Carnap [1] discusses 2 types of (probabilistic) confirmation.

- **Firmness.** \( E \) confirms \( f \) \( H \) iff
  \[
  \Pr(H \mid E) > t, \text{ where } t \geq 1/2.
  \]

- **Increase in Firmness.** \( E \) confirms \( i \) \( H \) iff
  \[
  \Pr(H \mid E) > \Pr(H).
  \]

Carnap also proposed (tentatively) a particular way of measuring the degree to which \( E \) confirms \( i \) \( H \):
\[
d(H, E) = \Pr(H \mid E) - \Pr(H).
\]

Confirms \( f \) & confirms \( i \) exhibit many theoretical divergences [15, 5, 10]. One of the most important of these divergences involves Hempel’s [13] **Special Consequence Condition**.

(SCC) If \( E \) confirms \( H_1 \) and \( H_1 \models H_2 \), then \( E \) confirms \( H_2 \).

Carnap [1, Ch. VI] discusses the fact that confirms \( f \) (generally) satisfies (SCC); but, confirms \( i \) does not.

Following Dretske [7], we may say that an epistemic operator \( \Theta(H, E) \) is a *penetrating operator* just in case \( \Theta(H, E) \) is always transmitted by deductive entailment.

Hempel’s (SCC) asserts that confirmation \( C(H, E) \) is a penetrating operator. Carnap shows that firmness \( C_f(H, E) \) is penetrating, while increase in firmness \( C_i(H, E) \) is not.

Dretske thought *knowledge* was not a penetrating operator (viz., that knowledge isn’t closed under entailment).

We will take no stand on knowledge closure here. But, it is worth noting that confirmation \( [C_i(H, E)] \) is a *propositional* relation, whereas knowledge is a *doxastic* relation (e.g., for one thing, \( E \) may not capture the agent’s *total evidence*).

Having said that, our discussion may be of some relevance to these broader epistemic questions, since some (putative) failures of knowledge transmission may involve (i.e., implicitly trade on) failures of \( C_i(H, E) \)-transmission.

Dretske [7] discusses an example he thinks shows that knowledge is not a penetrating operator.

**Zebra.** You’re at the zoo, and in the pen in front of you is a striped horse-like animal (which happens to be a zebra).

The sign on the pen says “Zebra.” Do you know it’s a zebra?

Dretske says: Well, what about the possibility that it’s just a mule painted to look like a zebra? Do you know that the animal is not a cleverly-disguised mule?

Let \( E \equiv \) your perceptual evidence (from observing the animal in the pen), \( H_1 \equiv \) the animal before you is a zebra, and \( H_2 \equiv \) the animal before you is not a cleverly-disguised mule.

Dretske seems to be suggesting (among other things) that, while \( E \) confirms \( H_1 \) and \( H_1 \models H_2 \), \( E \) does not confirm \( H_2 \). At least: \( E \) does not favor \( H_1 \) over \( \neg H_2 \) (and vice versa).

This basic Dretskean intuition leads to a simple sufficient condition for confirmation, \( i \)-transmission *failure*.
**Fact.** Suppose $E$ confirms, $H_1$ and $H_1 \vDash H_2$. Then, the following is a sufficient condition for the *failure* of confirmation,$^1$-transmission (i.e., for $E$ to *not* confirm,$^1$ $H_2$).

**Heavyweight.** $\Pr(E \mid H_1) = \Pr(E \mid \neg H_2)$.

- **Heavyweight** is a natural way to explicate the claim that evidence $E$ does not favor $H_1$ over $\neg H_2$ and *vice versa* [2].
- This way of understanding what Dretske means by “$\neg H_2$ is a heavy weight proposition” [6] is somewhat crude.
- For one thing, if $E$ confirms, $H_1$, then **Heavyweight** entails that $E$ disconfirms, $H_2$ — whether or not $H_1 \vDash H_2$.
- This makes **Heavyweight** not super interesting (for us). More interesting: conditions which (a) trade on $H_1 \vDash H_2$, and (b) are compatible with $E$ being *irrelevant to $H_2$*.
- We will examine some more interesting conditions (in these and other senses) shortly. First, we will discuss some other ways in which confirmation,$^1$-transmission can fail.

