When Obstinacy is a Better (Cognitive) Policy

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Abstract

For epistemic subjects like us, updating our credences incurs *epistemic* costs. Expending our limited processing power and working memory to properly update our credences by some information can come at the cost of not responding to other available information. It is thus desirable to flesh out and compare alternative ways of taking information into account in light of cognitive shortcomings like our own. This paper is a preliminary attempt to do so. I argue that it is better, in a range of "normal" circumstances and from the point of view of expected credal accuracy, for epistemic subjects like us *not to update* on available information that bears on propositions for which substantial evidence has been gathered than it is to update on information as it presents itself. In order to clarify the argument, and enable comparisons between information-response policies more generally, I develop a queue-theoretic model of learning for subjects with cognitive limitations. The model characterizes how policies for responding to information interact with a subject's limitations to yield confidences. Finally, I discuss implications of the discussion for work on confidence, outright belief, and the relationship between those two states. The comparison of information-response policies helps to (i) explain how some of the "biases" recorded in the social psychology literature might be cognitively valuable, (ii) clarify views that take outright belief to be a kind of epistemic plan that resists reconsideration, and (iii) assuage certain "demandingness" worries for the hypothesis that we are credal reasoners.

1 Introduction

Demonstrations to the effect that epistemic subjects should always update on *any* information they come across assume that updating is epistemically cost free for the subjects of interest.[1] But this is not true for epistemic subjects like us, whose capacities are far from being epistemically ideal. For creatures like us, taking available information into account taxes our processing resources and available working memory. Consequently, taking information into account incurs epistemic opportunity costs — when there is enough available information, resources spent taking some subset of our available information into account is time not spent processing other available information. Given this shortcoming, it is unclear whether the policy of responding to *any* relevant information as it becomes available is best *for us*. Our epistemic ends might be better served by adopting a distinct strategy. This paper explores that possibility.

2 Two information response policies

Reasoners who are not subject to cognitive limitations should process any available information relevant to propositions of interest. A subject with cognitive limitations like our own might reason like an ideal reasoner, and to the same effect when not cognitively overburdened, by instantiating the following policy:

> **(The Naïve Policy)** Take *any* relevant piece of available information into account for further processing just in case sufficient cognitive resources are currently available — no matter how much other information one is thinking through, or how much information is expected to arrive in the future, and whether or not it is expected to be evidentially weighty.

Is this policy a good one? Plausible assumptions are sufficient to establish that better are possible when it comes to subjects like us. Since we can only

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[1] See for example (Oddie, 1997), where it is argued that one does better from the point of view of epistemic value if one gathers *all and any* information that would make a difference to one's cognitive state when one updates by conditionalization — and under the assumption that updating is *cost free*. (Good, 1967) also makes the assumption explicitly as a part of a similar argument for the practical value of gathering information.

hold a limited amount of information before our minds and we have limited processing power, it is very likely that there will be circumstances in which we can only properly process a proper subset of our available information. From this, and the fact that our information is not usually misleading, we can conclude that it is better from the point of view of accuracy to process information that will have a greater impact on our confidences than a lesser impact when we cannot process all of the available information.

Since the naïve policy doesn't prioritize processing information by its impact, a subject will generally do better by acting in accordance with another policy that processes high-impact information at the expense of low-impact information. Moreover, as the amount of available information increases, the situations where we can only process a subset of our available information will become more common, amplifying the effect and justifying a more extreme prioritization of high-impact information over low-impact information.

One way to favor responding to high-impact information over low-impact information is to prioritize responding to pieces of information that bear on issues for which one has processed less weighty information rather than more. This is because inquiry is often subject to diminishing returns. Once substantial information has been gathered on a question, future information tends to make less of an impact — our attitude becomes more robust.

Some paradigmatic cases of this phenomenon, which I will single out as 'the "normal" cases', are those of inquiry focused on ordinary mid-sized dry-goods where *preliminary* observation provides substantial evidence. An example of a "normal" case, in this sense, is one where a subject is interested in whether there is peanut butter sandwich in the fridge, looks in the fridge, and observes that there is a peanut butter sandwich in it. After this preliminary observation, she can be near certain that there is a peanut butter sandwich in the fridge. In this case, picking up the sandwich, looking at it from another angle, smelling it, or tasting it won't usually make much more progress on the question. But, even in the rare case that there is also an almond butter sandwich in the fridge, so that the first impression is not decisive, any of *these* additional pieces of information will usually be weighty enough. Once a substantial body of information on a question has been taken into account, paying further attention to information relevant to it does not greatly improve the accuracy of our judgment on that issue.

One simple policy — or policy type, since it might be developed in several ways — that makes use of these observations is the following:

> **(The Obstinate Policy)** Disregard any available information as bearing on a proposition once substantial information for that proposition has been processed, otherwise proceed naïvely.

This policy prioritizes high-impact evidence over low-impact evidence in cases where inquiry is subject to diminishing returns. Assuming that more information presents itself than can be processed on average, it is reasonable to expect that subjects like us who adhere to it will be in a more accurate credal state than those who adhere to the naïve policy.

The considerations offered in support of the obstinate policy over the naïve one are general, but their imprecision makes it difficult to properly assess the argument. I now turn my attention developing a precise framework for comparing policies like those under discussion.

