

A Problem for Confirmation Measure Z

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1 Confirmation Measure Z

Crupi, Tentori, and Gonzalez [1] provide a very interesting set of theoretical and empirical arguments in favor of the following (piecewise) Bayesian measure of the degree to which evidence E confirms hypothesis H , relative background knowledge K .¹

$$Z(H, E | K) \stackrel{\text{def}}{=} \begin{cases} \frac{\Pr(H | E \& K) - \Pr(H | K)}{1 - \Pr(H | K)} & \text{if } \Pr(H | E \& K) \geq \Pr(H | K) \\ \frac{\Pr(H | E \& K) - \Pr(H | K)}{\Pr(H | K)} & \text{if } \Pr(H | E \& K) < \Pr(H | K) \end{cases}$$

I won't go into their arguments in favor of Z here. Instead, I will pose what I take to be a serious problem with Z . This will require a brief digression into the notion of independent evidence.

2 Independent Evidence Regarding a Hypothesis

Fitelson [2] offers the following Bayesian account of what it means for two pieces of evidence E_1 and E_2 to be *confirmationally independent*, regarding hypothesis H , according to a confirmation measure c .

Independence. E_1 and E_2 are *confirmationally independent regarding H* , according to c (viz., E_1 , E_2 are c -independent regarding H) iff both $c(H, E_1 | E_2) = c(H, E_1)$ and $c(H, E_2 | E_1) = c(H, E_2)$.²

Intuitively, E_1 and E_2 are confirmationally independent regarding H , according to c just in case the degree to which E_1 (E_2) confirms H (according to c) does not depend on whether E_2 (E_1) is already known.

As Fitelson shows, this notion can be applied in various useful confirmation-theoretic ways (e.g., to provide a Bayesian account of the value of varied/diverse evidence). I won't delve into **Independence** here. Rather, I will simply apply it to reveal that measure Z has a serious shortcoming when it comes to the handling of certain sorts of independent evidence.

3 A Problem for Measure Z

Sometimes, we have *conflicting evidence* regarding a hypothesis. That is to say, sometimes, the following property holds for a triple E_1 , E_2 , and H .

Conflict. E_1 and E_2 constitute *conflicting evidence regarding H* iff E_1 confirms H , while E_2 disconfirms H . Formally, E_1 and E_2 constitute conflicting evidence regarding H iff $\Pr(H | E_1) > \Pr(H)$ and $\Pr(H | E_2) < \Pr(H)$.

Intuitively, it should be possible for *some* triple E_1 , E_2 , and H to satisfy *both Independence and Conflict*. That is to say, intuitively, *there sometimes exists independent, conflicting evidence regarding some hypotheses*. More precisely, we have the following eminently plausible existence claim.

Existence. There exist some triples E_1 , E_2 , and H which satisfy *both Independence and Conflict*.

¹Several authors had discussed/endorsed measure Z before Crupi *et al.* See, for instance [3], [5], and [4]. However, Crupi *et al.* provide the most compelling and comprehensive theoretical and empirical arguments in its favor.

²Here, $c(H, E)$ is shorthand for $c(H, E | \top)$, where \top is a tautology. This can be read simply as "the degree to which E confirms H (unconditionally), according to c ."

Indeed, **Existence** strikes me as so plausible as to require no justification at all. Surprisingly, according to measure Z , **Existence** is *false*! That is, according to measure Z it is *impossible for any pair of evidence* E_1, E_2 *to be both independent regarding* H *and conflicting regarding* H (for any H).

Problem. According to measure Z , **Existence** is false.

Proof. Suppose, for *reductio*, that there *does* exist some triple E_1, E_2, H that satisfies *both Independence* (according to measure Z) *and Conflict*. Then, we may reason as follows.

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| (1) | $\Pr(H E_1) > \Pr(H)$ | Assumption (Conflict) |
| (2) | $\Pr(H E_2) < \Pr(H)$ | Assumption (Conflict) |
| (3) | $Z(H, E_1) = Z(H, E_1 E_2)$ | Assumption (Z-Independence) |
| (4) | $Z(H, E_2) = Z(H, E_2 E_1)$ | Assumption (Z-Independence) |
| (5) | $\frac{\Pr(H E_1) - \Pr(H)}{1 - \Pr(H)} = \frac{\Pr(H E_1 \& E_2) - \Pr(H E_2)}{1 - \Pr(H E_2)}$ | (1), (3), definition of Z |
| (6) | $\frac{\Pr(H E_2) - \Pr(H)}{\Pr(H)} = \frac{\Pr(H E_1 \& E_2) - \Pr(H E_1)}{\Pr(H E_1)}$ | (2), (4), definition of Z |

Now, let $x \stackrel{\text{def}}{=} \Pr(H | E_1)$, $y \stackrel{\text{def}}{=} \Pr(H | E_2)$, $z \stackrel{\text{def}}{=} \Pr(H)$, and $u \stackrel{\text{def}}{=} \Pr(H | E_1 \& E_2)$. Then, (5) and (6) can be rewritten as the following pair of algebraic equations.

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| (5) | $\frac{x - z}{1 - z} = \frac{u - y}{1 - y}$ |
| (6) | $\frac{y - z}{z} = \frac{u - x}{x}$ |

Algebraically (assuming only that x, y, z , and u are real numbers), (5) and (6) entail that *either* (7) $x = z$ *or* (8) $y = z$. But, this *contradicts* our assumption that *both* (1) $x > z$ *and* (2) $y < z$. \square

In closing, it is worth noting that it seems to be the piecewise nature of Z that causes **Problem**. For it can be shown that none of the non-piecewise-defined confirmation measures that have been discussed in the literature (see, *e.g.*, [1] for a survey) have this **Problem** (proof omitted).³

References

- [1] V. Crupi, K Tentori, and M. Gonzalez, 2007, "On Bayesian Measures of Evidential Support: Theoretical and Empirical Issues," *Philosophy of Science* 74(2): 229-52.
- [2] B. Fitelson, 2001, "A Bayesian Account of Independent Evidence with Applications," *Philosophy of Science* 68(S3): S123-S140.
- [3] N. Rescher, 1958, "A theory of evidence," *Philosophy of Science* 25(1): 83-94.
- [4] E. Shortliffe and B. Buchanan, 1975, "A model of inexact reasoning in medicine," *Mathematical biosciences* 23(3-4): 351-379.
- [5] H. Törnebohm, 1966, "Two Measures of Evidential Strength." In *Aspects of inductive logic*, J. Hintikka and P. Suppes (eds.), North-Holland Publishing Company, Amsterdam, pp. 81-95.

³Because **Problem** only rests on *ordinal* features, it will plague *any measure that is ordinally equivalent to* Z .