- The modern Bayesian conception of confirmation uses *probabilistic relevance* as its main conceptual tool.
- Keynes [31], and his teacher W.E. Johnson [28], were early proponents of the logical and epistemic importance of probabilistic relevance. But, they *mainly* talked about *high conditional probability* (more on that key ambiguity below).
- Nicod [35], taking Keynes as his point of departure, offered an instantial theory based explicitly on probabilistic relevance. "Positive instances raise the probability of laws."
- Later, Hempel [24] moved away from Nicodian *probabilistic relevance* instantial confirmation theory, in favor of an account based on *deductive* relations. This was a set-back!
- Largely because of (*a*) the early focus on high conditional probability, and (*b*) Hempel's deductive set-back, probabilistic relevance approaches took time to catch-on.
- Further complications were raised by Carnap [3], who will be the main histroical protagonist of today's lecture.

- In the first edition of LFP, Carnap [3] undertakes a precise probabilistic explication of the concept of confirmation. This is where modern confirmation theory was born (in sin).
- Carnap was interested not only in the qualitative confirmation relation. He also wanted explications of comparative and quantitative confirmation concepts.
  - **Qualitative**. E inductively supports H.
  - **Comparative**. E supports H *more strongly than* E' supports H'.
  - **Quantitative**. E inductively supports H *to degree* r.
- Carnap begins by clarifying the *explicandum* (the confirmation concept) in various ways, including:

  Qualitative. ($\star$) E gives some (positive) evidence for H.
- Note two things. First, ($\star$) sounds *epistemic* (not *logical*). Second, ($\star$) sounds like it involves (positive) *relevance*.
- Strangely, Carnap proceeds (in LFP_1) to offer a *logical* account of confirmation that does *not* involve relevance.
- These were the two original sins of Bayesian confirmation…

- In the 1st ed. of LFP, Carnap characterizes "the degree to which E confirms H" as $\mathfrak{c}(H,E) = \Pr(H \mid E)$, which leads to:

  Quantitative. $\Pr(H \mid E) = r$.

  Comparative. $\Pr(H \mid E) > \Pr(H' \mid E')$.

  Qualitative. $\Pr(H \mid E) > t$ (typically, with "threshold" $t > \frac{1}{2}$).
  - Doesn't sound like ($\star$). More on this dissonance below.
- Like Hempel, Carnap wanted a *logical* explication of confirmation (as a relation between sentences in $\mathcal{L}$s).
- For Carnap, this meant that the probability functions used in confirmation theory must *themselves* be "logical".
- This leads naturally to the Carnapian project of providing a "logical explication" of conditional probability $\Pr(\cdot \mid \cdot)$ *itself*.
- Here, Carnap (like Nicod) was influenced by Keynes [31], who believed there were "partial entailments". I'm skeptical (as are most modern Bayesians). See my [18] for discussion.
- Hempel's theory of confirmation satisfies the following:

(SCC) If E confirms H, then E confirms all consequences of H.

- In LFP_1, Carnap describes a counterexample to Hempel's (SCC), which presupposes a more ($\star$)-like **qualitative** conception of confirmation. There, he presupposes:

  Qualitative. E confirms H iff $\Pr(H \mid E) > \Pr(H)$.
- This *probabilistic relevance* conception *violates* (SCC), whereas the previous Pr-threshold conception *implies* (SCC).
- Popper [36] notes this tension in LFP. Largely in response to Popper, Carnap wrote a second edition of LFP [4], which includes a preface acknowledging an "*ambiguity*" in LFP_1:
  - **Firmness**. The degree to which E confirms$_f$ H:

    $$\mathfrak{c}_f(H,E) = \Pr(H \mid E).$$
  - **Increase in Firmness**. The degree to which E confirms$_i$ H:

    $$\mathfrak{c}_i(H,E) = \mathfrak{f}[\Pr(H \mid E), \Pr(H)]$$

  $\mathfrak{f}$ measures "the degree to which E *increases* the Pr of H."
- The 1st ed. of LFP was mainly about firmness, and the 2nd edition only adds the preface, which says very little about $\mathfrak{c}_i$. Specifically, no function $\mathfrak{f}$ is rigorously defended there.

