IN DEFENCE OF CLOSURE

By Richard Feldman

Closure principles for epistemic justification hold that one is justified in believing the logical consequences, perhaps of a specified sort, of things one justifiably believes. Unrestricted closure principles do not limit the principle to consequences having any particular property. Restricted closure principles say that only certain sorts of consequences of justified beliefs, e.g., 'obvious consequences', must be justified.

The denial of closure is supposed to be helpful in responding to sceptics.[1] Sceptics argue that the things we ordinarily take ourselves to know, e.g., that I see a table in front of me, imply anti-sceptical hypotheses, e.g., that I am not being deceived by an evil demon into falsely thinking that I see a table. Sceptics contend that we do not know these anti-sceptical propositions. That claim, combined with closure, implies that we do not know the propositions we ordinarily think we know. Denying closure evades this sceptical argument. Robert Audi, among others, has proposed examples designed to show that the denial of the closure principle is plausible.[2]

I believe that some version of the closure principle, restricted to known consequences, is surely true. Indeed, the idea that no version of this principle is true strikes me, and many other philosophers, as one of the least plausible ideas to come down the philosophical pike in recent years. I believe that much of the case against the principles has been refuted by Jonathan Vogel.[3] Audi has recently offered a new objection to closure principles. In this paper I shall take up his objection.

There are details about exactly how to formulate the closure principle that warrant attention. It is clearly not true that if you are justified in believing a proposition then you are justified in believing all its logical consequences. Those logical consequences will always include complex and distant consequences you do not even understand. Such consequences are not justified for you. So an unrestricted closure principle is false.

A more plausible closure principle, restricted to known consequences, is

C1. If S is justified in believing p and S knows that p logically implies q, then S is justified in believing q.

[1] See Robert Nozick, *Philosophical Explanations* (Harvard UP, 1981), pp. 204–11.

[2] Audi initially raised his objection in his *Belief, Justification and Knowledge* (Belmont: Wadsworth, 1988), pp. 76–8. He continues his argument in 'Justification, Deductive Closure and Reasons to Believe', *Dialogue*, 30 (1991), pp. 77–84. The latter is a response to objections raised by Catherine M. Canary and Douglas Odegard in 'Deductive Justification', *Dialogue*, 27 (1989), pp. 305–20.

[3] 'Are there Counter-examples to the Closure Principle?', in M. Roth and G. Ross (eds), *Doubting: Contemporary Perspectives on Skepticism* (Dordrecht: Kluwer, 1990), pp. 13–28.

I shall not worry here about the exact interpretation of 'logically implies' in (C1). As I interpret the phrase '*S* is justified in believing *p*', it does not imply that *S* does believe *p*. It just implies that *S* has adequate evidence for believing *p*. Examples in which there are long chains of propositions such that *S* knows that each implies its successor, but does not actually believe any proposition in the sequence, may be troublesome for (C1). (C1) implies that if the first proposition of the sequence is justified, then so is the last. If *S* does not believe the first proposition of the sequence, and has not 'put together' the chain of implications that leads to the last, it may be a mistake to think that the last proposition is justified for *S*.

A possible revision of (C1) changes the antecedent to require that *S* justifiably believe *p*. This means, roughly, that the proposition is justified for *S* and that *S* believes it on the basis of the justifying evidence. The resulting principle is thus

C2. If *S* justifiably believes *p* and *S* knows that *p* logically implies *q*, then *S* is justified in believing *q*.

This formulation avoids the problem raised for (C1). Notice that the consequent of (C2) does not imply that *S* does believe *q*.

A problem concerning the justification of belief in conjunctions confronts (C2). The widely disputed conjunction rule holds that, necessarily, if *S* is justified in believing each of two propositions, then *S* is justified in believing their conjunction. If the conjunction rule is false, then possibly there are cases in which *S* is justified in believing *p* and justified in believing that *p* implies *q*, but is not justified in believing the conjunction *p and p implies q*. If there are such cases, then perhaps they are counter-examples to closure principles formulated along the lines of (C2).