3 A queue-theoretic model

In this section, I present a model of credal updating for subjects for whom processing information incurs a cost in time spent and who have "working memories" of a limited capacity with which to store evidence as it is being processed, or awaiting processing. I begin with a simple statement of the idea and a general statement of the framework. Then, in §4, I model the two policies for responding to information over a range of "normal cases" using the framework and see how they compare from the point of view of expected accuracy.

The rough idea is straightforward. Assume there are n propositions whose truth values are of interest for a reasoner. Information relevant to those propositions will come in over time in a random way, with some probability of the next piece of information being relevant to one or another of these propositions. Our reasoners will have a "working memory" that can store a small number of pieces of information during processing and the amount of time it takes to process each piece of information stored in working memory will also be subject to random variation.[2] As a piece of information comes in, our reasoner can begin updating on that information as long as she has space in her working memory. Otherwise, the reasoner is assumed to have too much on her epistemic plate and the information must go unnoticed.

More formally, for each of those n propositions of interest, let each of $p_1, \dots, p_n$ pick out the truth on that matter, so that if whether q obtains is of interest and q is in fact false, then one of the p_i will be not-q. We treat the chance that the next piece of information relevant to some p_i arrives by future time t as a random variable X_i for $i \in \{1, 2, \dots, n\}$ with distribution $\mathcal{D}_i$ and λ_i its mean number of arrivals for a specified period. Our subjects are assumed to have a working memory capable of storing up to m pieces of information for processing. Then we treat how information is processed in a similar way. The chance that a piece of information stored in working memory will be processed by our reasoner by future time t is a random variable Y_μ with distribution $\mathcal{D}_\mu$, where μ is the mean number of pieces of information processed in a period of a specified duration.

The system can thus be depicted as in Figure 1. Here each X_i feeds its information into the working memory queue of length m. If the queue is full, any arriving piece of information is discarded. Otherwise, another spot in the queue becomes filled, reducing the available spots for further information by one. Finally, the number of empty spots in the queue increases by 1 according to the distribution Y_μ — i.e. pieces of information are processed, on average, at a rate of $\frac{1}{\mu}$ per period of interest.

This queue-theoretic framework provides a model of responding to information with memory and processing limitations. How exactly our rea-

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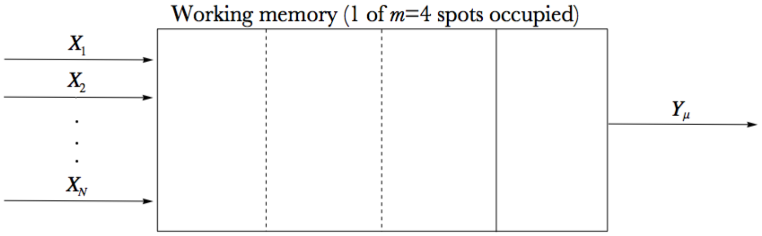
[2] I am thus assuming a multiple-tiered model of memory, or a "duplex model of memory", of the kind that is standardly assumed in the verbal and visual learning and memory traditions. See Christopher Cherniak's (1983) 'Rationality and the Structure of Human Memory' for a philosophical defense of this assumption. It is plausible that how much one can store in working memory depends not only on the count of pieces of information but also the relative informativeness of that information (Awh et al., 2007). The model can track total informativeness by letting the slots in the queue correspond to minimal units of information — analogous to bits in computer memory, but the correct picture might instead turn out to be a hybrid of the "slot based" and "total informativeness" models (Brady et al., 2011). These models are currently underdeveloped, making it difficult to differentiate and adjudicate between them. Consequently, I note the issue as a relevant complication only to set it aside.

Figure 1: The general model of epistemic inquiry.



soners update their credal states upon processing the information can then be specified externally, along with a way of scoring accuracy, to calculate the expected accuracy of adopting a policy for responding to information.

4 Comparing the policies

We can use this framework to compare the obstinate and naïve policies in a more nuanced way. Setting the parameters of the model to specific values — or, better, a range of values — and clarifying the policies within the framework, yields definite predictions about the expected accuracy of the cognitive state of a subject who adopts one of the precisified policies over the other.

Let us begin by formally specifying the impact of information on the subject's confidences. At a minimum, the non-skeptical premise of the preliminary argument for obstinacy requires that our subjects' experiences are usually not misleading and that their respective confidences will usually become more accurate as they update on more information. The initial argument for obstinacy also assumed that the epistemic returns of information for a given proposition are usually diminishing in the sense that as the information a subject has processed on a question becomes substantial further evidence is expected to rationally make less and less of an impact for that subject's confidences.

Here we will restrict the scope of the argument to the "normal cases", in which initial observations that bear on whether p provide substantial evidence and have a greater impact on a subject's confidence in p than observations made later in inquiry. Insofar as we are often preoccupied by "normal" cases, the simplification will not prevent us from drawing a general conclusion. However, we have to keep in mind the restriction when thinking about cases of theoretical, or scientific, inquiry where data is scarce and substantial data scarcer still. For all the more precise argument says, obstinacy may not be a better policy over this domain.

