- Let $\mathscr{T}$ be (classical, 1^{st}-order) arithmetic (see Appendix), and $\vdash$ be deducibility in $\mathscr{T}$. Carnap & Tarski proved [9].

  Diagonal Lemma (DL). For any predicate Ψ, there exists a sentence α s.t. $\vdash \alpha \leftrightarrow \Psi(\ulcorner\alpha\urcorner)$, where $\ulcorner\alpha\urcorner$ is a name of α.

- The two most famous applications of (DL) are (a) to prove the incompleteness of $\mathscr{T}$ (Gödel), and (b) to prove that the naïve theory of (arithmetic) truth is inconsistent (Tarski).
- For (a) $\Psi \overset{\text{def}}{=}$ "is not provable (in $\mathscr{T}$);" and, for (b) $\Psi \overset{\text{def}}{=}$ "is not true (on the standard interpretation of $\mathscr{T}$)." (a) assumes $\Box\alpha \overset{\text{def}}{=} \neg\Psi(\ulcorner\alpha\urcorner)$ satisfies the axioms of *provability logic* [25].
- Tarski's inconsistency proof assumes $\mathsf{T}(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} \neg\Psi(\ulcorner\alpha\urcorner)$ satisfies both directions of the **T–schema.** To wit:

  Release. $\vdash \neg\Psi(\ulcorner\alpha\urcorner) \to \alpha$. [*i.e.*, from $\mathsf{T}(\ulcorner\alpha\urcorner)$, infer α]

  Capture. $\vdash \alpha \to \neg\Psi(\ulcorner\alpha\urcorner)$. [*i.e.*, from α, infer $\mathsf{T}(\ulcorner\alpha\urcorner)$]

- I will be exploring — *via* focusing on the *epistemic role* of truth — the rejection of **Capture** for the truth predicate.

- Consider the following three conditions.

  Factivity. $\vdash \Phi(\ulcorner\alpha\urcorner) \to \alpha$.

  Classicality. The deducibility relation $\vdash$ obeys the laws of classical logic (*viz.*, classical first-order arithmetic).

  Normativity. If $\vdash \alpha$, then (one may infer) $\Phi(\ulcorner\alpha\urcorner)$.

  Theorem [18]. **Factivity**, **Classicality**, and **Normativity** are jointly inconsistent (*i.e.*, any classical theory $\mathfrak{T}$ of Φ satisfying **Factivity** & **Normativity** is s.t. $\mathfrak{T} \vdash \alpha$, *for all* α).

☞ One must reject at least one of these three assumptions.[1]

  - McGee [17] rejects **Factivity** (for the truth predicate).
  - Field [6], Priest [20], and others reject **Classicality**.

- I will explore the strategy of rejecting **Normativity**.

[1] Here, I am assuming (along with my opponents) the following.

Non-Triviality*. If, for all α, $\mathfrak{T} \vdash \alpha$, then $\mathfrak{T}$ ought (objectively, epistemically) to be rejected (i.e., we ought to reject trivial theories).*

In *this* sense, "logic is normative." I'll question other versions of this slogan.

- For any predicate Φ and sentence α, (DL) ensures the existence of a sentence provably equivalent to

  (C) $\Phi(\ulcorner C\urcorner) \to \alpha$.

- This allows us to prove our **Theorem**, Curry-style [22].

(1) $\Phi(\ulcorner C\urcorner)$	Assumption (for $\to$-introduction)
(2) C	1, **Factivity**
(3) $\Phi(\ulcorner C\urcorner) \to \alpha$	2, Definition of C
(4) α	1, 3, **Classicality** ($\to$-elimination)
(5) $\Phi(\ulcorner C\urcorner) \to \alpha$	1–4, **Classicality** ($\to$-introduction)
(6) C	5, Definition of C
(7) $\vdash C$	1-6, **Classicality** (we just proved C, classically)
(8) $\Phi(\ulcorner C\urcorner)$	7, **Normativity**
(9) α	5, 8, **Classicality** ($\to$-elimination)
(10) $\vdash \alpha$	1-9, **Classicality** (we just proved α, classically)
(11) $\Phi(\ulcorner\alpha\urcorner)$	10, **Normativity**
(12) $(\forall\alpha)\Phi(\ulcorner\alpha\urcorner)$	11, **Classicality** ($\forall$-introduction)
(13) $\Phi(\bot)$	12, **Classicality** ($\forall$-elimination)
(14) $\neg\Phi(\bot)$	**Factivity**, **Classicality**
(15) $\bot$	13, 14, **Classicality** ($\neg$-elimination) $\Box$

