A Problem for Confirmation Measure Z

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Abstract. In this paper, I present a serious problem for confirmation measure Z.

1 Confirmation Measure Z

Crupi, Tentori, and Gonzalez [1] provide a very interesting set of theoretical and empirical arguments in favor of the following (piecewise) Bayesian measure of the degree to which evidence E confirms hypothesis H, relative background knowledge K.[1]

$$Z(H, E \mid K) \stackrel{\text{def}}{=} \begin{cases} \dfrac{\Pr(H \mid E \,\&\, K) - \Pr(H \mid K)}{1 - \Pr(H \mid K)} & \text{if } \Pr(H \mid E \,\&\, K) \geq \Pr(H \mid K) \\[2ex] \dfrac{\Pr(H \mid E \,\&\, K) - \Pr(H \mid K)}{\Pr(H \mid K)} & \text{if } \Pr(H \mid E \,\&\, K) < \Pr(H \mid K) \end{cases}$$

I won't go into their arguments in favor of Z here. Instead, I will present what I take to be a serious problem with Z. This will require a brief digression into the notion of independent evidence.

2 Independent Evidence Regarding a Hypothesis

Fitelson [3] offers the following Bayesian account of what it means for two pieces of evidence E_1 and E_2 to be *confirmationally independent*, regarding hypothesis H, according to a confirmation measure $\mathfrak{c}$.

> **Independence**. E_1 and E_2 are *confirmationally independent regarding* H*, according to* $\mathfrak{c}$ (*viz*., E_1, E_2 are $\mathfrak{c}$*-independent regarding* H) iff both $\mathfrak{c}(H, E_1 \mid E_2) = \mathfrak{c}(H, E_1)$ and $\mathfrak{c}(H, E_2 \mid E_1) = \mathfrak{c}(H, E_2)$.[2]

Intuitively, E_1 and E_2 are confirmationally independent regarding H, according to $\mathfrak{c}$ just in case the degree to which E_1 (E_2) confirms H (according to $\mathfrak{c}$) does not depend on whether E_2 (E_1) is already known.

As Fitelson shows, this notion can be applied in various useful confirmation-theoretic ways (*e.g.*, to provide a Bayesian account of the value of varied/diverse evidence). I won't delve into **Independence** here. Rather, I will simply apply it to reveal that measure Z has a serious shortcoming when it comes to the handling of certain sorts of independent evidence.

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[1]Several authors had discussed/endorsed measure Z before Crupi *et. al.* See, for instance [7] and [8]. However, Crupi *et. al* provide the most compelling and comprehensive theoretical and empirical arguments in its favor.

[2]Here, $\mathfrak{c}(H, E)$ is shorthand for $\mathfrak{c}(H, E \mid \top)$, where $\top$ is a tautology. This can be read simply as "the degree to which E confirms H (unconditionally), according to $\mathfrak{c}$."

3 A Problem for Measure Z

Sometimes, we have *conflicting evidence* regarding a hypothesis. That is to say, sometimes, the following property holds for a triple E_1, E_2, and H.

> **Conflict**. E_1 and E_2 constitute *conflicting evidence* regarding H iff E_1 confirms H, while E_2 disconfirms H. Formally, E_1 and E_2 constitute conflicting evidence regarding H iff $\Pr(H \mid E_1) > \Pr(H)$ and $\Pr(H \mid E_2) < \Pr(H)$.

Intuitively, it should be possible for *some* triple E_1, E_2, and H to satisfy *both* **Independence** *and* **Conflict**. That is to say, intuitively, *there sometimes exists independent, conflicting evidence regarding some hypotheses.* More precisely, we have the following eminently plausible existence claim.

> **Existence**. There exist some triples E_1, E_2, and H which satisfy *both* **Independence** *and* **Conflict**.

Indeed, **Existence** strikes me as *so* plausible as to require little justification. Having said that, it is worth giving a simple example which illustrates the intuitive plausibility of **Existence.**[3] Here, I borrow the following example, which belongs to a class of examples used by Fitelson [3] to provide an intuitive illustration of **Independence** (with individual degrees of strength that can be tweaked *via* some simple parameters, which I have set here).

> **The Urn Example.** An urn has been selected at random from a collection of urns. Each urn contains some balls. In some of the urns (the H-urns) the proportion of white balls to non-white balls is $1/3$ and in all the other urns (the $\sim H$-urns) the proportion of white balls to non-white balls is $2/3$. The proportion of H-urns is $1/2$. Balls are to be drawn randomly from the selected urn, with replacement.
>
> Let H be the hypothesis that the proportion of white balls in the urn is $1/3$ (*viz.*, that the sampled urn is an H-urn). Let W_i state that the ball drawn on the i^{th} draw ($i \geq 1$) is white. Intuitively, $\sim W_1$ and W_2 are confirmationally independent regarding H, *i.e.*, the triple $\langle \sim W_1, W_2, H \rangle$ instantiates **Independence**.[4]

Surprisingly, according to measure Z, **Existence** is *false* (*a fortiori*). That is, according to measure Z, *it is conceptually impossible for any pair of evidence E_1, E_2 to be both independent regarding H and conflicting regarding H (for any hypothesis H).*

[3] I thank an anonymous referee for urging me to include an illustrative intuitive example of **Existence**.