We will capture the impact of information on a subject's confidences with these qualifications in the general model by assuming that the confidences that we expect our subjects to have in each of the true propositions $p_1, \ldots, p_n$ are increasing functions of the number of observations that they have taken into account as bearing on those propositions, modulo some local deviation. Of course, it is not assumed that the subjects will *know* that the respective p_i are whichever of p or $\neg p$ are true for each i among the n propositions of interest prior to inquiry.

For specificity, we make the conservative assumption that a subject's *expected* credence in p_i after taking into account i observations bearing on p_i follows the logistic function:

$$\mathbb{E}\left[\mathbb{P}\left[p_i \middle| i \text{ pieces of information relevant to } p_i\right]\right] = \frac{1}{1+e^{-i}}.$$

The function ignores local deviation in the quality of the information on the grounds that as long as it is as likely to push one's confidences towards the truth as away from the truth, that deviation will be washed out from the point of view of *expected* accuracy. Thus, a logistic function that incorporates deviation, as that in Figure 2 will produce the same results.[3]

This logistic function is a modest assumption in this context. It encodes the typical characteristics of the impact of observations on credence, like being in general increasing and being subject to diminishing returns. For

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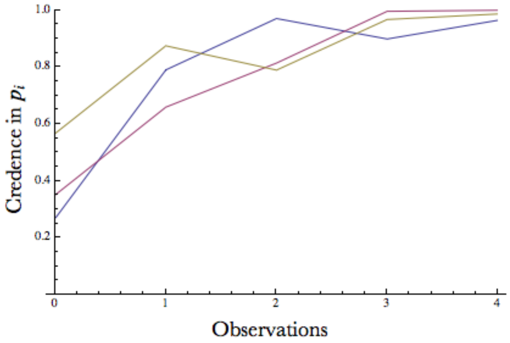
[3] The depicted progressions increase on average according to a logistic growth rate function centering the reasoner's initial confidence on .5 — i.e. $\frac{1}{1+e^{-i}}$, where i is the number of pieces of information that have been taken into account by the subject as bearing on the specified proposition. The fact that the data can be misleading, especially initially, is then accommodated by including some stochastic variation as follows: $\frac{1}{1+e^{-i+N}}$, where N is a normally distributed random quantity chosen independently at each time step so that the credence generally goes up, but sometimes regresses. The model of changes to a reasoner's credal state given successive observations can consequently be viewed as a kind of inverse "geometric Brownian motion". In the specific example, the three progressions were generated pseudo-randomly by the same function with N having mean 0 and variance 1.

Figure 2: Possible credence progressions in the truth on successive updating.



this particular function, the diminishing returns it encodes become pronounced after three or four observations. This seems plausible for hypotheses about everyday objects — the peanut butter sandwich example motivated convergence after one or two observations. By expanding the range of cases covered by the original argument, this function errs, if at all, on the side of modesty for "normal cases" by underestimating the rational impact of initial observation on confidence assignments to the propositions that make up the argument's focus. In any case, not much hangs on this choice since the effects of interest in the model are robust under plausible choices of impact functions for everyday inquiry.[4]

A second choice point concerns the time at which information in working memory should affect one's confidences in the model. One possibility would be to adjust a subject's confidences only *after* that information has been fully processed as it is removed from the queue. However, since the pieces of information placed in working memory need not be processed serially for ordinary reasoners, or even on a piece by piece basis, this stipulation would underestimate the information processed by a subject at a time by a small amount. Another option is to treat information as fully processed once it is placed in working memory. This stipulation would overestimate the information processed by a subject at a time, again by

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[4] What particular choices of impact functions *do* tend to effect is the required number of observations relevant to a proposition needed until the effects of further evidence become negligible. The logistic function is modest in this regard.

a small amount. The correct confidence distribution for a subject will lie somewhere between these options at a time.

Since it is questionable whether any precise trade-off between these two possible modeling choices will be meaningful, the results will be close under either choice, and the latter has the benefit of at least corresponding to the subject's state once all information obtained at the time has been fully processed, I use the later in what follows. Our reasoner's credences will be adjusted by an observation in the model at the time that it gets put into the queue instead of when that specific piece of information is removed from the queue. An added benefit this approach is that this allows us to simplify the bookkeeping in what follows by disregarding the order in which the pieces of information have been put into the working memory queue.

Our reasoners' cognitive states will be assessed from the point of view of their *accuracy*. Letting 1 represent the truth value of a true proposition, 0 the truth value of a false proposition, and credences range over the unit interval $[0, 1]$, we follow the prevalent tradition in formal epistemology, of judging a credence in p as being more accurate as the distance between that credence and the truth value of p decreases.[5]

Again to fix discussion, we will assume a version of the popular quadratic measure of *inaccuracy* for a total credal state S with respect to each proposition p_i in $\{p_1, \ldots, p_n\}$:

$$D(S) = \sum_i \left(1 - \mathbb{P}_t\left[p_i\right]\right)^2,$$

with $\mathbb{P}_t$ the subject's credence function at t.[6]

Finally, in order to assess the obstinate and naïve policies, the distributions of the waiting times between observations $\mathcal{D}_i$ for i in $\{1, \ldots, n\}$, the distribution of the time it takes to process an observation $\mathcal{D}_\mu$, and the amount

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[5] See (Joyce, 1998; Greaves and Wallace, 2006; Easwaran, 2013) for a representative sample of works in this tradition.