I take it that objections to closure are supposed to be distinct from objections to the conjunction rule. That is, I take it that Audi and other objectors to closure believe that there are cases in which *S* is justified in believing the conjunction *p and p implies q*, yet not justified in believing *q*. When Audi and like-minded critics of closure formulate their objections, they merely argue that belief in *q* is not justified. They never argue that the conjunction is not justified. In order to separate issues directly about closure from issues concerning the conjunction rule, I shall revise the closure principle so that its antecedent requires that the conjunction be justified:

C3. If *S* justifiably believes the conjunction *p and p logically implies q*, then *S* is justified in believing *q*.

In the remainder of this paper I shall examine (C3) and evaluate Audi's objection as an objection to it.

Audi presents an example designed to show that closure principles are acceptable only if they are restricted in some suitable way. The restrictions he has in mind are more extensive than those introduced by (C3). His example goes as follows:

> I add a column of ... figures, check my results twice, and thereby come to know, and justifiably believe, that the sum is 10,952. As it happens, I sometimes make mistakes, and my wife (whom I justifiably believe to be a better arithmetician) sometimes corrects me. Suppose that, feeling unusually confident, I now infer that if my wife says this is not the sum, she is wrong. From the truth that the sum is 10,952 it certainly

> follows that if she says it is not, she is wrong. If it *is* the sum, then if she denies it she is wrong. But even though I know and justifiably believe that this is the sum, can I, on this basis, *automatically* know or justifiably believe the *further* proposition that if she says that it is not the sum, she is wrong? Suppose my checking just twice is only enough to give me the *minimum* basis for justified belief and knowledge here. Surely I would then not have sufficient grounds for the further proposition that if she says the answer is wrong, she is wrong.[4]

Audi's example may have some initial appeal. Justifiably believing the mathematical fact does not seem to make me automatically justified in believing the conditional about my wife. However, one should be given pause by Audi's emphasis on two points: on the one hand, he says that the conditional 'certainly follows' from the mathematical fact; on the other hand, he says that the conditional is not justified, because it is a 'further proposition' for which I lack sufficient grounds. I shall argue that there is no interpretation of the conditional according to which it both follows from the mathematical fact and is not justified.

Let us be clear about the propositions that enter into the example. Suppose a mathematical fact that I justifiably believe is

A. 1,375 + 1,194 + 1,835 + 941 + 396 + 1,721 + 1,858 + 389 + 591 + 652 = 10,952

Assume I am justified in believing (A). The alleged consequence of (A) which I am supposedly not justified in believing is the conditional

B. If my wife says that (A) is wrong, then she is wrong.

Now, it is important for what follows to be clear about exactly what (B) says. Much of what follows will be about the consequences of adopting different interpretations of (B). In any case, for Audi's example to work, it must be that I am justified in believing the conjunction '(A) and (A) implies (B)', and also that I am not justified in believing (B). That I am justified in believing (A) is not at issue here.

It is important to realize that the antecedent of (B) does not refer to some column of numbers via a definite description that could refer to some other column of numbers. It refers instead to the very proposition expressed in (A). The consequent of (B) says that my wife is wrong in denying (A). In other words, it says, roughly, that (A) is true. So (B) is short for something like

B1. If my wife says that the sum of 1,375, 1,194, ... is not 10,952, then she is wrong when she says that the sum of 1,375, 1,194, ... is not 10,952 (i.e., if my wife says that (A) is false, then (A) is (nevertheless) true).

I shall speak of (B) in what follows, but keep in mind that (B) says what (B1) does. Audi says that the conditional 'certainly follows' from (A). Whether this is true depends upon exactly what sort of conditional it is.

Consider the possibility that (B) is a material conditional. (A) does imply (B) in that case. It would not be far wrong to see the inference as an instance of 'p, therefore if q then p'. Notice that (B) is equivalent to

[4] *Belief, Justification and Knowledge* p. 77, quoted in 'Justification, Deductive Closure and Reasons to Believe' p. 78.

B2. Either it is not the case that my wife says that (A) is false or my wife is mistaken when she says that (A) is false (i.e., either my wife does not say that (A) is false, or she denies it and it is true).

(A) does imply (B2). The pattern of inference is: *p*, therefore *q* or (not-*q* and *p*). Assume that, in the example, I justifiably believe that the inference is valid.

