A Problem for Confirmation Measure $Z$

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Abstract. In this paper, I present a serious problem for confirmation measure $Z$.

1 Confirmation Measure $Z$

Crupi, Tentori, and Gonzalez [1] provide a very interesting set of theoretical and empirical arguments in favor of the following (piecewise) Bayesian measure of the degree to which evidence $E$ confirms hypothesis $H$, relative background knowledge $K$.

$$Z(H, E | K) \equiv \begin{cases} \frac{\Pr(H | E \& K) - \Pr(H | K)}{1 - \Pr(H | K)} & \text{if } \Pr(H | E \& K) \geq \Pr(H | K) \\ \frac{\Pr(H | E \& K) - \Pr(H | K)}{\Pr(H | K)} & \text{if } \Pr(H | E \& K) < \Pr(H | K) \end{cases}$$

I won’t go into their arguments in favor of $Z$ here. Instead, I will present what I take to be a serious problem with $Z$. This will require a brief digression into the notion of independent evidence.

2 Independent Evidence Regarding a Hypothesis

Fitelson [3] offers the following Bayesian account of what it means for two pieces of evidence $E_1$ and $E_2$ to be confirmationally independent, regarding hypothesis $H$, according to a confirmation measure $c$.

**Independence.** $E_1$ and $E_2$ are confirmationally independent regarding $H$, according to $c$ (viz., $E_1, E_2$ are $c$-independent regarding $H$) iff both $c(H, E_1 | E_2) = c(H, E_1)$ and $c(H, E_2 | E_1) = c(H, E_2)$.

Intuitively, $E_1$ and $E_2$ are confirmationally independent regarding $H$, according to $c$ just in case the degree to which $E_1$ ($E_2$) confirms $H$ (according to $c$) does not depend on whether $E_2$ ($E_1$) is already known.

As Fitelson shows, this notion can be applied in various useful confirmation-theoretic ways (e.g., to provide a Bayesian account of the value of varied/diverse evidence). I won’t delve into Independence here. Rather, I will simply apply it to reveal that measure $Z$ has a serious shortcoming when it comes to the handling of certain sorts of independent evidence.

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1 I would like to thank Vincenzo Crupi, Graham Oddie, and two anonymous referees of this journal for their helpful feedback on previous versions of this paper.

2 Several authors had discussed/endorsed measure $Z$ before Crupi et. al. See, for instance [7] and [8]. However, Crupi et. al provide the most compelling and comprehensive theoretical and empirical arguments in its favor.

2 Here, $c(H, E)$ is shorthand for $c(H, E | \top)$, where $\top$ is a tautology. This can be read simply as “the degree to which $E$ confirms $H$ (unconditionally), according to $c$.”
3 A Problem for Measure Z

Sometimes, we have conflicting evidence regarding a hypothesis. That is to say, sometimes, the following property holds for a triple $E_1, E_2,$ and $H$.

**Conflict.** $E_1$ and $E_2$ constitute conflicting evidence regarding $H$ iff $E_1$ confirms $H$, while $E_2$ disconfirms $H$. Formally, $E_1$ and $E_2$ constitute conflicting evidence regarding $H$ iff $\Pr(H \mid E_1) > \Pr(H)$ and $\Pr(H \mid E_2) < \Pr(H)$.

Intuitively, it should be possible for some triple $E_1, E_2,$ and $H$ to satisfy both Independence and Conflict. That is to say, intuitively, there sometimes exists independent, conflicting evidence regarding some hypotheses. More precisely, we have the following eminently plausible existence claim.

**Existence.** There exist some triples $E_1, E_2,$ and $H$ which satisfy both Independence and Conflict.

Indeed, Existence strikes me as so plausible as to require little justification. Having said that, it is worth giving a simple example which illustrates the intuitive plausibility of Existence. Here, I borrow the following example, which belongs to a class of examples used by Fitelson to provide an intuitive illustration of Independence (with individual degrees of strength that can be tweaked via some simple parameters, which I have set here).

**The Urn Example.** An urn has been selected at random from a collection of urns. Each urn contains some balls. In some of the urns (the $H$-urns) the proportion of white balls to non-white balls is $1/3$ and in all the other urns (the $\sim H$-urns) the proportion of white balls to non-white balls is $2/3$. The proportion of $H$-urns is $1/2$. Balls are to be drawn randomly from the selected urn, with replacement.

Let $H$ be the hypothesis that the proportion of white balls in the urn is $1/3$ (viz., that the sampled urn is an $H$-urn). Let $W_i$ state that the ball drawn on the $i^{th}$ draw ($i \geq 1$) is white. Intuitively, $\sim W_1$ and $W_2$ are confirmationally independent regarding $H$, i.e., the triple $(\sim W_1, W_2, H)$ instantiates Independence.

