Symmetries and Asymmetries in Evidential Support

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Abstract. Several forms of symmetry in degrees of evidential support are considered. Some of these symmetries are shown not to hold in general. This has implications for the adequacy of many measures of degree of evidential support that have been proposed and defended in the philosophical literature.

Keywords: symmetry, evidence, confirmation, support, Bayesian, probability

1. Introduction

A plethora of non-equivalent measures of evidential support (or confirmation)[1] have been proposed and defended in the philosophical literature.[2] This plurality of measures of support is problematic, since it affects a great many arguments surrounding Bayesian confirmation theory (and inductive logic, generally).[3] Unfortunately, only a few arguments have been proposed which can serve to significantly narrow the field of alternative measures of support.[4] Typically, these arguments are relatively complex, and they tend to involve rather sophisticated premises and presuppositions.[5] In the present paper, we will show that the field of competing measures of inductive support can be drastically narrowed by appealing to just a few simple, intuitive considerations of symmetry. In particular, we will show that, simply by thinking about the following three relatively easy questions, we can eliminate all but a very small number of the competing measures of support.

– Does a piece of evidence E support a hypothesis H equally well as E's negation ($\neg E$) undermines, or countersupports, the same hypothesis H?

[1] See Eells (1985) and Eells and Fitelson (2000) on terminology: "evidence" vs. "confirmation". We suppose here that evidence E is not already known so that this terminology issue and "the problem of old evidence" are not issues here, and we will use "confirmation" and "evidence" interchangeably.

[2] See Kyburg (1983) for a survey of the many measures of inductive support (or confirmation) that have been proposed and defended over the years.

[3] See Festa (1999) and Fitelson (1999, 2001b) for discussions of the ramifications of the plurality of Bayesian measures of (incremental) confirmation.

[4] See, for instance, Carnap (1962, §67), Fitelson (2001a, 2001b), Good (1984), Heckerman (1988), Kemeny and Oppenheim (1952), and Milne (1996).

[5] See Fitelson (2001b) for a critical discussion of several of these arguments.

– Does a piece of evidence E support a hypothesis H equally well as H supports E?

– Does a piece of evidence E support a hypothesis H equally well as E undermines, or countersupports, the negation of H ($\neg H$)?

These are, of course, three quite different questions, concerning different kinds of possible symmetry of evidential support. Presently, we show that the first two of these three kinds of symmetry do not hold in general. This will have implications for the adequacy of many measures of confirmation (or evidential support) that have been proposed in the recent and not so recent literature on inductive logic. We will first, in section 2, define some of these measures of evidential support and, in terms of the idea of such quantitative measures, formulate the symmetry theses involved in the questions with which we began. In section 3, we will show — by way of some intuitive counterexamples — that the answers to the first two questions raised above are (clearly) both "No." The discussion in this section will not appeal to any of the measures of evidential support defined in the previous section, nor (at least this is our intent) the intuitions that lie behind them, the intent being to evaluate the symmetry theses independently of any such measure. In section 4, we will discuss the third kind of symmetry mentioned above. We will provide some (less than definitive) reasons for thinking that the answer to our third question is "Yes." Finally, in section 5 (and in the appendix), we detail the implications of these conclusions for the various standard measures of evidential support.

2. The Measures and the Symmetry Theses

Measures of evidential support (that are our topic) are supposed to quantify the degree to which a piece of evidence E provides, intuitively speaking, "evidence for or against" or "support for or against" a hypothesis H — in an incremental as opposed to a final or absolute way. They are supposed to capture what would be the impact of, rather than the final result of, the "absorption" of a piece of evidence. (We will sometimes use the terms "confirmation" and "evidential support" generically, to include disconfirmation and evidential irrelevance; and for a given evidence/hypothesis pair, a measure's value's being positive, negative, or 0 is supposed to correspond specifically to confirmation, disconfirmation, and evidential irrelevance, respectively.) Here we define and label the pertinent measures of evidential support (with comments and some relevant references given in footnotes, and where "Pr" denotes probability, on some appropriate interpretation, usually a "subjective," or "logical" interpretation):