- **Miners** [19, 13]. You are standing in front of two mine shafts (A and B). Flood waters are approaching. You know that ten miners are in one of the shafts, but you don't know which (*e.g.*, their location was determined by the result of a fair coin toss). You have enough sand bags to block one of the shafts. If the miners are in A (B), then blocking A (B) saves all 10 miners. If you block neither shaft, then only the lowest miner in the shaft will die. *Objectively*, you ought to block whichever shaft the miners are in. *Subjectively*, this is *not* the case (*i.e.*, *subjectively*, you may block neither shaft).
- **Gibbard's Coin** [10, 15]. A fair coin has been tossed (and you have no information about how it landed). *Objectively*, you ought (epistemically) to believe whichever hypothesis (Heads/Tails) is *true*. *Subjectively*, this is *not* the case (*i.e.*, *subjectively*, you may suspend judgment on Heads/Tails).

☞ There are (both prudential and epistemic) objective and subjective oughts. I'll focus on the objective, epistemic ought — in light of the (semantic & epistemic) paradoxes.

- I will be assuming a background theory strong enough to imply (DL), so as to support self-reference. And, I will be making use of the following eight (8) sentential predicates.
  - $\mathsf{T}(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} \ulcorner\alpha\urcorner$ is true.
  - $\mathsf{B}_S(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} S$ believes $\ulcorner\alpha\urcorner$.
  - $\mathsf{OB}_S(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} S$ ought to believe $\ulcorner\alpha\urcorner$.
    - Various senses of "ought" will be discussed.
  - $\mathsf{K}_S(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} S$ knows $\ulcorner\alpha\urcorner$.
  - ${}_m\mathsf{B}_S(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} S$ comes to believe $\ulcorner\alpha\urcorner$ *via* method m.
  - ${}_m\mathsf{K}_S(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} S$ comes to know $\ulcorner\alpha\urcorner$ *via* method m.
  - $\mathsf{K}^{\diamond}_S(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} S$ is *in a position to know* $\ulcorner\alpha\urcorner$.
    - Roughly, $(\exists m)\,[{}_m\mathsf{B}_S(\ulcorner\alpha\urcorner) \to {}_m\mathsf{K}_S(\ulcorner\alpha\urcorner)]$. More precisely, there is *no conceptual barrier* to S knowing α.
  - $\mathsf{K}^{\diamond}(\ulcorner\alpha\urcorner) \overset{\text{def}}{=} \ulcorner\alpha\urcorner$ is *knowable-in-principle*.
    - Roughly, $(\exists S)\mathsf{K}^{\diamond}_S(\ulcorner\alpha\urcorner)$. More precisely, there is *no conceptual barrier* to *some* agent knowing α.

Moore [23]. Jane is pondering the following sentence:

(M) It is raining, but Jane does not believe that it is raining.

As it happens, it is true that it is raining and Jane does not believe that it is raining. That is: $\mathsf{T}(\ulcorner M\urcorner)$. But, it is demonstrable (even for Jane) that Jane is not in a position to know (M). So, in a sense: $\mathsf{T}(\ulcorner M\urcorner)\ \&\ \neg\mathsf{OB}_{\text{Jane}}(\ulcorner M\urcorner)$.

- **Moore** differs from **Gibbard's Coin** in some crucial ways.
- While you *do not know* (H) is true (you only know H has a 50/50 chance), there are (in principle) ways for you to come to know H — there's *no conceptual barrier* to $\mathsf{K}_{\text{You}}(\ulcorner H\urcorner)$.
- In this sense, you are *in a position to know* H $[\mathsf{K}^{\diamond}_{\text{You}}(\ulcorner H\urcorner)]$. Moreover, you could (in this sort of example) even be *in a position to know **that** you are in a position to know* (H).
- ***Not so** for Jane and* (M). For she has a method m available to her — *a knowledge-yielding deduction of* $\neg\mathsf{K}_{Jane}(\ulcorner M\urcorner)$ — for coming to know that *she is **not** in a position to know* (M).