[6] Here the rule is appropriate since the truth value of the p is known to be 1 and it will be assumed that our subject's credences obey the complementation principle so that for any proposition p_i among the n — i.e. $\mathbb{P}[p_i] = 1 - \mathbb{P}[\neg p_i]$. A perfectly general account would allow an epistemic subject to choose any reasonable (in)accuracy measure. This isn't feasible in the present case, because of the numerical nature of the results — though they are also robust under other popular inaccuracy measures.

of working memory m possessed by a given subject must be specified. Here we restrict $\mathcal{D}_i$ and $\mathcal{D}_\mu$ to a class of distributions commonly used to model wait times for natural events. We will focus on situations in which any relevant event is as likely to occur within any time interval of equal length, and whether or not an interval in question was recently preceded by another incident. This amounts to assuming that the following are pairwise independent: (i) the waiting times for previous pieces of information relevant to a proposition, (ii) the variable waiting time for the next piece of information relevant to a proposition at a time, (iii) the variable waiting times between pieces of information pertaining to different propositions, and (iv) the variable processing times of pieces of information past, present, and future.[7]

Though these assumptions are simplifications, they are not unnatural or unmotivated at this level of generality. Constraints like the above make sense for processes like the number of people arriving at a bus stop during a work day or the wait times between rider arrivals, but not for either the number of buses which arrive throughout that interval or the wait times between buses — since they are scheduled. We are thus restricting our attention to cases in which information arrives more like people arrive at a bus stop than buses to that stop. In practice, these assumptions are used in modeling a wide range of systems like the number of photons that reach a telescope, the number of mutations in a given segment of DNA, and the number of phone calls arriving at a call center in a specified period. That said, the model can be applied to other (less tractable) choices of variables for the X_i and Y_μ too.

Under these assumptions, we can assess the two policies for responding to presented information for a wide range of values of the working memory m we possess, number of propositions of interest n, the average rate at which evidence for a given proposition of interest presents itself $\frac{1}{\lambda}$, and the

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[7] In this context, this is equivalent to the assumption that the number of pieces of information presented which are relevant to a proposition of interest and the number of pieces of information processed are independently Poisson distributed. Equivalently, that the waiting times are exponentially distributed. This entailment is well known, see (Billingsley, 1995, p. 190).

Another effect of this choice is that our system becomes, using Kendall's queuing-theory notation, a E/M/1/(m-1) queue. The *transient* — as opposed to long-run — behavior of this type of queue is not particularly well studied, thus the discussion might also be of some statistical interest.

average rate at which our observers fully process a given piece of information $\frac{1}{\mu}$. What range of values make sense? Setting m aside for the moment, it is not so much the values of n, λ, and μ, that are important from the point of view of the model, but rather the relationship between the average number of information arrivals on *any* question in the designated time period, $n\lambda$, and the average number of pieces of information that are processed in that period, μ. Consequently, I have fixed n at a manageable 3 and manipulated the relationship between the parameters by varying the choices of λ and μ given that $n = 3$. A robust range of conditions from cases in which information is on average processed much quicker than on average it arrives — $\mu = 10n\lambda$ — to cases in which much more information presents itself on average than can on average be processed — $10\mu = n\lambda$ — was examined.

The appropriate value for the total amount of working memory m available to a subject at a time will depend on the limitations of the subject, or subjects, of interest. The psychological literature and introspection suggest that we can consciously assess the evidential impact of very few pieces of information at a time, and probably only 3 or 4.[8] However, since a clear conception of what values of m are appropriate depends on subtle and substantive philosophical and psychological theorizing, I examined choices of m ranging from 1 up to a value of 5.

Call any event in which an observation is processed or a piece of information becomes available to our reasoner an 'epistemic event'. For this range of values, and over the short to medium term of thirty epistemic events, the obstinate policy of ignoring information does provably better from the point of view of expected credal accuracy than the naïve policy of taking information into account as it arrives whenever the average rate of information processing is less than two times the average rate at which it arrives. In many cases, the effect is even more pronounced. For instance, with smaller working memories of $m = 1$ or $m = 2$, the obstinate policy is provably preferable up to, and beyond, the boundary case of thirty epistemic events when information is processed on average *ten times faster* than it arrives. For a working memory of $m = 4$, the obstinate policy is preferable over the first thirty epistemic events up to and including the case where in-

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[8] See (Cowan, 2001, 2005) for a summary of the research on the limits of visual working memory. (Brady et al., 2011, pp. 1-5) provide a summary of recent results on the limits of visual working memory.

formation is processed four times faster than it, on average, arrives.[9,10]

These results give us a good idea of when the obstinate policy will be preferable to the naïve one. Whenever the amount of available information exceeds our processing power, and the distributional assumptions are approximately correct, the obstinate policy will be preferable. One interesting effect that the model illustrates is that even if *on average* we process information more quickly (and sometimes much more quickly) than it arrives the obstinate policy will be preferable to the naïve one. Sometimes a lot of information just happens by chance to come in all at once or by chance we process less information than average. Whenever either or both of these occur to a sufficient extent, it results in an information bottleneck. This effect makes prioritizing high-impact information important even in cases where information is fairly scarce and a subject processes incoming information with relative ease.