$$d(H, E) =_{df} \Pr(H \mid E) - \Pr(H).$$[6]

$$s(H, E) =_{df} \Pr(H \mid E) - \Pr(H \mid \neg E).$$[7]

$$r(H, E) =_{df} \log\left[\frac{\Pr(H \mid E)}{\Pr(H)}\right].$$[8]

$$\mathfrak{r}(H, E) =_{df} \Pr(H \,\&\, E) - \Pr(H) \cdot \Pr(E).$$[9]

$$l(H, E) =_{df} \log\left[\frac{\Pr(E \mid H)}{\Pr(E \mid \neg H)}\right].$$[10]

Where "c" stands for a measure of support (*e.g.*, d or s or r, *etc.*, as above), the first symmetry thesis described at the outset can be formulated as follows, where "ES" stands for "Evidence Symmetry."[11]

$$c(H, E) = -c(H, \neg E) \tag{ES}$$

The second symmetry thesis, which we call "Commutativity Symmetry" can be formulated as:

$$c(H, E) = c(E, H) \tag{CS}$$

[6] Among those who have used or defended the difference measure d are Earman (1992), Eells (1982), Gillies (1986) (see also Chihara and Gillies (1988), where Gillies tries to defend his use of d), Jeffrey (1992), and Rosenkrantz (1994).

[7] The name "s" is borrowed from Christensen (1999), where the measure s is applied to "the problem of old evidence." See also Joyce (1999).

[8] We use the logarithm of the ratio $\Pr(H \mid E)/\Pr(H)$ to ensure that the ratio measure r is $+/-/0$ if and only if E confirms/disconfirms/is confirmationally irrelevant to H. Since logarithms are strictly monotonic increasing functions, this will not change the ordinal structure imposed by r. Among those who have used or defended r (or measures ordinally equivalent to r) are Horwich (1982), Keynes (1921), Mackie (1969), Milne (1995,1996), Schlesinger (1995), and Pollard (1999).

[9] The relevance measure $\mathfrak{r}$ is introduced by Carnap (1962, §67). It is unclear whether Carnap intended to argue that $\mathfrak{r}$ was a superior measure of *the degree to which E (incrementally) confirms H*. It seems that Carnap (1962, §67) was mainly using $\mathfrak{r}$ for the purpose of establishing certain *qualitative* results concerning probabilistic relevance. While Carnap (1962, page 361) does suggest that he prefers $\mathfrak{r}$ over d and r, he seems to be thinking of $\mathfrak{r}$ as a measure of the *mutual dependence* between E and H. For *this* purpose, the kinds of symmetries exhibited by $\mathfrak{r}$ (which make $\mathfrak{r}$ a *poor* measure of *confirmation*) may be desirable. See appendix §D below.

[10] As with r, we use the logarithm of the likelihood ratio $\Pr(E \mid H)/\Pr(E \mid \neg H)$ to ensure that the measure l is $+/-/0$ if and only if E confirms/disconfirms/is confirmationally irrelevant to H. Among those who have used or defended l (or measures ordinally equivalent to l) are Kemeny and Oppenheim (1952), Good (1984), Heckerman (1988), Pearl (1988), Schum (1994), and Fitelson (2001a, 2001b).

[11] Each of these symmetry conditions is (implicitly) *universally quantified*.

Finally, the third symmetry property, which we call "Hypothesis Symmetry," can be formulated as:

$$c(H, E) = -c(\neg H, E) \tag{HS}$$

Of course, the difference between (ES) and (HS) is just in the placement of the logical negation symbol (in front of evidence E or in front of the hypothesis H, respectively, on the right hand sides). (ES) says that a piece of evidence E would confirm (or disconfirm) a hypothesis to the same degree that E's negation would disconfirm (or confirm) the same hypothesis, while (HS) says that evidence E would confirm (or disconfirm) H equally well as the same evidence would disconfirm (or confirm) the negation of H. It is easy to show that (ES) and (HS) are equivalent, given the assumption of (CS).[12]