---

**Confirm**

- We will say that a probabilistic condition $X$ is *sufficient* for confirmation,$^1$-transmission, just in case the following holds.

  **Sufficiency.** There are no probability functions $\Pr(\cdot)$ s.t.

  $(S_1)$ $\Pr(H_1 \& \neg H_2) = 0$, and these are the only zeros of $\Pr(\cdot)$.

  $(S_2)$ $\Pr(H_1 \mid E) > \Pr(H_1)$. [E confirms, $H_1$, wrt $\Pr(\cdot)$]

  $(S_3)$ $\Pr(\cdot)$ satisfies $X$.

  $(S_4)$ $\Pr(H_2 \mid E) \leq \Pr(H_2)$. [E does not confirm, $H_2$, wrt $\Pr(\cdot)$]

- Kotzen [14] has an illuminating discussion of confirmation,$^1$ transmission in which he identifies the following sufficient condition for confirmation,$^1$ transmission.

  **Dragging.** $\Pr(H_2) < \Pr(H_1 \mid E)$.

- It is easy to see why **Dragging** is sufficient for transmission.

  **Proof.** $(S_1)$ implies $\Pr(H_2 \mid E) \succeq \Pr(H_1 \mid E)$, $(S_2)$ and Dragging then imply $\Pr(H_2 \mid E) > \Pr(H_2)$, which contradicts $(S_4)$. □

---

**Ace.** You are going to draw a single card at random from a standard deck. Let $E$ be the card is black, $H_1$ be the card is the ace of spades, and $H_2$ be the card is an ace.

- In **Ace**, $E$ confirms, $H_1$, since $\Pr(H_1 \mid E) = 1/26 > 1/52 = \Pr(H_1)$. Moreover, $H_1 \vDash H_2$. However, $E$ is *irrelevant to $H_2$*, since $\Pr(H_2 \mid E) = 2/26 = 4/52 = \Pr(H_2)$. [Note: $H_1 \vDash E$ in **Ace**.]

- Much more extreme failures of confirmation,$^1$ transmission are possible. To wit, there are cases such that (see Extras 14)

  1. $E$ *strongly* confirms, $H_1 [d(H_1, E) \gg 0]$.

  2. $H_1 \vDash H_2$ [more precisely, $\Pr(H_2 \mid H_1) = 1$].

  3. $E$ *strongly* disconfirms, $H_2 [d(H_2, E) \ll 0]$.  

  $^1$There are limits on how badly (SCC) can fail (in this sense). Specifically, if we understand $x \gg y$ as $x - y \geq t$, then we must have $t < 1/2$ in (1) & (3).

---

**Non-confirmation of Exhaustive Alternatives (NEA).**

$E$ does not confirm,$^1$ $H_2 \supset H_1$ [viz., $d(H_2 \supset H_1, E) \leq 0$].

- We call this **Non-confirmation of Exhaustive Alternatives** because it involves the non-confirmation of a claim which asserts that $\neg H_2$ and $H_1$ are *exhaustive alternatives*.

- For instance, in **Zebra**, $H_2 \supset H_1$ asserts that the animal before you is *either* a cleverly-dressed mule or a zebra.

- In **Zebra**, whether $E$ supports the exhaustivity of $H_1$ and $\neg H_2$ (as alternative hypotheses) seems probative (perhaps this relates to whether $\neg H_2$ is a “relevant alternative”?).

- Anyhow, in **Zebra**, $E$ *may not* confirm,$^1$ $H_2 \supset H_1$. And, if it doesn’t, then it turns out that $E$ must (also) confirm,$^1$ $H_2$.  

---

**Dragging.** It is easy to see why **Dragging** is sufficient for transmission.

**Proof.** $(S_1)$ implies $\Pr(H_2 \mid E) \succeq \Pr(H_1 \mid E)$. $(S_2)$ and Dragging then imply $\Pr(H_2 \mid E) > \Pr(H_2)$, which contradicts $(S_4)$. □
• A probabilistic condition $X$ (e.g., $\neg$Heavyweight) is necessary for confirmation transmission just in case

**Necessity.** There are no probability functions $\Pr(\cdot)$ s.t.

$(S_1)$ $\Pr(H_1 \land \neg H_2) = 0$, and these are the only zeros of $\Pr(\cdot)$.