As a consequence, the model suggests that the obstinate policy might be beneficial even if we implement certain other strategies for overcoming our limitations. In particular, it might be beneficial if in response to overabundant information we implement a strategy that involves increasing the speed at which we process information at the cost of a higher variance in the results, or even at a slight cost in accuracy. Heuristic, or otherwise quick-and-dirty, reasoning of this kind may increase the rate at which information is processed relative to the amount arriving, but the previous observation shows that obstinacy is still a better policy than the naïve one under modest increases in processing speed. Obstinacy is compatible with, and may be a good supplement to, an array of strategies to overcome our limitations.

5 Applications

Some authors (correctly) hold that facts about our cognitive limitations should structure theorizing about our world-directed cognitive states like

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[9] Of course, adopting the obstinate policy is *guaranteed* to do worse in the longterm since, in the longterm, the policy leads to indiscriminately ignoring all information with probability 1. From the current perspective, it is important that our reasoner, like us, epistemically operates in the present and near future rather than in the long run.

[10] Details of the proofs can be found in the technical appendix §7.

outright belief and credence. In this section, I argue that the above framework and results help to clarify some of this theorizing and explain why some of our cognitive behavior which seems deleterious might be beneficial. §5.1 presents some influential work in psychology documenting obstinacy effects in our reasoning processes. While these are often presented as “cognitive biases” to be overcome, the results above explain how they are useful. §5.2 shows how the result fills in some of the much needed details for accounts of belief as a kind of epistemic plan. In §5.3, I argue that the results concerning the obstinate policy undercut certain “demandingness” objections to the possibility that we are credal reasoners.

5.1 Obstinacy effects in psychology

Work in social psychology has revealed a few ways in which we resemble obstinate reasoners. So-called ‘primacy effects’ provide one example. In Cameron Peterson and Wesley DuCharme’s now classic (1967) study on the phenomenon, subjects were told the distribution of colored chips in two urns. They were then presented with a series of data that they were told corresponded to draws from exactly one of the urns, with replacement. After each datum was presented, the subjects recorded their confidence in the hypothesis that the draws were from the first urn. The information presented to subjects favored the hypothesis that the draws were from the first urn rather than the second for the first 30 observations then, symmetrically, the next 30 “draws” favored the converse. The last forty “draws” also favored the hypothesis that the draws were from the second urn.

If they were acting as perfect Bayesian reasoners, setting their confidence that the draws were from the first urn upon receiving information to their prior confidence in that proposition conditional on that information, the subjects would have had a confidence of .5 or less in the hypothesis that the data resulted from draws of the first urn after their initial sixty observations. In fact, for most subjects, high confidence in the first hypothesis persisted well beyond sixty observations — over half of the subjects failed to reduce their confidence in the first urn hypothesis below .5 over the total 100 observations, by which point a Bayesian updater would have had a confidence below .05. Information provided later in the experiment had much less of an impact on their confidences than it would have if the reasoners took it into account by updating in an ideal Bayesian way. Other experiments reveal a similar tendency.[11]

Documented belief "persistence effects" constitute another way in which we resemble obstinate reasoners. The experiments that best illustrate belief persistence are one's in which subjects retain an elevated confidence in a proposition after any additional support for that proposition is undercut (as opposed to merely being outweighed as in the Peterson and DuCharme experiment just discussed). In one experiment, Lee Ross et al. presented subjects with pairs of purported suicide notes, told them that each pair contained one real and one fabricated note, and asked them to say which one they thought it was (1975). After each response was elicited, subjects were told whether or not they were correct — the response was in fact predetermined by the researchers and independent of their performance. One test group received mostly positive responses, and another negative.

After the test, the subjects were debriefed and told that the information that they had received was predetermined and independent of their actual choices. Nevertheless, those that received mostly positive responses still thought that they did much better at the task and would be better at it in the future then those who were told that their choices were mostly wrong. The effect was also present when an outside party, after watching the experiment and similarly debriefed, was asked to rate the participant on how well they performed and how they might be expected to perform in the future. In their discussion of the result, Ross et al. conclude that "the relevance, reliability, and validity of dubiously relevant, reliable, or valid information is resolved as a function of its consistency with the attributor's [sic.] initial impression" (1975, p. 889). Impressions, they take it, can be sustained by the evidence filtering effects that accompany them.

Both primacy and persistence effects are usually presented as epistemically deleterious cognitive biases. The above argument and model suggest that, at least in circumstances like those the model describes, subjects who exhibit these "biases" might have a more accurate picture of the world as a result.

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[11] (Baron, 2007) contains a good summary of "primacy" results like the one described.

5.2 Clarifying the belief-as-plan view

According to the belief-as-plan view, belief is best thought of by analogy to Micheal Bratman's notion of intention (Bratman, 1985).[12] Just as intention is, on this view, a kind of *practical* coordination point or a stable point that constrains future action, belief is a kind of *epistemic* coordination point or a stable point that shapes our representation of the world, guiding theoretical deliberation and inquiry. By forming a belief in a proposition, we become disposed to treat that proposition as true in reasoning.