Just to round out the list of what may initially seem to be natural symmetry theses for measures of evidential support, we list here (what we call) Total Symmetry:

$$c(H, E) = c(\neg H, \neg E) \tag{TS}$$

It is easy to see that (TS) follows from the conjunction of (ES) and (HS).[13]

3. Definitive Negative Answers to our First Two Questions

Our first two questions can now be expressed as follows: "Do (ES) and (CS) hold in general?" The examples described in this section will show that the (intuitive) answer to both of these questions is "No."[14]

Suppose that after years of research — having carefully examined thousands and thousands of ravens, nonravens, black things, and nonblack things — we have become virtually convinced, but not absolutely certain, of the hypothesis, H^*, that all ravens are black. Suppose also, as is somewhat standard, that positive instances of the hypothesis — that is, black ravens — still confirm, though to a very small degree, the hypothesis, as do nonblack nonravens and black nonravens. Suppose even that, in accordance with Hempelian confirmation theory, the information about a newly found object, not previous tested for color or

[12] *Proof*: Assuming (CS) and (HS), we have: $c(H, E) =_{\text{by (CS)}} c(E, H) =_{\text{by (HS)}} -c(\neg E, H) =_{\text{by (CS)}} -c(H, \neg E)$, showing that (ES) follows from (CS) and (HS). Assuming (CS) and (ES), we have: $c(H, E) =_{\text{by (CS)}} c(E, H) =_{\text{by (ES)}} -c(E, \neg H) =_{\text{by (CS)}} -c(\neg H, E)$, showing that (HS) follows from (CS) and (ES).

[13] *Proof*: $c(H, E) =_{\text{by (HS)}} -c(\neg H, E) =_{\text{by (ES)}} --c(\neg H, \neg E) = c(\neg H, \neg E)$.

[14] Examples like these are described in Eells (2000) for the narrower purpose of comparing two specific measures of confirmation, namely d and s.

ravenhood, that it is not a nonblack raven would confirm H^*, though again of course only to a minute degree.[15] Call this evidence E^*. Since H^* is already so highly confirmed, evidence E^* should be no surprise and would seem to provide little evidence in favor of H^*. However, $\neg E^*$ (the information about that object that it is a nonblack raven) would seem to disconfirm H^* very strongly (and of course conclusively). $\neg E^*$ would be a surprise, and intuitively, to us at least, would seem to provide very strong evidence against H^*, and stronger evidence against H^* than E^* would provide in favor of H^*. That is, we should have: $c(H^*, E^*) \ll |c(H^*, \neg E^*)| = -c(H^*, \neg E^*)$, so that this is, we claim, a counterexample to Evidence Symmetry.

Here is a more quantifiable, and thus perhaps clearer, counterexample to Evidence Symmetry. A card is randomly drawn from a standard deck. Let E^{**} be the evidence that the card is the seven of spades, and let H^{**} be the hypothesis that the card is black. We take it to be intuitively clear that E^{**} is not only conclusive, but also strong, evidence in favor of H^{**}, whereas $\neg E^{**}$ (that the card drawn is not the seven of spades) is close to useless, or close to "informationless," with regard to the color of the card. Again we have an intuitive counterexample to Evidence Symmetry, and we should have: $c(H^{**}, E^{**}) \gg |c(H^{**}, \neg E^{**})| = -c(H^{**}, \neg E^{**})$.

We note that the first counterexample involves conclusive disconfirmation while the second involves conclusive confirmation. This of course is simply due to taking the "evidence" in the first example to be E^* rather than $\neg E^*$, and in the second example to be E^{**} rather than $\neg E^{**}$. We also note that the conclusiveness feature of the examples (that $\neg E^*$ logically implies $\neg H^*$, and E^{**} logically implies H^{**}) is not what is at the heart of the counterexamples. To see this, simply consider a modification of the examples where E^* and E^{**} are reports of color/ravenhood and suit/rank, respectively, of very reliable, but fallible, assistants.