Audi's argument against closure depends upon the further claim that I am not justified in believing (B). He does not explicitly argue for the conclusion that I am not justified in believing the conditional. We are just supposed to see that this is a reasonable assessment in this case. The point, emphasized twice in the passage quoted, is that (B) is a 'further proposition', one not justified by my evidence for (A). However, taken as a material conditional, (B) is not in any interesting sense a 'further' proposition. Of course, it is a different proposition from (A). It asserts less. (B) can be made true by (A)'s being true and it can also be made true by my wife's not denying (A). My reasons for asserting (B) are at least as good as my reasons for asserting (A). After asserting (A), I am not sticking myself further out on a limb by asserting (B), when (B) is interpreted as a material conditional. So I see no reason at all to deny justification for the material conditional. As we shall see, there may be other propositions in the vicinity that are not justified, but this one seems well justified.

It might be thought that the argument here covertly appeals to the very principle at issue.[5] The reasoning seems to be that if (A) is justified, then, since it so obviously implies (B), as a material conditional (B) is also justified. There may be some justice to the charge. However, Audi needs a reason to think that (B) is not justified when it is interpreted as a material conditional. Intuitions about the epistemic status of 'further propositions' do not support his claim. Furthermore, as we shall see, there is more plausibility to thinking that (B) is not justified when it is given other interpretations. Thus there is little to be said on behalf of the claim that (B) is not justified when it is a material conditional and quite a bit to be said on behalf of the claim that it is justified.

Consider next the possibility that (B) is supposed to express a subjunctive conditional. It is initially plausible to think that I am not justified in believing in the truth of

B3. If my wife were to say that (A) is false, then she would be wrong.

Since my wife is the better arithmetician, (B3) may seem unreasonable. However, the issue here is somewhat complicated. It will be easiest to approach it by considering first a different example.

Suppose I justifiably believe

G. George Washington was the first president of the United States.

I know that my wife knows more about American history than I do, and I consider the conditional

[5] Canary and Odegard point out (p. 316) that my evidence for (A) provides me with good reason to think that my wife, a superior arithmetician, did not deny (A). Audi raises the point discussed in this paragraph in reply (1991 p. 81).

H. If my wife were to say that George Washington was not the first president, then she would be wrong.

One might doubt that I am justified in believing (H). Perhaps so, but this is no counter-example to closure. (G) does not imply (H). A possible-worlds analysis makes this clear. If (G) implies (H), then in every world in which (G) is true, (H) is also true. Suppose (G) is true in some world *W*. It does not follow that (H) is true in *W*. It could be that the closest world to *W* in which the antecedent of (H) is true is a world in which Washington was not the first president, and my wife would be right, not wrong, in saying he was not the first president. Even if in 'normal situations' if (G) is true then (H) is also true, (G) does not logically imply (H). Some (G)-worlds are such that the closest world in which my wife denies (G) are worlds in which her denial is correct. So (H) simply does not follow from (G).

Let us return now to Audi's example. One might think that the same reasoning applies. The result would be that (A) does not imply (B3) and the example fails. However, this is not right. The problem is brought about by the fact that (B3) involves a mathematical proposition which is, if true, necessarily true. Consider any world in which (A) is true, i.e., any world. Next, consider the nearest world in which the antecedent of (B3) is true. The consequent of (B3) is true in that world. Facts about my wife and her mathematical expertise do not matter here. Since (A) is true in every world, she is mistaken in every world in which she denies it and thus mistaken in the closest world in which she denies it. Therefore, (B3) is not only true, but necessarily true. Indeed, every counterfactual with the same consequent as (B3) must be true.

If (B3) is a necessary truth, then perhaps (A) logically implies it. It does in the sense in which everything implies all necessary truths. If I recognize this, then I am justified in thinking that (B3) follows from (A). So we cannot get out of Audi's example, in the way we could get out of the George Washington example, by denying that the implication holds.