One of the central mechanisms by which belief plays this role, on this view, is by being disposed to *resist reconsideration*. The thought parallels the rough argument for obstinacy presented above. It is cognitively costly to remain responsive to every epistemic contingency. After weighty enough evidence for a proposition is taken into account, proponents of the belief-as-plan view argue, we usually expect that further information for that proposition will be largely epistemically inconsequential. There is little to gain by responding to further information regarding that proposition. Thus, instead of being disposed to respond to such information, it would be better to be disposed to disregard it in order to make up gains on other issues — to resist reconsideration on well-established propositions by being blind to information that bears on it.[13]

According to the belief-as-plan view, that is what believing does. Believing a proposition firms up one's epistemic stance towards the believed proposition. It trades off the epistemic flexibility of being able to constantly fine-tune one's epistemic stance towards a proposition in order make up epistemic gains on other issues. This should sound familiar. The obstinate policy, remember, recommends disregarding available information as bear-

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[12] A defense of the belief-as-plan view can be found at (Holton, 2013). We follow the details of his presentation here. Similar positions are suggested by Ross and Schroeder (2012) and Weisberg (ms).

[13] It is important that the stance towards future evidence more closely resembles blindness than deliberate ignorance. Deliberately ignoring information is irrational. On the other hand, if one is blind to information then, arguably, that information is not raised to the level of evidence to begin with — at least on the prominent evidentialist construal where evidence are the propositions of which you are aware (Feldman, 1988). In addition, if the resistance to reconsideration were a deliberate or conscious matter the proposal would be self-undermining, since consciously considering how further information bears on a proposition already incurs the cost of reconsideration.

ing on a proposition when a substantive information for that proposition has been gathered. The motivation for the belief-as-plan view parallels the rough defense of the obstinate policy over the naïve one and stands to be clarified along the same lines as the model clarified the rough argument for obstinacy.

The model and case study can be used to clarify the belief-as-plan view by filling in its details if either (i) credence and outright belief "march in step",[14] in the sense that belief is always accompanied by high confidence, or (ii) if outright belief reduces to credence.

If outright belief reduces to credence, then the application is direct. Belief's resistance to reconsideration can be understood as credal obstinacy and the obstinate policy is one way of filling in the details of the mechanism underling the formation of belief-as-plan states. A benefit of filling in the details this way is that it explains why forming beliefs in accordance with the credal analogue of the belief-as-plan view, the obstinate policy, is cognitively valuable since in normal circumstances credences formed in this way are expected to be more accurate than those formed using a naïve policy.

The model can also be used to unpack the details of our mental lives on the belief-as-plan picture if credence and belief merely "march in step". If beliefs resist reconsideration according to the belief-as-plan view, then the high credences that accompany those beliefs will have to likewise "march in step" and resist reconsideration in the same cases. But, then whatever one's account of how one gets into a belief-as-plan state, one will need an account of the mechanism by which credence gets into a state of resisting reconsideration too. The obstinate policy is a good way of spelling out that mechanism. If one updates in accordance with the obstinate policy while outright belief and credence "march in step", then the obstinate policy can provide an explanation of why forming plan like beliefs promotes overall representational accuracy at the level of outright belief.

By following the obstinate policy at the credal level, a subject increases the chances that the available information will propel more of her credences towards the truth than if she were to update naïvely — that is the effect driving the results in the case study. But, given the "march in step"

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[14] This locution, and observation, is due to Scott Sturgeon (2008). The thought is implicit in any account of belief that reduces belief to a variety of substantial confidence.

phenomenon, one consequence of this is that following the obstinate policy will make it possible to form more outright beliefs in truths than if one updated naïvely. So, the obstinate policy promotes a credal state that is more fertile for forming accurate outright beliefs than the naïve policy. The hypothesis that credences adhere to the obstinate policy, together with the "march in step" claim, supports the idea that forming belief-as-plan states is good for overall representational accuracy.

In sum, many find the view that outright belief reduces to confidence plausible. But, even if it is false, the "march in step" is hard to deny. It is difficult to imagine what it could be like to believe a proposition without being more confident than not that that proposition obtains. In either case, the queue-theoretic model and case study provide a plausible way to unpack the cognitive mechanisms at the heart of the belief-as-plan view, and how entering into such states should be cognitively beneficial.

5.3 Demandingness worries

The possibility of information response policies like the obstinate policy helps to alleviate, if not eliminate, certain "demandingness" objections to views that take us to be credal reasoners. In this vein, Gilbert Harman (1986, ch. 3), and more recently Richard Holton (2013, pp. 2-3, 10-2), have argued that we are not the kinds of creatures that explicitly reason with credences because doing so would outstrip our mental capacities — reasoning with credences is *too cognitively costly* for creatures like us.[15] According to one line of thought these authors advance, in order to be good credal reasoners we would have to be willing to take unrealistically many things into epistemic account. They both begin by assuming that credences are the kinds of states that are responsive to *any* non-trivial evidence. But, we are not the kinds of creatures that are capable of readily responding