Of course, these examples also tell intuitively against Total Symmetry (*e.g.*, E^{**}, that the card is the seven of spades, is highly informative, confirmatory, and of course also conclusive for the card's being black, H^{**}, while on the other hand the card's not being the seven of spades is nearly silent on whether the card is nonblack).

[15] We do not mean to endorse Hempelian confirmation theory, the positive instance criterion, or any of the other standard assumptions underlying traditional discussions of the ravens paradox. We are simply using the ravens example to illustrate why (ES) is unintuitive. If the reader is uncomfortable with the Hempelian lore in this example, they may prefer the example below which does not appeal to anything of the kind.

Moreover, the very same examples can be used to show that Commutativity Symmetry (CS) is not generally true either. It seems clear from these examples that a piece of evidence E can confirm a hypothesis H to a much different degree than H confirms E. Consider for example whether the observation that a card is the seven of spades confirms the proposition that the card is black equally well as the proposition that the card is black confirms the proposition that the card is the seven of spades. With initial uncertainty about the value of the card, we consider the seven of spades, as evidence, to be more highly informative and confirmatory of the blackness of the card, as hypothesis, than the blackness of the card, as evidence, is for the card's being the seven of spades in particular. Other examples like this (against both (CS) and (ES)) can easily be multiplied, where X logically implies (or just confers probability 1 on) Y but not vice versa, though again the extremeness of logical implication of (or conferring probability 1 on) Y is not what is crucial to the examples for the purposes of evaluating (CS) (or (ES)). So, the examples in this section undergird firm, negative answers to both of our first two questions, about (ES) and (CS).

4. Toward an Affirmative Answer to our Third Question

Turning now to Hypothesis Symmetry, we cannot, of course, offer an argument for the thesis as concrete as the counterexamples offered against Evidence Symmetry and Commutativity Symmetry. We note first that we are comparing the evidential significance of E for H to the evidential significance of E for the negation of H, where H and $\neg H$ are mutually exclusive and collectively exhaustive competitors. So it is of course natural at least that the significance of E for H should be of the opposite sign $(+/-)$ as the significance of E for $\neg H$: when there are exactly two such competitors, it is natural to think of E's confirming or disconfirming one of them only "at the (positive or negative) expense of" its single competitor.[16] As to the magnitudes of degrees of support and countersupport, it would seem that, intuitively speaking (which, again, is the rule of this section), there is "only so much credence to pass around" — and a constant amount to be divided between exclusive and exhaustive hypotheses — so that whatever enhancement of credence one of the two such alternatives enjoys (from a particular piece of evidence E) should be exactly the amount that is taken away from the

[16] We are not the first to have intuitions that accord with (HS). Kemeny and Oppenheim (1952, page 309) impose (HS) as one of their (twelve) conditions of adequacy for measures of inductive support. Their reasons for requiring (HS) are quite similar to (and, we think, no more or less definitive than) ours.

other (where else could it go, or be taken from?).[17] While we do not claim to have a definitive argument in favor of Hypothesis Symmetry, we do find (HS) appealing, and we have not been able to think of any intuitive counterexamples to it.[18]

5. Narrowing the Field

The table below summarizes verdicts concerning each of the five measures of evidential support defined in section 2. Each cell contains two answers: the answer on the left says whether (as a matter of mathematical/logical fact) the relevant (row) measure satisfies the relevant (column) symmetry thesis, and the answer on the right says whether, based on the considerations advanced above, we think the answer on the left is "good news" (☺) or "bad news" (☹) for the relevant measure.

measure	(ES)	(HS)	(CS)	(TS)
d	no / ☺	yes / ☺	no / ☺	no / ☺
s	yes / ☹	yes / ☺	no / ☺	yes / ☹
r	no / ☺	no / ☹	yes / ☹	no / ☺
$\mathfrak{r}$	yes / ☹	yes / ☺	yes / ☹	yes / ☹
l	no / ☺	yes / ☺	no / ☺	no / ☺

We have already argued for the ☺/☹ answers on the rights, and we have nothing further to add here in support of these verdicts. The yes/no answers on the lefts are verified in the appendix, where we credit others for having previously noted some of these facts (basically, the proofs of the "yes" answers rest on easy probabilistic considerations, and the "no" answers are verified by the cards counterexample described above).