Notice, however, how the reasoning goes here: I know that (A) implies (B3) because I know that (B3) is a necessary truth and thus that it follows from everything. However, if I know that (B3) is a necessary truth, then I know that it is true and I am justified in believing it. And we then have no counter-example to the closure principle. The counter-example was supposed to be that I justifiably believed '(A) and (A) implies (B3)', but was not justified in believing (B3). However, the only way to defend the claim that I am justified in believing that (A) implies (B3) depended on the assumption that I was justified in believing that (B3) was true, indeed that it was necessarily true. There may, of course, be cases in which I am justified in believing (A) but not (B3). Those will be situations in which I do not know that (A) implies (B3). We have yet to find a case in which I justifiably believe (A) and also that (A) implies (B3), yet am not justified in believing (B3).

We have seen that Audi's example fails if (B) is either a material or a subjunctive conditional. There are other possible interpretations of the conditional in Audi's example. I do not have the space to consider them here. However, I believe that in every case it will turn out either that I am justified in believing the conditional or that I am not justified in believing that it follows from (A). In no case will there be a counter-example to the closure principle.[6]

I want to conclude by considering briefly some remarks Audi makes about conditionals and suppositions. In his book he says that, to be justified in believing the conditional, 'I need grounds good enough not to be outweighed by the supposition that she (the better arithmetician) says that 10,952 is not the sum'. In the subsequent paper he tentatively offers an explanation:

> When, in exploring whether I am justified in believing a conditional, I suppose its antecedent, I as it were add that proposition to my evidence base. If, with this addition to that base (and other factors remaining equal), I am not justified in believing the conditional – for instance, because relative to the new base, I am not justified in believing the consequent – then I am not justified in believing the conditional.[7]

I must admit that I do not see how evidence for a proposition can be affected by suppositions concerning counter-evidence. I do not believe that suppositions affect one's evidence at all. Evidence can only be affected by other evidence. So I do not see why Audi thinks we can determine whether a conditional is actually justified by seeing if it would be justified if the supposition of its antecedent were added to one's evidence. It does make sense to ask what would be justified if one's evidence were different in various ways. Thus, we can sensibly consider what would happen if the antecedent of the conditional in Audi's example were added to one's evidence. But I do not see how that affects the actual epistemic status of the conditional.

In thinking about Audi's idea, it is useful to distinguish two issues: one concerns the truth-conditions for a conditional, and the other concerns the conditions under which one is justified in believing it, its justification-conditions. What Audi explicitly says here is that the justification-conditions for a conditional include being justified in believing the consequent if one adds the antecedent to one's evidence base. He does not claim that this matter concerning the addition of the antecedent to one's evidence base has anything to do with the truth-conditions for the conditional.

If Audi does think that (B) expresses a conditional which is true only if the consequent would be justified if the antecedent were added to my evidence base, then (A) surely does not imply (B). This is because (A) clearly does not imply anything about what I would be justified in believing if my wife were to say that (A) is false. (A), remember, is just a mathematical statement. So let us assume that Audi's suggestion is that (B) is a conditional which is justified for *S* only if this condition is met. Thus, he suggests:

> If *S* is justified in believing the proposition expressed by 'If *p* then *q*', then if *p* were added to *S*'s evidence base, *S* would be justified in believing *q*.

[6] For discussion of indicative conditionals, see Roy Sorenson, 'Dogmatism, Junk Knowledge and Conditionals', *The Philosophical Quarterly*, 38 (1988), pp. 433–54. Although Sorenson does not discuss Audi's examples, his account of indicative conditionals provides a reason to think that one might mistakenly think that (B) is unjustified when it is interpreted as an indicative conditional. His idea is that (B) is justified, and known, but it amounts to 'junk knowledge' because it would not survive learning the truth of its antecedent.

[7] *Belief, Justification and Knowledge* p. 77; 'Justification, Deductive Closure and Reasons to Believe' p. 81.

Let us assume that the sort of conditional Audi takes (B) to express is one for which this condition holds. This apparently rules out the possibility that the conditional is either a material or a subjunctive conditional. To see that Audi's condition does not apply to material conditionals, consider a situation in which I know that p is true. I can then know that the material conditional if $\sim p$ then q is true. But I would not be justified in believing q if $\sim p$ were added to my evidence. To see that the condition does not hold for subjunctives, consider the conditional 'If I were at home, I would be in my garden'. Assume I know that to be true. *Simply* adding to my evidence base the proposition 'I am at home' does not make me justified in believing that I am in my garden. Adding just that evidence does not add any evidence about what I am doing. Of course, if I were at home, my evidence would change much more radically. If I were at home, then I would be in my garden and my perceptual evidence would presumably change accordingly.