- Let $R \overset{\text{def}}{=}$ it is raining. We (incl. Jane) can deduce as follows:

(1) $\mathsf{K}_{\text{Jane}}(M)$	Assumption (for *reductio*)
(2) $\mathsf{K}_{\text{Jane}}(R\ \&\ \neg\mathsf{B}_{\text{Jane}}(R))$	1, Definition of M, R
(3) $\mathsf{K}_{\text{Jane}}(R)\ \&\ \mathsf{K}_{\text{Jane}}(\neg\mathsf{B}_{\text{Jane}}(R))$	2, $\mathsf{K}_S(p\ \&\ q) \to \mathsf{K}_S(p)\ \&\ \mathsf{K}_S(q)$
(4) $\mathsf{K}_{\text{Jane}}(R)$	3, Logic (&E)
(5) $\mathsf{K}_{\text{Jane}}(\neg\mathsf{B}_{\text{Jane}}(R))$	3, Logic (&E)
(6) $B_{\text{Jane}}(R)$	4, $K(p) \to B(p)$
(7) $\neg B_{\text{Jane}}(R)$	5, $K(p) \to p$
(8) $\neg\mathsf{K}_{\text{Jane}}(M)$	1–7, Logic (*reductio*) □

- Because $\mathsf{K}_{\text{Jane}}\left(\neg\mathsf{K}^{\diamond}_{\text{Jane}}(\ulcorner M\urcorner)\right)$, there is a (stronger, but still subjective) sense in which *she* ought not believe (M).
- While Jane's ignorance regarding (M) is *ineliminable in principle*, this is still not the objective "ought" I am after.
- Jane's believing (M) need not be an *incorrect* believing. In this sense, *even Jane* (objectively) ought to believe (M).

Untrue Believer. Bill is pondering this instance of (C):

(P) Bill does not truly believe $\ulcorner P\urcorner$, *viz.*, $\neg\,[\mathsf{B}_{\text{Bill}}(\ulcorner P\urcorner)\ \&\ \mathsf{T}(\ulcorner P\urcorner)]$.

Assuming **Release** (*viz.*, **Factivity**), (P) is *deducible*. But, Bill *cannot truly believe* (P). Hence, he can't *know* (P). In this sense, he is similar to Mary (wrt M). But, unlike Mary (wrt M), Bill's believing (P) must be an *incorrect* believing.

- Here is a simple deduction of (P).

(1) $\neg P$	Assumption (for $\to$ I)
(2) $\mathsf{B}_{\text{Bill}}(\ulcorner P\urcorner)\ \&\ \mathsf{T}(\ulcorner P\urcorner)$	1, Definition of P, Logic (DN)
(3) $\mathsf{T}(\ulcorner P\urcorner)$	2, Logic
(4) P	3, **Release** (**Factivity** of T)
(5) $\neg P \to P$	1–4, Logic ($\to$ I)
(6) P	5, Logic □

- Suppose for *reductio* $\mathsf{B}_{\text{Bill}}(\ulcorner P\urcorner)\ \&\ \mathsf{T}(\ulcorner P\urcorner)$. Assuming **Release**, this implies $\mathsf{T}(\ulcorner P\urcorner)$ *and* $\neg\mathsf{T}(\ulcorner P\urcorner)$. $\therefore\ \mathsf{B}_{\text{Bill}}(\ulcorner P\urcorner) \to \neg\mathsf{T}(\ulcorner P\urcorner)$.
- *Almost all* agents can correctly believe (P). My objective ought (OB) is *universal*. So, in this case, I say $\neg\mathsf{OB}(\ulcorner P\urcorner)$.

- Note: my objective ought $\mathsf{OB}(\ulcorner \alpha \urcorner)$ is not coextensional with knowability-in-principle $K^{\diamond}(\ulcorner \alpha \urcorner)$. Fitch cases [4] show this.
- Let t be some mundane truth which happens not to be known (by anyone). And, consider the claim

  (F) $t \,\&\, \neg(\exists S) K_S(t)$.

- Intuitively, nothing (conceptually) prevents anyone from *correctly believing* (F). But, there is a conceptual barrier to anyone *knowing* (F). So, I would say $\mathsf{OB}(F)$ and $\neg K^{\diamond}(F)$.
- $\therefore \mathsf{OB}(\ulcorner \alpha \urcorner) \not\Rightarrow K^{\diamond}(\ulcorner \alpha \urcorner)$, and ($P$) shows $K^{\diamond}(\ulcorner \alpha \urcorner) \not\Rightarrow \mathsf{OB}(\ulcorner \alpha \urcorner)$.
- Here is a more precise characterization of OB.

  $\mathsf{OB}(\ulcorner \alpha \urcorner)$ iff every agent (objectively) ought to believe α (*i.e.*, iff all agents can, in principle, correctly believe α).

- I want OB to satisfy **Factivity**. So, assuming **Classicality**, I must reject **Normativity**. That is, I must reject the claim that *deducibility implies objective correctness of belief*.