We note that the measures d and l are the only ones with a perfect score (and that the violations of (ES) and (CS) by these measures for the examples of section 3 are inequalities going in the correct directions

[17] We have heard of athletic coaches, and bosses, exhorting people to "give 110%," but this of course is beside the point. Also, we are ignoring here proposals of interval valued credences, on which both the formulation and the evaluation of (HS), as well as of (ES) and (CS), would be different.

[18] If (CS) were generally true, then we could turn our intuitive (ES) counterexamples into (HS) counterexamples. However, as we have seen, both (ES) and (CS) seem to fall prey to the same kinds of intuitive counterexamples.

urged in section 3, as verified in the appendix, A.1, A.3, E.1, and E.3, for the cards example). In particular, s and $\mathfrak{r}$ are ruled-out because of their satisfaction of (ES), and r and $\mathfrak{r}$ are ruled-out because of their satisfaction of (CS).[19]

In closing, we point out that the measures $\Pr(E \mid H) - \Pr(E)$ (Mortimer 1988) and $\Pr(E \mid H) - \Pr(E \mid \neg H)$, (Nozick 1981) are both ruled-out by their satisfaction of (ES), and the measure $\Pr(E \mid H)/\Pr(E)$ (Kuipers 2000) is ruled-out by its satisfaction of (CS) (proofs omitted). We suspect other measures of evidential support appearing in the literature will also be affected by the present symmetry considerations.

Appendix

There are twenty (5×4) theorems implicit in the table above. We address them in five parts below, one for each measure of confirmation, each part establishing four theorems.

A. Theorems pertaining to the difference measure d

A.1. d violates Evidence Symmetry (ES)

The cards example suffices to show this. In the cards example, we have (suppressing the **'s here and throughout the appendix) the following four atomic probabilities: $\Pr(H \;\&\; \neg E) = 25/52$, $\Pr(H \;\&\; E) = 1/52$, $\Pr(\neg H \& E) = 0$, $\Pr(\neg H \& \neg E) = 1/2$. Therefore, in the cards example, we have: $d(H, E) = 1/2 \gg 1/102 = -d(H, \neg E)$.

A.2. d satisfies Hypothesis Symmetry (HS)

$$\begin{aligned} \textit{Proof}: d(\neg H, E) &= (1 - \Pr(H \mid E)) - (1 - \Pr(H)) \\ &= -[\Pr(H \mid E) - \Pr(H)] \\ &= -d(H, E). \end{aligned}$$

A.3. d violates Commutativity Symmetry (CS)

In the cards example, we have: $d(H, E) = 1/2 \gg 1/52 = d(E, H)$.

[19] Although we have given ☹'s to measures that violate (HS) (and ☺'s to those that satisfy it), we wish to stress that *we do not need* (HS) *to rule-out s, r, or $\mathfrak{r}$.* As we explained above, we do not claim to have a knock-down argument in favor of (HS). But, so long as there is no knock-down argument *against* (HS), d and l will remain the only measures which pass through the present "symmetry filter."

A.4. d VIOLATES TOTAL SYMMETRY (TS)

In the cards example, we have: $d(H, E) = 1/2 \neq 1/102 = d(\neg H, \neg E)$.

B. Theorems pertaining to Christensen's measure s

B.1. s SATISFIES EVIDENCE SYMMETRY (ES)

$$\begin{aligned} \textit{Proof}:\ s(H, E) &= \Pr(H \mid E) - \Pr(H \mid \neg E) \\ &= -[\Pr(H \mid \neg E) - \Pr(H \mid \neg\neg E)] \\ &= -s(H, \neg E). \end{aligned}$$

B.2. s SATISFIES HYPOTHESIS SYMMETRY (HS)

$$\begin{aligned} \textit{Proof}:\ s(\neg H, E) &= (1 - \Pr(H \mid E)) - (1 - \Pr(H \mid \neg E)) \\ &= -[\Pr(H \mid E) - \Pr(H \mid \neg E)] \\ &= -s(H, E). \end{aligned}$$

B.3. s VIOLATES COMMUTATIVITY SYMMETRY (CS)

In the cards example, we have: $s(H, E) = 26/51 \gg 1/26 = s(E, H)$.