[4] Fitelson [3] would be committed to a claim far stronger than mere **Existence** here (note: **Conflict** is *obviously* true in this case). He would be committed to the stronger claim that $\sim W_1$ and W_2 *should have equal and opposite degrees of confirmation, which exactly cancel each other out*, so that the total degree to which the conjunction $\sim W_1 \,\&\, W_2$ confirms H is *zero*. This is because he accepts the likelihood-ratio measure of degree of confirmation, which satisfies (not only **Existence**, but) a strong independence-additivity requirement. Of course, we do not need to go along with Fitelson [3] on that stronger/more specific claim. All we need this example to do is make **Existence** somewhat plausible (*viz.*, *not a conceptual impossibility*). As I point out below, among all the measures of confirmation that have been proposed and defended in the literature, Z is the *only* measure that entails the conceptual impossibility of **Existence**. Indeed, a plenitude of examples satisfying **Existence** are easily described, for all other confirmation measures in the literature.

Problem. According to measure Z, **Existence** is (analytically) false.

Proof. Suppose, for *reductio*, that there *does* exist some triple E_1, E_2, H that satisfies *both* **Independence** (according to measure Z) *and* **Conflict**. Then, we may reason as follows.

(1) $\Pr(H \mid E_1) > \Pr(H)$ — Assumption (**Conflict**)

(2) $\Pr(H \mid E_2) < \Pr(H)$ — Assumption (**Conflict**)

(3) $Z(H, E_1) = Z(H, E_1 \mid E_2)$ — Assumption (Z-**Independence**)

(4) $Z(H, E_2) = Z(H, E_2 \mid E_1)$ — Assumption (Z-**Independence**)

(5) $$\frac{\Pr(H \mid E_1) - \Pr(H)}{1 - \Pr(H)} = \frac{\Pr(H \mid E_1 \,\&\, E_2) - \Pr(H \mid E_2)}{1 - \Pr(H \mid E_2)}$$ (1), (3), definition of Z

(6) $$\frac{\Pr(H \mid E_2) - \Pr(H)}{\Pr(H)} = \frac{\Pr(H \mid E_1 \,\&\, E_2) - \Pr(H \mid E_1)}{\Pr(H \mid E_1)}$$ (2), (4), definition of Z

Now, let $x \overset{\text{def}}{=} \Pr(H \mid E_1)$, $y \overset{\text{def}}{=} \Pr(H \mid E_2)$, $z \overset{\text{def}}{=} \Pr(H)$, and $u \overset{\text{def}}{=} \Pr(H \mid E_1 \,\&\, E_2)$. Then, (5) and (6) can be rewritten as the following pair of algebraic equations.

(5) $$\frac{x - z}{1 - z} = \frac{u - y}{1 - y}$$

(6) $$\frac{y - z}{z} = \frac{u - x}{x}$$

Algebraically (assuming only that x, y, z, and u are real numbers), (5) and (6) entail that *either* (7) $x = z$ *or* (8) $y = z$. But, this *contradicts* our assumption that *both* (1) $x > z$ *and* (2) $y < z$. $\square$

In closing, it is worth noting that it seems to be the piecewise nature of Z that causes **Problem.** For it can be shown that none of the non-piecewise-defined confirmation measures that have been discussed in the literature (see, *e.g.*, [1] and [2] for recent surveys) have this **Problem** (proof omitted). Finally, because **Problem** only rests on *ordinal* features, it will plague *any measure that is ordinally equivalent to* Z.[5]

[5]An anonymous referee points out that the following (formally similar) piecewise confirmation measure (on which see [5], [6], and [2] for further discussion), which takes its theoretical inspiration from [9], is *not* ordinally equivalent to Z.

$$Z^{\star}(H, E \mid K) \overset{\text{def}}{=} \begin{cases} \dfrac{\log[\Pr(H \mid E \,\&\, K)] - \log[\Pr(H \mid K)]}{-\log[\Pr(H \mid K)]} & \text{if } \Pr(H \mid E \,\&\, K) \geq \Pr(H \mid K) \\[2ex] \dfrac{\log[\Pr({\sim}H \mid E \,\&\, K)] - \log[\Pr({\sim}H \mid K)]}{-\log[\Pr({\sim}H \mid K)]} & \text{if } \Pr(H \mid E \,\&\, K) < \Pr(H \mid K) \end{cases}$$

I have been unable to determine whether $Z^{\star}$ also falsifies **Existence** (because $Z^{\star}$ involves *logarithms*, this question cannot be answered using standard algebraic techniques [4]). This is an interesting open question.

References

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[5] A. Mura, 2006, "Deductive probability, physical probability, and partial entailment." In M. Alai and G. Tarozzi (*eds.*), *Karl Popper philosopher of science* (*pp.* 181–202).

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