(B) must therefore be some other sort of conditional, one for which the justification condition holds. I am not sure that there are any such conditionals, but even if there are, their mere existence does not make the case for Audi. For his example to work, it must also be the case that (A) logically implies (B), so interpreted. (Strictly, I must be justified in believing that (A) implies (B).) Audi never discusses in his book or his paper any conditionals that meet these two conditions, and I do not see any reason to think that anyone would be justified in believing that (A) implies (B) where (B) is given such an interpretation. So I do not see any basis for accepting the counter-example on the basis of the claim that some conditionals satisfy this justification condition.

It is worth noting a possible confusion here.[8] For any proposition I know (or justifiably believe), there are other things such that I can know that learning them would undermine my knowledge. The simplest case is the denial of the proposition itself. Suppose I know p. It does not follow that, were I to learn that p is false, I would still be justified in believing p. Similarly, if I were to learn that all my evidence for p is false, I would not still be justified in believing p. My justification for p may also fail to survive my learning that my wife said that p is false. Neither p nor the fact that I am justified in believing p implies otherwise. The inclination to deny justification for the conditional in Audi's example may derive from the correct realization that my justification for (A) would not survive my learning of my wife's testimony. But (A) does not imply otherwise. So this fact does not undermine the closure principle.

In my view, the claim that there is no true restricted closure principle along the lines of (C3) is not remotely plausible. I have argued that there is no interpretation of Audi's example that undermines (C3). There is, furthermore, a general reason to doubt that any such example will work. Imagine I were actually in the situation supposed to be mine in Audi's example. If the epistemic situation were as Audi says, then I could reasonably think: (A) is true and (A) implies (B), but (B) is not true. To say, 'Yes, I accept that (A) is true and that (A) implies (B), but I draw the line at (B); I do not commit myself to that', is to be patently unreasonable. It is to refuse to accept what you know to be the consequences of your beliefs. That is the sort of thing we

[8] Canary and Odegard make this point as well: see p. 316.

routinely counsel our first-year students not to do. We ought not let examples such as Audi's convince us that it is reasonable to do what we wisely advise against.[9]

University of Rochester

[9] I am grateful to Robert Audi, Earl Conee, Ted Sider, Steven Hales and Keith DeRose for helpful discussions of this paper. A version of it was presented at the 1994 Western Division meeting of the American Philosophical Association. Robert Audi commented.

DEDUCTIVE CLOSURE, DEFEASIBILITY AND SCEPTICISM: A REPLY TO FELDMAN

BY ROBERT AUDI

If entailment is truth-preserving and justification is a positive status *vis à vis* truth, one might expect justification for believing a proposition to transmit to certain propositions entailed by it. One might think, e.g., that if S is justified in believing p, and also justified in believing that p entails q, then S is justified in believing q – let us call this the transmission by justifiably presumed entailment principle [*the entailment principle* for short]. In previous work I have attacked various closure principles, in part with a view to rebutting scepticism. In his paper 'In Defence of Closure' (above, pp. 487–94), Richard Feldman's probing, very valuable study of some of my efforts indicates that there is far more to be said on the matter. I shall try both to assess his main points and to move towards a better understanding of when justification transmits and when it does not. I should add that I believe he would take his main points to apply to the principle just stated, which is among those I have rejected, though he intends to be discussing the issue in terms of his principle (C3), the principle that if S is justified in believing the conjunction *p and p logically implies q*, then S is justified in believing q (p. 488). I shall compare the principles later.

I

Let us start with the material conditional interpretation of my example: suppose I have only minimal justification for believing the arithmetical proposition

A. The sum of the relevant column of figures is 10,952.

The question is whether it is plausible to claim that I am justified in believing that

B. Either my wife does not say that (A) is false, or she denies it and it is true.