Liar [3]. Consider the following instance of (C):

(L) It is not the case that $\ulcorner L \urcorner$ is true, *i.e.*, $\neg \mathsf{T}(\ulcorner L \urcorner)$.

Using the Liar (and assuming T satisfies both **Release** and **Capture**), we can deduce a contradiction, as follows.

(1) $\mathsf{T}(\ulcorner L \urcorner)$	Assumption (for → I)
(2) L	1, **Release**
(3) $\neg\mathsf{T}(\ulcorner L \urcorner)$	2, Definition of L
(4) $\mathsf{T}(\ulcorner L \urcorner) \to \neg\mathsf{T}(\ulcorner L \urcorner)$	1–3, Logic (→ I)
(5) $\neg\mathsf{T}(\ulcorner L \urcorner)$	4, Logic
– – – – – – – – – – – – – – – –	
(6) $\neg\mathsf{T}(\ulcorner L \urcorner)$	Assumption (for → I)
(7) L	6, Definition of L
(8) $\mathsf{T}(\ulcorner L \urcorner)$	7, **Capture**
(9) $\neg\mathsf{T}(\ulcorner L \urcorner) \to \mathsf{T}(\ulcorner L \urcorner)$	6–8, Logic (→ I)
(10) $\mathsf{T}(\ulcorner L \urcorner)$	9, Logic
– – – – – – – – – – – – – – – –	
(11) $\mathsf{T}(\ulcorner L \urcorner) \,\&\, \neg\mathsf{T}(\ulcorner L \urcorner)$	10, 5, Logic (&I) □

- I'm fine with the first sub-derivation, which yields $\neg\mathsf{T}(\ulcorner L \urcorner)$. That relies only on **Release**, which is epistemically kosher.
- The second sub-derivation is where I balk. Specifically, the application of **Capture** at step (8) is *not* kosher.
- ☞ Because T is factive, (L) is deducible. However, objectively, *ought* one believe (L)? Would/could believing (L) be a *correct* believing? I'm inclined to say "No." [5]
- This suggests the following *restriction* on **Capture**.

  ($\mathbb{C}$) *If* $\mathsf{OB}(\ulcorner \alpha \urcorner)$, *then* (one may) infer $\mathsf{T}(\ulcorner \alpha \urcorner)$ from α.

- In words, ($\mathbb{C}$) says **Capture** is kosher — *viz.*, inferring $\mathsf{T}(\ulcorner \alpha \urcorner)$ from α (or $\vdash \alpha$) is kosher — *provided* that $\mathsf{OB}(\ulcorner \alpha \urcorner)$.
- For example, since $\neg\mathsf{OB}(\ulcorner L \urcorner)$, ($\mathbb{C}$) does not sanction the use of **Capture** to derive a contradiction regarding (L).
- Of course, any approach to the **Liar** must beware *revenge*.

- Because I accept **Release**, ($\mathbb{C}$) is meant to imply:

  ($\mathbb{T}$) $\mathsf{OB}(\ulcorner \alpha \urcorner) \to (\alpha \leftrightarrow \mathsf{T}(\ulcorner \alpha \urcorner))$.[2]

- As Bacon [1] explains, a certain type of revenge plagues any strategy that (generally) restricts the T-schema in this way.
- As applied to our approach, Bacon's argument implies the existence a sentence γ such that $\gamma \,\&\, \neg\mathsf{OB}(\ulcorner \gamma \urcorner)$ is deducible. In fact, here is a specific sentence that does the trick:

  (I) It is not the case that (I) ought (objectively) to be believed, *viz.*, $\neg\mathsf{OB}(\ulcorner I \urcorner)$.

- Because OB is factive, (I) is deducible. And, since (I) is equivalent to $\neg\mathsf{OB}(\ulcorner I \urcorner)$, $I \,\&\, \neg\mathsf{OB}(\ulcorner I \urcorner)$ is also deducible.
- Bacon also shows if ($\mathbb{T}$) holds generally, then **Normativity** (of OB) must fail, on pain of contradiction. We already reject **Normativity** [b/c **Theorem**], so this is no problem for us.

[2] Indeed, I accept: $\mathsf{OB}(\ulcorner \alpha \urcorner) \to (\alpha \,\&\, \mathsf{T}(\ulcorner \alpha \urcorner))$, *i.e.*, $\mathsf{OB}(\ulcorner \alpha \urcorner) \to \mathsf{T}(\ulcorner \alpha \urcorner)$.