$(S_2)$ $\Pr(H_1 \, | \, E) > \Pr(H_1)$. [E confirms, $H_1$, wrt $\Pr(\cdot)$]

$\neg(S_3)$ $\Pr(\cdot)$ does not satisfy $X$.

$\neg(S_4)$ $\Pr(H_2 \, | \, E) > \Pr(H_2)$. [E confirms, $H_2$, wrt $\Pr(\cdot)$]

• Kotzen [14, p. 70] voices skepticism about the existence of an interesting necessary and sufficient condition for confirmation transmission. We think we've found one.

**Relative Disconfirmation of Exhaustive Alternatives (RDEA).**

$E$ confirms, $H_1$ more strongly than $E$ confirms, $H_2 \supset H_1$, according to Carnap’s $d$ [i.e., $d(H_1, E) > d(H_2 \supset H_1, E)$].

The confirmation $E$ provides for $H_1$ transmits to $H_2$ iff $E$ raises $H_1$’s probability more (as measured by $d$) than it does the claim that $H_1$ and $\neg H_2$ are exhaustive alternatives.

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• Here’s a summary of which measures $c(H, E)$ of the degree to which $E$ confirms $H$ imply sufficiency/necessity of

(RDEA$_c$) $c(H_1, E) > c(H_2 \supset H_1, E)$

for confirmation transmission.

<table>
<thead>
<tr>
<th>$c$</th>
<th>Is (RDEA$_c$) Sufficient?</th>
<th>Is (RDEA$_c$) Necessary?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>$r$</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>$z$</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>$l$</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>$s$</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

The $s$-measure [3, 9, 8] also satisfies our quantitative

**Theorem** (see Extras 13). See Extras 17–18 for probability models establishing the four “No”s in the above table.

[Our proofs of the “Yes”s for (RDEA$_c$)/(RDEA$_z$) are complex (omitted).]

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• To prove our results, we’ll use the following algebraic representation, and the approach described in [12].

<table>
<thead>
<tr>
<th>State ($s_i$)</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$E$</th>
<th>$\Pr(s_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>$\Pr(s_1) = a_1$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>$\Pr(s_2) = a_2$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>$\Pr(s_3) = a_3$</td>
</tr>
<tr>
<td>$s_4$</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>$\Pr(s_4) = a_4$</td>
</tr>
<tr>
<td>$s_5$</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>$\Pr(s_5) = a_5$</td>
</tr>
<tr>
<td>$s_6$</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>$\Pr(s_6) = a_6$</td>
</tr>
<tr>
<td>$s_7$</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>$\Pr(s_7) = a_7$</td>
</tr>
<tr>
<td>$s_8$</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>$\Pr(s_8) = a_8$</td>
</tr>
</tbody>
</table>

The fact that (RDEA) is necessary and sufficient for transmission of confirmation is a corollary of the following general, quantitative result (see Extras 12 for a proof of it).

**Theorem.** If $\Pr(H_2 \mid H_1) = 1$, then

$$d(H_2, E) = d(H_1, E) - d(H_2 \supset H_1, E).$$

• **Theorem** implies both (i) (RDEA) \iff transmission and (ii) (NEA) \iff transmission, and it (iii) gives the $d$-degree to which $H_2$ is confirmed by $E$, whenever $H_1 = H_2$.

• This result — and its qualitative corollary — depends on how we choose to measure degree of confirmation. Specifically, here are 4 other measures of degree of confirmation [11, 4].

$$r(H, E) \equiv \frac{\Pr(H|E)}{\Pr(H)} = \frac{\Pr(H|E) + \Pr(H|\neg E)}{\Pr(H|E) - \Pr(H|\neg E)}$$

$$l(H, E) \equiv \frac{\Pr(E|H)}{\Pr(E|H)} = \frac{\Pr(E|H) + \Pr(E|\neg H)}{\Pr(E|H) - \Pr(E|\neg H)}$$

$$z(H, E) \equiv \frac{d(H, E)}{d(H|E)}$$

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• This involves (a) translating the desired result into algebra, and (b) showing it corresponds to a theorem of algebra (or that it does not), assuming $a_i \in [0, 1]$ and $\sum a_i = 1$. 

