simplest cases will be the subject of the second lecture; the following lectures will be devoted to the most delicate question of this study: that of understanding the subjectivistic explanation of the use we make of the results of observation, of past experience, in our predictions of the future.

This point of view is only one of the possible points of view, but I would not be completely honest if I did not add that it is the only one that is not in conflict with the logical demands of my mind. If I do not wish to conclude from this that it is "true", it is because I know very well that, as paradoxical as it seems, nothing is more subjective and personal than this "instinct of that which is logical" which each mathematician has, when it comes to the matter of applying it to questions of principle.

## CHAPTER I
## *The Logic of the Probable*

Let us consider the notion of probability as it is conceived by all of us in everyday life. Let us consider a well-defined event and suppose that we do not know in advance whether it will occur or not; the doubt about its occurrence to which we are subject lends itself to comparison, and, consequently, to gradation. If we acknowledge only, first, that one uncertain event can only appear to us (a) equally probable, (b) more probable, or (c) less probable than another; second, that an uncertain event always seems to us more probable than an impossible event and less probable than a necessary event; and finally, third, that when we judge an event $E'$ more probable than an event $E$, which is itself judged more probable than an event $E''$, the event $E'$ can only appear more probable than $E''$ (transitive property), it will suffice to add to these three evidently trivial axioms a fourth, itself of a purely qualitative nature, in order to construct rigorously the whole theory of probability. This fourth axiom tells us that inequalities are preserved in logical sums: if $E$ is incompatible with $E_1$ and with $E_2$, then $E_1 \vee E$ will be more or less probable than $E_2 \vee E$, or they will be equally probable, according to whether $E_1$ is more or less probable than $E_2$, or they are equally probable. More generally, it may be deduced from this[2] that

(2) See [34], p. 321, note 1.



two inequalities, such as

$$E_1 \text{ is more probable than } E_2,$$
$$E_1' \text{ is more probable than } E_2',$$

can be added to give

$$E_1 \vee E_1' \text{ is more probable than } E_2 \vee E_2',$$

provided that the events added are incompatible with each other ($E_1$ with $E_1'$, $E_2$ with $E_2'$). It can then be shown that when we have events for which we know a subdivision into possible cases that we judge to be equally probable, the comparison between their probabilities can be reduced to the purely arithmetic comparison of the ratio between the number of favorable cases and the number of possible cases (not because the judgment then has an objective value, but because everything substantial and thus subjective is already included in the judgment that the cases constituting the division are equally probable). This ratio can then be chosen as the appropriate index to measure a probability, and applied in general, even in cases other than those in which one can effectively employ the criterion that governs us there. In these other cases one can evaluate this index by comparison: it will be in fact a number, uniquely determined, such that to numbers greater or less than that number will correspond events respectively more probable or less probable than the event considered. Thus, while starting out from a purely qualitative system of axioms, one arrives at a quantitative measure of probability, and then at the theorem of total probability which permits the construction of the whole calculus of probabilities (for conditional probabilities, however, it is necessary to introduce a fifth axiom: see note 8, p. 109).

One can, however, also give a direct, quantitative, numerical definition of the degree of probability attributed by a given individual to a given event, in such a fashion that the whole theory of probability can be deduced immediately from a very natural condition having an obvious meaning. It is a question simply of making mathematically precise the trivial and obvious idea that the degree of probability attributed by an individual to a given event is revealed by the conditions under which he would be disposed to bet on that event.[3] The axiomatization whose general outline we have

(3) Bertrand ([1], p. 24) beginning with this observation, gave several examples of subjective probabilities, but only for the purpose of contrasting them with "objective probabilities". The subjectivistic theory has been developed according to the scheme of bets in the exposition (Chap. I and II) in my first paper of 1928 on this subject. This was not published in its original form, but was summarized or partially developed in [27], [34], [35], etc.