beta)^{n+1}\, P(E_{n+1})$, containing the probability of E_{n+1} (see the second line in Equation (12)). Clearly, the value of this remainder term cannot be computed unless we know this probability. Does this mean that Fumerton and other foundationalists would be right if they were to claim that Equation (12) can only be solved if we assume that the value of $P(E_{n+1})$ is known and hence that E_{n+1} is noninferentially justified?

The answer is negative. To see this, let us consider the infinite case. The standard way to investigate the convergence of an infinite series is first to look at a finite series of, say, $n+1$ terms only, with a remainder term, and then to investigate what happens as n tends to infinity. Applying this procedure to Equation (12), we observe that, since $0 < \alpha - \beta < 1$, the factor $(\alpha - \beta)^{n+1}$ becomes smaller and smaller as n becomes larger and larger. In the formal limit that n goes to infinity, we find that the series has an infinite number of terms, and that the terms in the second and third lines of Equation (12) that contain the unknown $P(E_{n+1})$ tend to zero, and hence disappear completely.

In the limit of an infinite number of terms in the series, corresponding to an indefinite iteration of Equation (7), we find

$$P(E_0) = \frac{\beta}{1-\alpha+\beta} = \frac{0.3}{1-0.9+0.3} = 0.75$$

with the values given above for α and β, 0.9 and 0.3. Thus, even after an infinite number of steps in the inferential justification, the value of $P(E_0)$ can be exactly calculated: it is 0.75. The justification is, although infinite, perfectly computable and completable.

One might object that the argument developed above hinges on a very special case. For in demonstrating that an infinite regress *can* make sense, and that justification by an infinite chain of reasoning *can* indeed be carried out, I have assumed that the conditional probabilities α and β remain the same throughout the entire process. Such an assumption is rarely fulfilled. It is very unusual that the degree with which a proposition E_0 is made probable by E_1 is identical to the degree with which E_1 is made probable by E_2, and E_2 is made probable by E_3, and so on, *ad infinitum*. The fact that, in addition, the probabilities $P(E_0|\neg E_1)$, $P(E_1|\neg E_2)$, $P(E_2|\neg E_3)$, etc. are also identical only underlines the special nature of this case.

My answer to such an objection would be twofold. First I would point out that one counterexample is enough to refute the foundationalist's claim that *all* inferential chains of reasoning must come to an end. Similarly, one counterexample is enough to confute the claim of the infinitists that *no* infinite chain can be actually completed. The fact that this counterexample presupposes special situations is not really relevant.

Second, it is relatively easy to construct counterexamples without making the assumption that α and β are the same for each step. For suppose that the conditional probabilities $P(E_m|E_{m+1})$ and $P(E_m|\neg E_{m+1})$ *do* change as the index m varies, which we indicate by adding the index m to α and β:

$$P(E_m|E_{m+1}) = \alpha_m \qquad \text{and} \qquad P(E_m|\neg E_{m+1}) = \beta_m$$

The second line of Equation (12), where α and β are the same, is a special case of

$$(13)\quad \begin{aligned} P(E_0) = {} & \beta_0 + (\alpha_0 - \beta_0)\,\beta_1 + \\ & (\alpha_0 - \beta_0)(\alpha_1 - \beta_1)\,\beta_2 + \ldots + \\ & (\alpha_0 - \beta_0)(\alpha_1 - \beta_1)\ldots(\alpha_{n-1} - \beta_{n-1})\,\beta_n + \\ & (\alpha_0 - \beta_0)(\alpha_1 - \beta_1)\ldots(\alpha_n - \beta_n)\,P(E_{n+1}) \end{aligned}$$

where a_m and β_m vary with m. Equation (13) can be proved by mathematical induction, but we will not stop to do that here. Now, since $1 > \alpha - \beta > 0$, we might erroneously think that

(14) $1 > \alpha_m - \beta_m > 0$

would be enough to make the remainder term in the last line of (13) vanish in the limit that n goes to infinity, and hence might provide us with counterexamples of the sort that we are looking for. However, condition (14) does not guarantee the vanishing of the remainder term, let alone does it ensure summability of the entire series. We need something stronger, for example the constraint that $\alpha_m - \beta_m$ be uniformly bounded from above by a constant, c, that is strictly less than 1:

(15) $1 > c > \alpha_m - \beta_m > 0$

Under constraint (15), Equation (13) does the trick. For now not only does its remainder term tend to zero as n tends to infinity, it is also completable: we can find instances of β_n and $\alpha_n - \beta_n$ such that the sum that (13) expresses can be explicitly calculated. These instances are our counterexamples, and there are infinitely many of them.[2,3]

Faculty of Philosophy
University of Groningen
Oude Boteringestraat 52
9712 GL Groningen
The Netherlands
Jeanne.Peijnenburg@rug.nl

JEANNE PEIJNENBURG

References

Fumerton, Richard 2004: 'Epistemic Probability'. *Philosophical Issues*, 14, pp. 149–64.
——2006a: *Epistemology*. Malden, MA: Blackwell.
——2006b: 'Foundationalist Theories of Epistemic Justification'. *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta, <http://plato.stanford.edu/archives/spr2006/entries/justep-foundational>
Klein, Peter 1998: 'Foundationalism and the Infinite Regress of Reasons'. *Philosophy and Phenomenological Research*, 58, pp. 919–25.

[2] Here is one: if $\alpha_n - \beta_n = a/(n+1)$ and $\beta_n = b/(n+1)$, where a and b are positive constants, then $P(E_0) = b(e^a - 1)/a$.

[3] I thank an anonymous referee and Tom Stoneham, in his role as associate editor of *Mind*, for stimulating comments. Thanks also to David Atkinson for help with mathematical details.