Surprisingly, according to measure Z, Existence is false (a fortiori). That is, according to measure Z, it is conceptually impossible for any pair of evidence $E_1, E_2$ to be both independent regarding $H$ and conflicting regarding $H$ (for any hypothesis $H$).

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3I thank an anonymous referee for urging me to include an illustrative intuitive example of Existence.

4Fitelson would be committed to a claim far stronger than mere Existence here (note: Conflict is obviously true in this case). He would be committed to the stronger claim that $\sim W_1$ and $W_2$ should have equal and opposite degrees of confirmation, which exactly cancel each other out, so that the total degree to which the conjunction $\sim W_1 \& W_2$ confirms $H$ is zero. This is because he accepts the likelihood-ratio measure of degree of confirmation, which satisfies (not only Existence, but) a strong independence-additivity requirement. Of course, we do not need to go along with Fitelson on that stronger/more specific claim. All we need this example to do is make Existence somewhat plausible (viz., not a conceptual impossibility). As I point out below, among all the measures of confirmation that have been proposed and defended in the literature, Z is the only measure that entails the conceptual impossibility of Existence. Indeed, a plenitude of examples satisfying Existence are easily described, for all other confirmation measures in the literature.
Problem. According to measure Z, Existence is (analytically) false.

Proof. Suppose, for reductio, that there does exist some triple \( E_1, E_2, H \) that satisfies both independence (according to measure Z) and conflict. Then, we may reason as follows.

1. \( \Pr(H \mid E_1) > \Pr(H) \)  
   Assumption (Conflict)
2. \( \Pr(H \mid E_2) < \Pr(H) \)  
   Assumption (Conflict)
3. \( Z(H, E_1) = Z(H, E_1 \mid E_2) \)  
   Assumption (Z-independence)
4. \( Z(H, E_2) = Z(H, E_2 \mid E_1) \)  
   Assumption (Z-independence)
5. \( \frac{\Pr(H \mid E_1) - \Pr(H)}{1 - \Pr(H)} = \frac{\Pr(H \mid E_1 \& E_2) - \Pr(H \mid E_2)}{1 - \Pr(H \mid E_2)} \)  
   (1), (3), definition of Z
6. \( \frac{\Pr(H \mid E_2) - \Pr(H)}{\Pr(H)} = \frac{\Pr(H \mid E_1 \& E_2) - \Pr(H \mid E_1)}{\Pr(H \mid E_1)} \)  
   (2), (4), definition of Z

Now, let \( x \equiv \Pr(H \mid E_1), y \equiv \Pr(H \mid E_2), z \equiv \Pr(H), \) and \( u \equiv \Pr(H \mid E_1 \& E_2) \). Then, (5) and (6) can be rewritten as the following pair of algebraic equations.

5. \( \frac{x - z}{1 - z} = \frac{u - y}{1 - y} \)
6. \( \frac{y - z}{z} = \frac{u - x}{x} \)

Algebraically (assuming only that \( x, y, z, \) and \( u \) are real numbers), (5) and (6) entail that either (7) \( x = z \) or (8) \( y = z \). But, this contradicts our assumption that both (1) \( x > z \) and (2) \( y < z \).

In closing, it is worth noting that it seems to be the piecewise nature of Z that causes Problem. For it can be shown that none of the non-piecewise-defined confirmation measures that have been discussed in the literature (see, e.g., [1] and [2] for recent surveys) have this Problem (proof omitted). Finally, because Problem only rests on ordinal features, it will plague any measure that is ordinally equivalent to Z.\(^3\)

\(^3\)An anonymous referee points out that the following (formally similar) piecewise confirmation measure (on which see [3], [6], and [2] for further discussion), which takes its theoretical inspiration from [9], is not ordinally equivalent to Z.

\[
Z^*(H, E \mid K) \equiv \begin{cases} 
\frac{\log[\Pr(H \mid E \& K)] - \log[\Pr(H \mid K)]}{\log[\Pr(H \mid K)] - \log[\Pr(H \mid E \& K)]} & \text{if } \Pr(H \mid E \& K) \geq \Pr(H \mid K) \\
\frac{\log[\Pr(H \mid K)] - \log[\Pr(H \mid E \& K)]}{\log[\Pr(H \mid K)] - \log[\Pr(H \mid E \& K)]} & \text{if } \Pr(H \mid E \& K) < \Pr(H \mid K)
\end{cases}
\]

I have been unable to determine whether \( Z^* \) also falsifies Existence (because \( Z^* \) involves logarithms, this question cannot be answered using standard algebraic techniques [4]). This is an interesting open question.
References