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[15] Both Harman and Holton have additional arguments for the conclusion that we are not credal reasoners. It is less clear that the above discussion can help with those arguments so, while I don't find them compelling, I do not take them up in what follows.

to all of the available information.[16] Consequently, the line of thought continues, the amount of cognitive processing that being a credal reasoner would require makes it implausible that we are credal reasoners.[17]

However, the assumption upon which this general argument relies — that credences are the kind of states that are responsive to any non-trivial information — is implausible. It marks another place that a theory of ideal reasoning, in this case Bayesian updating, is being misinterpreted as description of conscious deliberation. If we take our cognitive limitations seriously in the way that both Harman and Holton suppose we should then, contrary to what Bayesian models of ideal credal reasoning might suggest, credal states should not be expected to be responsive to absolutely any non-trival information. Some credences might resist reconsideration in the same way that Holton takes belief states to fail to respond to some pieces of information as a matter of brute disposition. The obstinate policy serves as a simple proof of concept of how this might be so while illustrating one way that we stand to profit by instantiating a policy of disregarding information relevant to the propositions to which we assign some credence. The objection is no more persuasive when leveled against accounts of reasoning with credences than it is when leveled against accounts of reasoning with belief.

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[16] Holton and Harman give different reasons for why we cannot readily respond to all of the available information. Harman worries that doing so would require credal reasoners to have implausibly many conditional confidences waiting in the wing. Holton worries that doing so would make our mental lives unmanageably unstable, in that we would have to be constantly recalculating our credences in established propositions.

[17] I am here focusing on the general features shared by each author's objection rather than the specifics over which they differ. There are further worries for the specifics — many of which are clearly articulated in (Staffel, 2012)'s criticisms of (Harman, 1986)'s version of the objection.

It should also be noted that this objection should not be confused with the kind of "demandingness" objection which tries to establish that we cannot be credal reasoners on the grounds that the *probabilistic computation* that it entails is too taxing. I have nothing new to say about this objection, and largely agree with (Staffel, 2012)'s criticism of the objection — which turns, in part, on the possibility that we employ credal heuristics rather than engage in explicit probabilistic calculations when we reason credally.

6

For limited creatures like us, properly responding to information comes at a cognitive cost. In this paper, I laid out a queue-theoretic framework for precisely assessing how different policies for responding to information interact with some of our limitations to influence the cognitive value of our total credal state. Two simple policies for responding to information, 'the naïve policy' and 'the obstinate policy', were assessed within this framework under modest assumptions about our cognitive limitations, in a range of common or "normal" cases, and under defensible simplifying assumptions. Under these conditions it is provable that, from the point of view of expected credal accuracy, it is better for epistemic subjects like us *not to update* on available information that bears on propositions for which substantial evidence has been gathered than it is to update on information as it presents itself.

The conditions assumed in the model are most appropriate when the propositions under investigation concern the familiar properties of ordinary mid-sized dry-goods. Given the central role that hypotheses of this kind play in everyday inquiry, the model helps to explain why some of our non-ideal techniques for responding to evidence are nonetheless useful. By pointing out that credences might be formed in accordance with the obstinate policy, the above picture undercuts a "demandingness" objection to the possibility that we are credal reasoners and helps to give substance to the view that takes belief to be a kind of plan.

7 Technical Appendix

This section provides the details of the result documented in §4. In order to derive the results, it was assumed that the waiting times between observations relevant to the truth of a proposition p_i, X_i for $i \in \{1, \ldots, n\}$ are identically but independently exponentially distributed with mean rate of arrival λ. Likewise, the waiting time for a proposition in the queue to be processed, Y_μ, is independently exponentially distributed at a (possibly distinct) mean rate of μ pieces of information per time period. That is, the X_i

and Y_μ have the following probability density function:

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{1}{\theta}x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

where θ is the mean rate at which pieces of information arrive for the given p_i or are processed, respectively.

Now, let $t_1, t_2, \ldots$ be the sequence of times at which an *epistemic event* occurs, that is, either a piece of information relevant to a proposition is presented according to an X_i or a piece of information is processed from the queue according to Y_μ. Let $S_{p_1,p_2,p_3,\ldots,q}$, with $p_k, q \in \mathbb{N}$ and $q \leq m$ be the state in which our reasoner has made p_1 observations relevant concerning the truth of the first proposition of interest, p_2 observations relevant to the second proposition of interest, ..., and for which q of m states of working memory are currently being expended to process observations. A proposition and two corollaries follow from these definitions and observations:

Proposition 1. The probability, at time t, that a given $\mathcal{V}$ chosen from $\{X_1, \ldots, X_n, Y_\mu\}$ will occur next, $\mathbb{P}_t\left[\mathcal{V} = min\{X_1, \ldots, X_n, Y_\mu\}\right]$, is $\frac{\lambda_v}{\lambda_\mu + n\lambda}$, where$\lambda_v$ is the mean of $\mathcal{V}$. By the no-memory property of $X_1, \ldots, X_n, Y_\mu$, this is so no matter which events have occurred before t.[18]

This, in turn, yields the following probabilities for transitioning between states when the naïve policy for responding to presented information is operative:

Corollary 2 (Naïve transition probabilities). Where '$S_i \rightarrow_{\mathbb{P}} S_j$' is the probability of transitioning from state S_i to S_j, and $p_k, q \in \mathbb{N}$:

1. $S_i \rightarrow_{\mathbb{P}} S_j = \frac{\lambda}{\lambda_\mu + n\lambda}$ if $q < m$ and $S_i = S_{\ldots,p_k,***,q}$ while $S_j = S_{\ldots,p_k+1,***,q+1}$;
2. $S_i \rightarrow_{\mathbb{P}} S_j = \frac{\lambda_\mu}{\lambda_\mu + n\lambda}$ if $S_i = S_{\ldots,q}$ while $S_j = S_{\ldots,q-1}$;
3. $S_i \rightarrow_{\mathbb{P}} S_j = \frac{n\lambda}{\lambda_\mu + n\lambda}$ if $S_i = S_j = S_{\ldots,q}$ with $q = m$;

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[18] See (Ross, 2007, p. 294) for a standard proof of the result. It is worth noting that the assumptions support the more general proposition in which the means of the distributions of observations are not assumed to be equal. In that case, $\mathbb{P}_t\left[\mathcal{V} = min\{X_1, \ldots, X_n, Y_\mu\}\right] = \frac{\lambda_v}{\sum \lambda}$, where λ ranges over the means of $X_1, \ldots, X_n$ and Y_μ.

4. $S_i \rightarrow_{\mathbb{P}} S_j = 0$, otherwise.

Moreover, the state-transition probabilities under the obstinate policy are given by the following:

Corollary 3 (Obstinate transition probabilities). Where '$S_i \rightarrow_{\mathbb{P}} S_j$' is the probability of transitioning from state S_i to S_j, information for a proposition of interest is ignored after ω observations, and $p_k, q \in \mathbb{N}$:

1. $S_i \rightarrow_{\mathbb{P}} S_j = \frac{\lambda}{\lambda_\mu + n\lambda}$ if either
    (a) $q < m$, $p_k < \omega$ and $S_i = S_{\ldots,p_k,***,q}$ while $S_j = S_{\ldots,p_k+1,***,q+1}$; or
    (b) $q < m$, $p_k = \omega$, and $S_i = S_j$.
2. $S_i \rightarrow_{\mathbb{P}} S_j = \frac{\lambda_\mu}{\lambda_\mu + n\lambda}$ if $S_i = S_{\ldots,q}$ while $S_j = S_{\ldots,q-1}$;
3. $S_i \rightarrow_{\mathbb{P}} S_j = \frac{n\lambda}{\lambda_\mu + n\lambda}$ if $S_i = S_j = S_{\ldots,q}$ with $q = m$;
4. Otherwise, $S_i \rightarrow_{\mathbb{P}} S_j = 0$.

The above observations allow us to compute the probability that an individual is in a given state after a specified number E of epistemic events for either of the discussed information-response policies. The result follows by application of a version of the Chapman-Kolmogorov Equations. In particular, letting $S_i \rightarrow_{\mathbb{P},E} S_j$ be the probability that beginning in state S_i one arrives at S_j after E epistemic events, the probabilities follow by looking at each possible way that one can end up in a state S_j from S_i

$$S_i \rightarrow_{\mathbb{P},a+b=E} S_j = \sum_{k=0}^{\infty} (S_i \rightarrow_{\mathbb{P},a} S_k) \cdot (S_k \rightarrow_{\mathbb{P},b} S_j).$$

Specifying the starting state yields a distribution of probabilities for the possible states one might be in after E epistemic events according to the relevant method of responding to available information.

With these details in place, fixing m, n, λ, μ and the epistemic subject's starting state allows the subject to calculate the expected (in)accuracy of their credal state S at a time t after a given number of epistemic events E. Recall that for a total credal state S in propositions $p_1, \ldots, p_n$, the total inaccuracy $D(S)$ of that state given by a version of the quadratic scoring rule is $\sum_i (1 - \mathbb{P}_t[p_i])^2$ with $\mathbb{P}_t$ the subject's credence function at t.

Now, letting $S_1, S_2, \ldots$ be the possible states describing the pieces of information taken into account as above and the length of the queue at t, $o_i(S)$ be the number of propositions with i observations specified in the subjects total credal state S, t_0 be the current time, $\mathbb{P}$ — without index — be our reasoner's credence function at t_0, and $t \geq t_0$ be the time at which some given number of observations has occurred, the expected inaccuracy of our reasoner's credal state at t is:

$$\begin{aligned}
\mathbb{E}\left[D(S)\right] &= \mathbb{E}\left[\mathbb{E}\left[D(S) \middle| S_i\right]\right], \text{by the Law of Total Expectation.} \\
&= \mathbb{E}\left[\sum_j o_j(S_i)\left(1 - \frac{1}{1+e^j}\right)\right] \\
&= \sum_i \mathbb{P}\left[S_i\right] \sum_j o_j(S_i)\left(1 - \frac{1}{1+e^j}\right)
\end{aligned}$$

Filling in the state transitions probabilities for either policy yields the expected accuracy of that policy at a time at which any specified number of epistemic events has occurred, as desired.[19]

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[19] A Mathematica notebook for calculating the expected value of epistemic states in the test case for any number of epistemic events is available on request.

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