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**If** $\Pr(H_2 \mid H_1) = 1$, then $\alpha_3 = \alpha_4 = 0$. And, we have:

\[
\begin{align*}
    d(H_2, E) &= \frac{\alpha_1 + \alpha_5}{\alpha_1 + \alpha_5 + \alpha_7} - (\alpha_1 + \alpha_2 + \alpha_5 + \alpha_6), \\
    d(H_1, E) &= \frac{\alpha_1}{\alpha_1 + \alpha_5 + \alpha_7} - (\alpha_1 + \alpha_2), \\
    d(H_2 \supset H_1, E) &= \frac{\alpha_1 + \alpha_7}{\alpha_1 + \alpha_5 + \alpha_7} - (1 - (\alpha_5 + \alpha_6))
\end{align*}
\]

- Then, the following reasoning establishes our **Theorem**:

\[
\begin{align*}
    d(H_1, E) - d(H_2 \supset H_1, E) &= \left[ 1 - \frac{\alpha_7}{\alpha_1 + \alpha_5 + \alpha_7} \right] - (\alpha_1 + \alpha_2 + \alpha_5 + \alpha_6) \\
    &= \left[ 1 - \Pr(\neg H_1 \& \neg H_2 \mid E) \right] - \Pr(H_2) \\
    &= \Pr(H_1 \lor H_2 \mid E) - \Pr(H_2) \\
    &= \frac{\alpha_1 + \alpha_3}{\alpha_1 + \alpha_3 + \alpha_7} - (\alpha_1 + \alpha_2 + \alpha_5 + \alpha_6) \\
    &= d(H_2, E) \quad \Box
\end{align*}
\]

**Here is a model (all models were found with PrSAT [12]) on which** $(S_1), \, d(H_1, E) = 0.49$ and $d(H_2, E) = -0.49$. This is about as extreme a failure of (SCC) as possible (see fn. 1).

<table>
<thead>
<tr>
<th>State ($s_i$)</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$E$</th>
<th>$\Pr(s_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>$\Pr(s_1) = \frac{450}{57600}$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>$\Pr(s_2) = \frac{126}{57600}$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>$\Pr(s_3) = 0$</td>
</tr>
<tr>
<td>$s_4$</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>$\Pr(s_4) = 0$</td>
</tr>
<tr>
<td>$s_5$</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>$\Pr(s_5) = \frac{1}{57600}$</td>
</tr>
<tr>
<td>$s_6$</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>$\Pr(s_6) = \frac{5611}{57600}$</td>
</tr>
<tr>
<td>$s_7$</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>$\Pr(s_7) = \frac{449}{57600}$</td>
</tr>
<tr>
<td>$s_8$</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>$\Pr(s_8) = \frac{63}{57600}$</td>
</tr>
</tbody>
</table>

- **If** $\Pr(H_2 \mid H_1) = 1$, then $\alpha_3 = \alpha_4 = 0$. And, we have:

\[
\begin{align*}
    s(H_2, E) &= \frac{\alpha_1 + \alpha_5}{\alpha_1 + \alpha_5 + \alpha_7} - \frac{\alpha_2 + \alpha_6}{\alpha_2 + \alpha_5 + \alpha_7}, \\
    s(H_1, E) &= \frac{\alpha_1}{\alpha_1 + \alpha_5 + \alpha_7} - \frac{\alpha_2}{\alpha_2 + \alpha_5 + \alpha_7}, \\
    s(H_2 \supset H_1, E) &= \frac{\alpha_1 + \alpha_7}{\alpha_1 + \alpha_5 + \alpha_7} - \frac{\alpha_2 + \alpha_6}{\alpha_2 + \alpha_5 + \alpha_7}
\end{align*}
\]

- Then, the following establishes the $s$-version of **Theorem**.