- $\mathfrak{c}_i$ is more similar to (∗) than $\mathfrak{c}_f$ is. To see this, note that we can have $\Pr(H \mid E) > r$ *even if E **lowers** the probability of H*.
- Example: Let H be the hypothesis that John does *not* have HIV, and let E be a *positive* test result for HIV from a highly reliable test. Plausibly, in such cases, we could have both:
  - $\Pr(H \mid E) > t$, for just about any threshold value t, but
  - $\Pr(H \mid E) < \Pr(H)$, since E *lowers* the probability of H.
- So, if we adopt Carnap's $\mathfrak{c}_f$-explication, then we must say that E confirms H in such cases. But, in (∗)-terms, this implies E provides some *positive evidential support for H*!
- I take it we don't want to say *that*. Intuitively, what we want to say here is that, while H is (still) *highly probable given E*, (nonetheless) E provides (strong!) evidence ***against*** H.
- Rather than *ambiguity*, I'd say this reflects *confusion* about the nature of the concept (∗) Carnap was trying to explicate.
- Carnap [4] concedes that $\mathfrak{c}_i$ is "more interesting" than $\mathfrak{c}_f$.
- Contemporary Bayesians would agree with this. They've since embraced a probabilistic relevance conception [38].

(EQC) If E confirms H, then E confirms anything equivalent to H.
(EC) If E entails H, then E confirms H.
(CC) If E confirms both H and H', then H and H' are consistent.
(M) If E confirms H, then any E' stronger than E confirms H.
(SCC) If E confirms H, then E confirms any H' weaker than H.
(CCC) If E confirms H, then E confirms any H' stronger than H.

	EQC	EC	CC	M	SCC	CCC
Firmness	Yes	Yes[3]	Yes[4]	No	Yes	No
Increase in Firmness	Yes	Yes[5]	No	No	No	No

- Four counterexamples for increase in firmness:

(CC) E = card is black, H = card is A♠, H' = card is J♣.
(M) E = card is black, H = card is A♠, E' = card is J♣.
(SCC) E = card is black, H = card is A♠, and H' = card is *some* ace.
(CCC) E = card is A♠, H = card is *some* ace, and H' = card is A♦.

[3] Provided that $\Pr(E) \neq 0$.
[4] Provided that the "threshold" value $t > \frac{1}{2}$.
[5] Provided that $\Pr(H) \in (0,1)$, and $\Pr(E) \in (0,1)$.

- *Many* candidate functions $\mathfrak{f}$ satisfy the *relevance* constraint:

($\mathcal{R}$) $\mathfrak{f}[\Pr(H \mid E), \Pr(H)] \gtrless 0$ iff $\Pr(H \mid E) \gtrless \Pr(H)$

- I'll say much more about the plethora of Pr-relevance measures, below. But, for now, back to *Carnapian* $\mathfrak{c}_i$.
- From an inductive-logical point of view, confirmation measures should *quantitatively generalize* entailment:

($\mathcal{D}$) Provided that both E and H are *contingent* claims $\mathfrak{c}_i(H,E)$ should be *maximal* if $E \vDash H$, and *minimal* if $E \vDash {\sim}H$. [Note: $\Pr(H \mid E)$ satisfies *this*, but *not* $\mathcal{R}$.]

- Kemeny & Oppenheim [30] used this consideration (and others) to argue that the best explication of $\mathfrak{c}_i(H,E)$ is:

$$F(H,E) = \frac{\Pr(E \mid H) - \Pr(E \mid {\sim}H)}{\Pr(E \mid H) + \Pr(E \mid {\sim}H)}$$

- F can be expressed as a function $\mathfrak{f}$ of $\Pr(H \mid E)$ and $\Pr(H)$, and it satisfies $\mathcal{R}$, $\mathcal{D}$, and various other IL desiderata.
- One can use F to define **comparative** $[F(H,E) > F(H',E')]$ and **qualitative** $[F(H,E) > 0]$ confirmation$_i$ concepts.

- Bayesianism assumes that the *epistemic* degrees of belief (that is, the *credences*) of rational agents are *probabilities*.
- Let $\Pr(H)$ be the degree of belief that a rational agent a assigns to H at some time t (call this a's "prior" for H).
- Let $\Pr(H \mid E)$ be the degree of belief that a would assign to H (just after t) were a to learn E at t (a's "posterior" for H).
- Toy Example: Let H be the proposition that a card sampled from some deck is a ♠, and E assert that the card is black.
- Making the standard assumptions about sampling from 52-card decks, $\Pr(H) = \frac{1}{4}$ and $\Pr(H \mid E) = \frac{1}{2}$. So, (learning that) E (or supposing that E) *raises the probability of H*.
- Following Popper [36], Bayesians define confirmation in a way that is *formally* very similar to Carnap's $\mathfrak{c}_i$-explication.
- For Bayesians, E confirms H for an agent a at a time t iff $\Pr(H \mid E) > \Pr(H)$, where Pr captures a's credences at t.
- While this is *formally* very similar to Carnap's $\mathfrak{c}_i$, it uses credences as opposed to "logical" probabilities [38], [18].