B.4. s SATISFIES TOTAL SYMMETRY (TS)

Proof: This is an easy consequence of s's satisfaction of both (ES) and (HS) (see note 13 above).

C. Theorems pertaining to the ratio measure r

C.1. r VIOLATES EVIDENCE SYMMETRY (ES)

In the cards example, $r(H, E) = \log(2) \neq \log(51/50) = -r(H, \neg E)$.

C.2. r VIOLATES HYPOTHESIS SYMMETRY (HS)

In the cards example, $r(H, E) = \log(2) \neq +\infty = -r(\neg H, E)$. Both Good (1987) and Fitelson (1999) mention that r has this undesirable property (in contrast to both d and l).

C.3. r SATISFIES COMMUTATIVITY SYMMETRY (CS)

$$\begin{aligned} \textit{Proof}:\ r(H, E) &= \log[\Pr(H \mid E)/\Pr(H)] \\ &= \log[\Pr(E \mid H)/\Pr(E)] \qquad \text{(by Bayes' Theorem)} \\ &= r(E, H). \end{aligned}$$

C.4. r VIOLATES TOTAL SYMMETRY (TS)

In the cards example, $r(H, E) = \log(2) \neq \log(52/51) = r(\neg H, \neg E)$.

D. Theorems pertaining to Carnap's relevance measure $\mathfrak{r}$

Carnap (1962, §67) proves that his relevance measure $\mathfrak{r}$ obeys all four of the symmetry properties discussed in this paper (he also seems to have been aware of the relevant theorems pertaining to the measures d and r). Carnap thinks this is a virtue of his measure. Apparently, Carnap likes all of this symmetry for two reasons: (1) he's concerned mainly with representing quantitatively a (completely symmetric) *qualitative* relevance relation, and (2) $\mathfrak{r}$ is more "convenient" (mathematically, we suppose) because it exhibits such robust mathematical symmetry. It is worth noting, however, that Carnap (1962, pages *xvi–xvii* and page 361) seems to think that the difference measure d agrees with intuitions in applications to *confirmation*, despite its lack of total and utter mathematical symmetry. Carnap seems to be less sympathetic to the ratio measure r (which he calls the "relevance quotient"), but he does not discuss either of the measures s or l.

E. Theorems pertaining to the likelihood-ratio measure l

E.1. l VIOLATES EVIDENCE SYMMETRY (ES)

In the cards example, we have: $l(H, E) = +\infty \gg \log(26/25) = -l(H, \neg E)$.

E.2. l SATISFIES HYPOTHESIS SYMMETRY (HS)

$$\begin{aligned} \textit{Proof}:\ l(H, E) &= \log[\Pr(E \mid H)/\Pr(E \mid \neg H)] \\ &= -\log[\Pr(E \mid \neg H)/\Pr(E \mid \neg\neg H)] \\ &= -l(\neg H, E). \end{aligned}$$

E.3. l violates Commutativity Symmetry (CS)

In the cards example, we have: $l(H, E) = +\infty \gg \log(51/25) = l(E, H)$.

E.4. l violates violates Total Symmetry (TS)

In the cards example, we have: $l(H, E) = +\infty \neq \log(26/25) = l(\neg H, \neg E)$.

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References

Carnap, R.: 1962, *Logical Foundations of Probability*. Chicago: University of Chicago Press, second edition.

Chihara, C. and D. Gillies: 1988, 'An Interchange on the Popper-Miller argument'. *Philosophical Studies* **54**, 1–8.

Christensen, D.: 1999, 'Measuring Confirmation'. *Journal of Philosophy* **XCVI**, 437–61.

Earman, J.: 1992, *Bayes or Bust: A Critical Examination of Bayesian Confirmation Theory*. Cambridge: MIT Press.

Eells, E.: 1982, *Rational Decision and Causality*. Cambridge: Cambridge University Press.