- What *shall* we say about (I)?
  - It is deducible. Moreover, on reflection, it *seems right*. So, (I) seems to have *something* (epistemically) going for it.
- But, we don't want to be in a position where we *believe* (I), since this would be to believe a claim, and also to believe that (objectively) we shouldn't believe that very claim.
- So, we need some positive epistemic attitude to take toward (I) that does not commit us to believing (I), or even to the implication that one ought (objectively) to believe (I).
- Several philosophers have argued recently that we (philosophers, generally) need such an attitude — for reasons that have nothing to do with the paradoxes.
- Fleisher [8] and Barnett [2] independently argue that such an attitude is needed to properly handle/understand, *e.g.*, the phenomenon of deep (philosophical) disagreement.
- I think (I) also calls out for just such an attitude.

- It is sometimes said that logic is *epistemically normative* for thought/reasoning/inference/credence, *etc.* [11, 6, 15].
- It is clear that this normative force (if there be such) cannot apply (generally) to *subjective* epistemic ought(s) [24].
- One might have hoped that logic would be normative with respect to the *objective* epistemic ought. But, there are reasons for skepticism (deducibility $\not\Rightarrow$ objective correctness).
- **Untrue Believer**, *e.g.*, seems to be a clear counterexample to even very weak forms of epistemic closure, such as:

  Closure [12]. If S knows Q and S competently deduces P from Q — *while maintaining their knowledge of* Q — then S (thereby) knows P (*via* said competent deduction).

- After all, Bill can competently deduce P from Q (where Q is the factivity of T) while maintaining his knowledge that Q. But, this does not (*cannot*) put Bill in a position to know P.
- I'm inclined to conclude that *logic is not normative* [14, 7].

First-Order Logical Axioms & Rules	Arithmetic Axioms ($\mathscr{T}$)
$\vdash \alpha \to (\beta \to \alpha)$	s is the successor function
$\vdash (\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$	0 is the number zero
$\vdash (\alpha \,\&\, \beta) \to \alpha$	$+$ is the addition function
$\vdash (\alpha \,\&\, \beta) \to \beta$	$\cdot$ is the multiplication function
$\vdash \alpha \to (\beta \to (\alpha \,\&\, \beta))$	t's are terms; R is an n-ary relation;
$\vdash \alpha \to (\beta \to (\alpha \,\&\, \beta))$	and, f is an n-ary function
$\vdash \alpha \to (\alpha \vee \beta)$	$\vdash (\forall x)\,[s(x) \neq 0]$
$\vdash \beta \to (\alpha \vee \beta)$	$\vdash (\forall x)(\forall y)\,[s(x) = s(y) \to x = y]$
$\vdash (\alpha \to \gamma) \to ((\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma))$	$\vdash (\forall x)\,[x + 0 = x]$
$\vdash (\alpha \to \beta) \to ((\alpha \to \neg\beta) \to \neg\alpha)$	$\vdash (\forall x)(\forall y)\,[x + s(y) = s(x + y)]$
$\vdash \neg\alpha \to (\alpha \to \beta)$	$\vdash (\forall x)\,[x \cdot 0 = 0]$
$\vdash \alpha \vee \neg\alpha$	$\vdash (\forall x)(\forall y)\,[x \cdot s(y) = (x \cdot y) + x]$
$\alpha, \alpha \to \beta \vdash \beta$	
	$\vdash t = t$
	$\vdash \&_{i=1}^{n}(t_i = t_i') \to \left[R(t_i, \ldots, t_n) \to R(t_i', \ldots, t_n')\right]$
with t a term; $\phi(x/t)$ a kosher substitution;	$\vdash \&_{i=1}^{n}(t_i = t_i') \to \left[f(t_i, \ldots, t_n) = f(t_i', \ldots, t_n')\right]$
and, χ a formula, where x is not free in χ:	
$\vdash \phi(t) \to (\exists x)\phi(x)$	with ψ a formula, where x is free in ψ:
$\vdash (\forall x)\phi(x) \to \phi(t)$	$\vdash \{\psi(0) \,\&\, (\forall x)\,[\psi(x) \to \psi(s(x))]\} \to (\forall x)\psi(x)$
$\vdash (\forall x)\,[\chi \to \phi(x)] \to [\chi \to (\forall x)\phi(x)]$	
$\vdash (\forall x)\,[\phi(x) \to \chi] \to [(\exists x)\phi(x) \to \chi]$	
$\phi \vdash (\forall x)\phi$	

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