- There are *many logically equivalent* (but *syntactically* distinct) ways of saying E confirms H, in the Bayesian sense.
- Here are the three most common ways:
  - E confirms H iff $\Pr(H \mid E) > \Pr(H)$. $[\frac{1}{2} > \frac{1}{4}]$
  - E confirms H iff $\Pr(E \mid H) > \Pr(E \mid {\sim}H)$. $[1 > \frac{1}{3}]$
  - E confirms H iff $\Pr(H \mid E) > \Pr(H \mid {\sim}E)$. $[\frac{1}{2} > 0]$
- By taking differences or ratios of the left/right sides of such inequalities, various confirmation *measures* $\mathfrak{c}(H, E)$ emerge.
- A plethora of such confirmation measures have been used in the literature of Bayesian confirmation theory. See my thesis [12] for a survey. Here are the four most popular $\mathfrak{c}$'s:
  - $d(H,E) \overset{\text{def}}{=} \Pr(H \mid E) - \Pr(H)$
  - $r(H,E) \overset{\text{def}}{=} \log\left[\dfrac{\Pr(H \mid E)}{\Pr(H)}\right] \doteq \dfrac{\Pr(H \mid E) - \Pr(H)}{\Pr(H \mid E) + \Pr(H)}$
  - $l(H,E) \overset{\text{def}}{=} \log\left[\dfrac{\Pr(E \mid H)}{\Pr(E \mid {\sim}H)}\right] \doteq \dfrac{\Pr(E \mid H) - \Pr(E \mid {\sim}H)}{\Pr(E \mid H) + \Pr(E \mid {\sim}H)} = F(H,E)$
  - $s(H,E) \overset{\text{def}}{=} \Pr(H \mid E) - \Pr(H \mid {\sim}E)$

- Question: do these (and other) measures disagree only *conventionally*, or do they disagree in substantive ways?
- Note: mere *numerical* differences between measures are not important, since they need not affect *ordinal* judgments of what is more/less well confirmed than what (by what).
- If two measures $\mathfrak{c}_1$ and $\mathfrak{c}_2$ agree on *all comparisons*, then we say that $\mathfrak{c}_1$ and $\mathfrak{c}_2$ are *ordinally equivalent* ($\mathfrak{c}_1 \doteq \mathfrak{c}_2$). That is:

$$\mathfrak{c}_1 \doteq \mathfrak{c}_2 \overset{\text{def}}{=} \mathfrak{c}_1(H,E) \geq \mathfrak{c}_1(H',E') \text{ iff } \mathfrak{c}_2(H,E) \geq \mathfrak{c}_2(H',E')$$

- **Fact**. *No two* of $\{d, r, l, s\}$ are ordinally equivalent.
- OK, but do they disagree on *important* applications or in *important* cases? Unfortunately, they disagree *radically*.
- **Fact**. *Almost every* argument/application of Bayesian confirmation in the literature is valid for *only some* choices of d, r, l, s. I call this *the problem of measure sensitivity*.
- Note: things only get *worse* if you consider still *other* relevance measures (and there are many others out there).

- Here is an incomplete list of examples of the problem of measure-sensitivity. I'm happy to discuss any of these.
  - Hempel's Ravens Paradox [22], [17]
  - Goodman's "Grue" Paradox [8], [39], [20]
  - The Tacking Problem [37], [7], [14], [21]
  - The Confirmational Value of Evidential Variety [25], [13], [1]
  - The Old Evidence Problem [5], [29], [9], [15]
  - The Likelihood Principle/Law [33], [19], [40]
  - The Monty Hall Problem [2]
  - The Virtue of Unification [34], [32]
  - Earman's Old Evidence *Critique* of Bayesianism [7], [16]
  - Gillies's Popper-Miller *Critique* of Bayesianism [23]
- See [11] and [12] for further examples and discussion.
- We need some *normative principles* to narrow the field …

- Consider the following two propositions concerning a card c, drawn at random from a standard deck of playing cards:

  E: c is the ace of spades. H: c is *some* spade.
- I take it as intuitively clear and uncontroversial that:
  - The degree to which E confirms H $\neq$ the degree to which H confirms E, since $E \vDash H$, but $H \nvDash E$. $[\mathfrak{c}(H,E) \neq \mathfrak{c}(E,H)]$
  - The degree to which E confirms H $\neq$ the degree to which ${\sim}E$ *dis*confirms H, since $E \vDash H$, ${\sim}E \nvDash {\sim}H$. $[\mathfrak{c}(H,E) \neq -\mathfrak{c}(H,{\sim}E)]$
- Therefore, *no adequate measure of confirmation* $\mathfrak{c}$ *should be such that either* $\mathfrak{c}(H,E) = \mathfrak{c}(E,H)$ *or* $\mathfrak{c}(H,E) = -\mathfrak{c}(H,{\sim}E)$ for all E and H and for all probability functions Pr. I'll call these two symmetry desiderata S_1 and S_2, respectively.
- Note: for all H, E, and for all Pr, $r(H,E) = r(E,H)$ and $s(H,E) = -s(H,{\sim}E)$. That is, r violates S_1 and s violates S_2.
- *Both* d and l satisfy these S-desiderata. This narrows the field to d and l [10]. We can narrow the field further still …

- If we think of inductive logic as a *quantitative generalization* of deductive logic, then the following *logical* desideratum seems natural (it's also implicit in the previous example):
  - (†) **Quantitative Rendition.** $\mathfrak{c}(H, E)$ should be *maximal* when $E \vDash H$ and $\mathfrak{c}(H, E)$ should be *minimal* when $E \vDash \sim H$.
  - (†) **Comparative Rendition.** If $E \vDash H$ but $E' \nvDash H'$, then the following inequality should hold: $\mathfrak{c}(H, E) \geq \mathfrak{c}(H', E')$.
- The measure d violates these desiderata. For, when $E \vDash H$:

$$d(H, E) = \Pr(H \mid E) - \Pr(H) = 1 - \Pr(H) = \Pr(\sim H)$$

- So, if the prior probability of H is sufficiently high, then (according to d) E will confirm H *very weakly, even if* $E \vDash H$.
- From an inductive-logical point of view, this is absurd, since *the logical strength of a valid argument should not depend on how probable its conclusion is* (or on its truth-value).
- Indeed, of all the Bayesian measures of confirmation that have been used in the literature (*so far* [6]!), only l (or its ordinal equivalents) satisfy our three desiderata: S_1, S_2, (†).

- Seven properties of $\mathfrak{c}(H, E)$, for contingent H, E, H', E', X:

(1) If $E \vDash H$ and $E \nvDash H'$, then $\mathfrak{c}(H, E) \geq \mathfrak{c}(H', E)$. [19]

(2) If $\Pr(E \mid H) > \Pr(E \mid H')$, then $\mathfrak{c}(H, E) > \mathfrak{c}(H', E)$. [19]

(3) If $\Pr(H \mid E) > \Pr(H \mid E')$, then $\mathfrak{c}(H, E) > \mathfrak{c}(H, E')$. [17]

(4) $\mathfrak{c}(H, E) = \mathfrak{c}(E, H)$. [10]

(5) $\mathfrak{c}(H, E) = -\mathfrak{c}(H, \sim E)$. [10]

(6) $\mathfrak{c}(H, E) = -\mathfrak{c}(\sim H, E)$. [10]

(7) If $H \vDash E$, then $\mathfrak{c}(H, E) > \mathfrak{c}(H \& X, E)$, for any X. [14]

	Does Measure have property?						
Relevance Measure	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$d(H, E)$	No	No	Yes	No	No	Yes	Yes
$r(H, E)$	No	Yes	Yes	Yes	No	No	No
$l(H, E)$	Yes	No	Yes	No	No	Yes	Yes
$s(H, E)$	No	No	No	No	Yes	Yes	Yes

[1] L. Bovens and S. Hartmann, *Bayesian networks and the problem of unreliable instruments*, Philosophy of Science, 2002.

[2] D. Bradley and B. Fitelson, *Monty hall, doomsday, and confirmation*, *Analysis*, 2003. URL: `http://fitelson.org/monty.pdf`.

[3] R. Carnap, *Logical foundations of probability*, University of Chicago, 1950.

[4] ———, *Logical foundations of probability* (2nd ed), University of Chicago, 1962.

[5] D. Christensen, *Measuring confirmation*, Journal of Philosophy, 1999.

[6] V. Crupi and K. Tentori and M. Gonzalez, *On Bayesian theories of evidential support: Normative and descriptive considerations*, manuscript, 2006.

[7] J. Earman, *Bayes or bust?*, MIT, 1992.