Eells, E.: 1985, 'Problems of Old Evidence'. *Pacific Philosophical Quarterly* **66**, 283–302.

Eells, E.: 2000, 'Review: *The Foundations of Causal Decision Theory*, by James M. Joyce'. *The British Journal for the Philosophy of Science* **51**, 893–900.

Eells, E. and B. Fitelson: 2000, 'Measuring Confirmation and Evidence'. *Journal of Philosophy* **XCVII**(12), 663–672.

Festa, R.: 1999, 'Bayesian Confirmation'. In: M. Galavotti and A. Pagnini (eds.): *Experience, Reality, and Scientific Explanation*. Dordrecht: Kluwer Academic Publishers, pp. 55–87.

Fitelson, B.: 1999, 'The Plurality of Bayesian Measures of Confirmation and the Problem of Measure Sensitivity'. *Philosophy of Science* **66**, S362–S378.

Fitelson, B.: 2001a, 'A Bayesian Account of Independent Evidence with Applications'. *Philosophy of Science* (to appear).

Fitelson, B.: 2001b, 'Studies in Bayesian Confirmation Theory'. Ph.D. thesis, University of Wisconsin–Madison.

Gillies, D.: 1986, 'In Defense of the Popper-Miller Argument'. *Philosophy of Science* **53**, 110–113.

Good, I.: 1984, 'The Best Explicatum for Weight of Evidence'. *Journal of Statistical Computation and Simulation* **19**, 294–299.

Good, I.: 1987, 'A Reinstatement, in Response to Gillies, of Redhead's Argument in Support of Induction'. *Philosophy of Science* **54**, 470–72.

Heckerman, D.: 1988, 'An Axiomatic Framework for Belief Updates'. In: L. Kanal and J. Lemmer (eds.): *Uncertainty in Artificial Intelligence 2*. New York: Elsevier Science Publishers, pp. 11–22.

Horwich, P.: 1982, *Probability and Evidence*. Cambridge: Cambridge University Press.

Jeffrey, R.: 1992, *Probability and the Art of Judgment*. Cambridge: Cambridge University Press.

Joyce, J.: 1999, *The Foundations of Causal Decision Theory*. Cambridge: Cambridge University Press.

Kemeny, J. and P. Oppenheim: 1952, 'Degrees of Factual Support'. *Philosophy of Science* **19**, 307–324.

Keynes, J.: 1921, *A Treatise on Probability*. London: Macmillan.

Kuipers, T.: 2000, *From Instrumentalism to Constructive Realism*. Dordrecht: Kluwer.

Kyburg, H.: 1983, 'Recent Work in Inductive Logic'. In: T. Machan and K. Lucey (eds.): *Recent Work in Philosophy*. Lanham: Rowman & Allanheld, pp. 87–150.

Mackie, J.: 1969, 'The Relevance Criterion of Confirmation'. *The British Journal for the Philosophy of Science* **20**, 27–40.

Milne, P.: 1995, 'A Bayesian Defence of Popperian Science?'. *Analysis* **55**, 213–215.

Milne, P.: 1996, '$\log[P(h/eb)/P(h/b)]$ is the One True Measure of Confirmation'. *Philosophy of Science* **63**, 21–26.

Mortimer, H.: 1988, *The Logic of Induction*. Paramus: Prentice Hall.

Nozick, R.: 1981, *Philosophical Explanations*. Cambridge: Harvard University Press.

Pearl, J.: 1988, *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*. San Francisco: Morgan Kauffman.

Pollard, S.: 1999, 'Milne's Measure of Confirmation'. *Analysis* **59**, 335–337.

Rosenkrantz, R.: 1994, 'Bayesian Confirmation: Paradise Regained'. *The British Journal for the Philosophy of Science* **45**, 467–476.

Schlesinger, G.: 1995, 'Measuring Degrees of Confirmation'. *Analysis* **55**, 208–212.

Schum, D.: 1994, *The Evidential Foundations of Probabilistic Reasoning*. New York: John Wiley & Sons.