\[
\begin{align*}
    s(H_1, E) - s(H_2 \supset H_1, E) &= \frac{\alpha_1}{\alpha_1 + \alpha_5 + \alpha_7} - \frac{\alpha_2}{\alpha_2 + \alpha_5 + \alpha_7} - \frac{\alpha_2 + \alpha_6}{\alpha_2 + \alpha_5 + \alpha_7} + \frac{\alpha_1 + \alpha_7}{\alpha_1 + \alpha_5 + \alpha_7} \\
    &= \frac{\alpha_1 - \alpha_2 - \alpha_2 + \alpha_6}{\alpha_1 + \alpha_5 + \alpha_7} - \frac{\alpha_7}{\alpha_1 + \alpha_5 + \alpha_7} \\
    &= \Pr(\neg H_1 \& \neg H_2 \mid E) - \Pr(\neg H_1 \& \neg H_2 \mid E) \\
    &= \left[ 1 - \Pr(H_1 \lor H_2 \mid E) \right] - \left[ 1 - \Pr(H_1 \lor H_2 \mid E) \right] \\
    &= \frac{\alpha_1 - \alpha_2 + \alpha_6}{\alpha_1 + \alpha_5 + \alpha_7} - \frac{\alpha_7}{\alpha_1 + \alpha_5 + \alpha_7} \\
    &= s(H_2, E) \quad \Box
\end{align*}
\]

**Here is a probability model on which** $(S_1), \, (S_2)$, and (NEA) are true, but **Dragging** is false (this shows NEA $\neq$ Dragging).

<table>
<thead>
<tr>
<th>State ($s_i$)</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$E$</th>
<th>$\Pr(s_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>$\Pr(s_1) = \frac{256}{512}$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>$\Pr(s_2) = \frac{28}{512}$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>$\Pr(s_3) = 0$</td>
</tr>
<tr>
<td>$s_4$</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>$\Pr(s_4) = 0$</td>
</tr>
<tr>
<td>$s_5$</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>$\Pr(s_5) = \frac{64}{512}$</td>
</tr>
<tr>
<td>$s_6$</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>$\Pr(s_6) = \frac{5}{512}$</td>
</tr>
<tr>
<td>$s_7$</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>$\Pr(s_7) = \frac{128}{512}$</td>
</tr>
<tr>
<td>$s_8$</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>$\Pr(s_8) = \frac{31}{512}$</td>
</tr>
</tbody>
</table>
Here is a probability model on which \((S_1), (S_2), \) and Dragging are true, but (NEA) is false (this shows Dragging \(\neq\) NEA).

<table>
<thead>
<tr>
<th>State ((s_i))</th>
<th>(H_1)</th>
<th>(H_2)</th>
<th>(E)</th>
<th>(Pr(s_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1)</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>(\frac{128}{256})</td>
</tr>
<tr>
<td>(s_2)</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>(\frac{5}{256})</td>
</tr>
<tr>
<td>(s_3)</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>0</td>
</tr>
<tr>
<td>(s_4)</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>(s_5)</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>(\frac{12}{256})</td>
</tr>
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<td>T</td>
<td>F</td>
<td>(\frac{10}{256})</td>
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<td>(s_7)</td>
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<td>T</td>
<td>(\frac{64}{256})</td>
</tr>
<tr>
<td>(s_8)</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>(\frac{37}{256})</td>
</tr>
</tbody>
</table>

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Here is a probability model on which \((S_1), (S_2), \) \(\neg\) (RDEA_1), \(\neg\) (RDEA_2), and \(\neg\) (S_4) are all true. This shows that neither (RDEA_1) nor (RDEA_2) is necessary for transmission.

<table>
<thead>
<tr>
<th>State ((s_i))</th>
<th>(H_1)</th>
<th>(H_2)</th>
<th>(E)</th>
<th>(Pr(s_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1)</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>(\frac{64}{512})</td>
</tr>
<tr>
<td>(s_2)</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>(\frac{5}{512})</td>
</tr>
<tr>
<td>(s_3)</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>0</td>
</tr>
<tr>
<td>(s_4)</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>0</td>
</tr>
<tr>
<td>(s_5)</td>
<td>F</td>
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<td>(s_6)</td>
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<td>(\frac{45}{512})</td>
</tr>
<tr>
<td>(s_7)</td>
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<td>F</td>
<td>T</td>
<td>(\frac{128}{512})</td>
</tr>
<tr>
<td>(s_8)</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>(\frac{14}{512})</td>
</tr>
</tbody>
</table>

Yablo & Fitelson

When Confirmation Transmits 17