[8] E. Eells, *Rational decision and causality*, Cambridge, 1982.

[9] E. Eells and B. Fitelson, *Measuring confirmation and evidence*, Journal of Philosophy, 2001. URL: `http://fitelson.org/dccfin.pdf`.

[10] ———, *Symmetries and asymmetries in evidential support*, Philosophical Studies, 2002. URL: `http://fitelson.org/symmetry.pdf`.

[11] B. Fitelson, *The plurality of Bayesian measures of confirmation and the problem of measure sensitivity*, Philosophy of Science, 1999. URL: `http://fitelson.org/psa.pdf`.

[12] ———, *Studies in Bayesian confirmation theory*, Ph.D. thesis, University of Wisconsin–Madison, 2001. URL: `http://fitelson.org/thesis.pdf`.

[13] ———, *A Bayesian account of independent evidence with applications*, Philosophy of Science, 2001. URL: `http://fitelson.org/psa2.pdf`.

[14] ———, *Putting the irrelevance back into the problem of irrelevant conjunction, Philosophy of Science*, 2002. URL: `http://fitelson.org/ic.pdf`.

[15] ———, *Review of [29], Mind*, 2003. URL: `http://fitelson.org/joyce.pdf`.

[16] ———, *Earman on Old Evidence and Measures of Confirmation*, 2004. URL: `http://fitelson.org/oe.pdf`.

[17] ———, *The paradox of confirmation*, Philosophy Compass (B. Weatherson and C. Callender, *eds.*), Blackwell (online publication), 2005. URL: `http://fitelson.org/ravens.htm`.

[18] ———, *Confirmation Theory As a Branch of Inductive Logic: Some Historical and Philosophical Reflections*, APA Eastern lecture, 2005. URL: `http://fitelson.org/apa2005_script.pdf`.

[19] ———, *Likelihoodism, Bayesianism, and relational confirmation*, forthcoming in *Synthese*, 2006. URL: `http://fitelson.org/synthese.pdf`.

[20] ———, *Goodman's 'New Riddle'*, 2006. URL: `http://fitelson.org/grue.pdf`.

[21] B. Fitelson and J. Hawthorne, *Re-solving irrelevant conjunction with probabilistic independence*, Philosophy of Science, 2005. URL: `http://fitelson.org/ic2.pdf`.

[22] ———, *How Bayesian confirmation theory handles the paradox of the ravens*, forthcoming in *Probability in Science*, 2006. URL: `http://fitelson.org/ravens.pdf`.

[23] D. Gillies, *In defense of the Popper-Miller argument*, Philosophy of Science, 1986.

[24] C. Hempel, *Studies in the logic of confirmation*, Mind, 1945.

[25] P. Horwich, *Probability and evidence*, Cambridge, 1982.

[26] C. Howson and P. Urbach, *Scientific reasoning: The Bayesian approach*, Open Court, 1993.

[27] R. Jeffrey, *Probability and the art of judgment*, Cambridge, 1992.

[28] W.E. Johnson, *Logic*, Cambridge University Press, 1921.

[29] J. Joyce, *The foundations of causal decision theory*, Cambridge, 1999.

[30] J. Kemeny and P. Oppenheim, *Degrees of factual support*, Philosophy of Science, 1952.

[31] J. Keynes, *A treatise on probability*, Macmillan, London, 1921.

[32] Lange. M, *Bayesianism and unification: A reply to W. Myrvold*, Philosophy of Science, 2003.

[33] P. Milne, $\log[p(h/eb)/p(h/b)]$ *is the one true measure of confirmation*, Philosophy of Sci., 1996.

[34] W. Myrvold, *A Bayesian account of the virtue of unification*, Philosophy of Science, 2003.

[35] J. Nicod, *The Logical Problem of Induction*, (1923) in Geometry and induction, University of California Press, Berkeley, California, 1970.

[36] K. Popper, *Degree of confirmation*, British Journal of Philosophy of Science, 1954.

[37] R. Rosenkrantz, *Bayesian confirmation: Paradise regained*, British Journal of Phil. of Sci., 1994.

[38] W. Salmon, *Confirmation and relevance*, in Induction, Probability, and Confirmation: Minnesota Studies in the Philosophy of Science, vol. 6., Maxwell and Anderson *eds.*, 1975.

[39] E. Sober, *No model, no inference: A bayesian primer on the grue problem*, in Grue! The New Riddle of Induction (D. Stalker, ed.), Open Court, 1994.

[40] D. Steel, *A Bayesian way to make stopping rules matter*, Erkenntnis, 2003.