

Likelihoods, Counterfactuals, and Tracking

Branden Fitelson

Department of Philosophy
Group in Logic and the Methodology of Science
&
Cognitive Science Core Faculty
University of California-Berkeley

branden@fitelson.org
http://fitelson.org/

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- Today, I will be comparing likelihood vs counterfactual renditions of tracking *vis-à-vis* their *logical* properties.
- Specifically, I will mainly be focusing on the following counterfactual vs likelihood *components* of tracking:
 - **Likelihood Tracking** [4]. S tracks_{*l*} p just in case:
 - (L₁) $\Pr(Bp \mid p) > t$.
 - (L₂) $\Pr(\sim B\sim p \mid p) > t$.
 - (L₃) $\Pr(\sim Bp \mid \sim p) > t$.
 - **Counterfactual Tracking** [3]. S tracks_{*c*} p just in case:
 - (C₁) $p \square \rightarrow Bp$.
 - (C₂) $p \square \rightarrow \sim B\sim p$.
 - (C₃) $\sim p \square \rightarrow \sim Bp$.
- For simplicity, I will assume that we are talking about a specific agent S in a fixed (actual) belief context, relative to a fixed probability function \Pr , and a fixed threshold $t > 1/2$.
- Some of my comparisons will involve tracks_{*l*} vs tracks_{*c*}, and some will involve the underlying conditions (L_{*i*}) vs (C_{*i*}).

- The first issue involves the consequences for tracks_{*l*} vs tracks_{*c*} of assuming that S 's beliefs are consistent.
 - **Consistency.** $Bp \Rightarrow \sim B\sim p$.
- If we impose the constraint of Consistency, then conditions (L₂) and (C₂) become redundant and may be dropped.
- This relies on the following two properties of likelihoods and counterfactuals (for just about any theories thereof):
 - If $\Pr(Bp \mid p) > t$ and $Bp \Rightarrow \sim B\sim p$, then $\Pr(\sim B\sim p \mid p) > t$. [In other words, (L₁) and Consistency jointly entail (L₂).]
 - If $p \square \rightarrow Bp$ and $Bp \Rightarrow \sim B\sim p$, then $p \square \rightarrow \sim B\sim p$. [In other words, (C₁) and Consistency jointly entail (C₂).]
- This *similarity* leads to a simplification of both accounts for consistent agents. But, it also obscures a crucial *difference*:
 - $(p \square \rightarrow Bp \ \& \ p \square \rightarrow \sim B\sim p) \Rightarrow p \square \rightarrow (Bp \ \& \ \sim B\sim p)$.
 - $[\Pr(Bp \mid p) > t \ \& \ \Pr(\sim B\sim p \mid p) > t] \not\Rightarrow \Pr(Bp \ \& \ \sim B\sim p \mid p) > t$.
- As such, the counterfactual account does not need to add an extra condition to ensure that $p \square \rightarrow (Bp \ \& \ \sim B\sim p)$, but the likelihood account needs to add $\Pr(Bp \ \& \ \sim B\sim p \mid p) > t$.

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- Lewis's account of $\Box \rightarrow$ implies the following:
 - **Centering.** $p \ \& \ q \Rightarrow p \ \Box \rightarrow \ q$.
- This is because Lewis's semantics for $\Box \rightarrow$ assumes:
 - (i) The actual world is *the* closest world to itself.
 - (ii) $p \ \Box \rightarrow \ q$ is (actually) true iff the closest ($p \ \& \ q$)-world is closer (to the actual world) than the closest ($p \ \& \ \sim q$)-world.
- This has unfortunate consequences for a (Lewisian) full counterfactual tracking relation, which also includes:
 - (C₄) Bp .
 - (C₅) p .
- Given (C₄) and (C₅), Centering implies (C₁), which makes (C₁) redundant assuming a Lewisian semantics for $\Box \rightarrow$.
- Nozick [3] and other tracking theorists [2] are well aware of this problem, which has led them to reject Centering.
- Nozick suggests revising (ii) so as to require only that q be true in some "sphere" of *close enough* p -worlds.
- While this avoids Centering and the resulting redundancy of (C₁), it still suffers another sort of (Lewisian) redundancy.

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- Assume we change Lewis's semantics by revising (ii) as:
 - (ii') $p \ \Box \rightarrow \ q$ is true iff all *close enough* p -worlds are q -worlds.
- This avoids Centering because some *close enough* p -worlds can be $\sim q$ -worlds, *even if* the actual world is a ($p \ \& \ q$)-world.
- But, (C₅) – the truth of p – remains redundant in such a (full) counterfactual tracking account, because of (C₃) and (C₄).
- To see why this is the case, note that (i) and (ii') share the following property with the Lewisian semantics for $\Box \rightarrow$
 - (†) $p \ \Box \rightarrow \ q \Rightarrow p \ \supset \ q$.
- Assuming (†), it's easy to derive (C₅) from (C₃) and (C₄).
 - **Proof.** By (†), we have $\sim p \ \Box \rightarrow \ \sim Bp \Rightarrow \sim p \ \supset \ \sim Bp$. So, by (C₃), we may infer $\sim p \ \supset \ \sim Bp$, *i.e.*, $Bp \ \supset \ p$. Then, by (C₄) and *modus ponens* for \supset , we may infer p , *i.e.*, (C₅). \square
- So, any such semantics for $\Box \rightarrow$ must allow for the actual world *not* to be included in its own "sphere" in order to avoid this redundancy. Thus, (i) must be *radically* revised.
- This seems odd. Here, we seem to have an advantage of likelihood-tracking, which suffers neither redundancy.

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- At this point, one might be tempted to try *probabilistic* constraints on "closeness". Gunderson [2] combines (ii) and
 - (iii) The closest ($p \ \& \ q$)-world is closer than the closest ($p \ \& \ \sim q$)-world iff $\Pr(q \mid p) > 1/2$ and $\Pr(q \mid p) \gg \Pr(q \mid \sim p)$.
- Depending on how one interprets " \gg ", one gets different semantical consequences for $\Box \rightarrow$. Two possibilities are:
 - (LD) $\Pr(q \mid p) \gg \Pr(q \mid \sim p)$ iff $\Pr(q \mid p) - \Pr(q \mid \sim p) > d$.
 - (LR) $\Pr(q \mid p) \gg \Pr(q \mid \sim p)$ iff $\Pr(q \mid p) / \Pr(q \mid \sim p) > r$.
- If we use (LD), then (assuming Consistency), the resulting account of tracking_c *reduces to (is equivalent to)* tracks_l.
- If we use (LR), then (assuming Consistency), the resulting tracking_c relation is *strictly logically stronger than* tracks_l.
- There are compelling independent reasons for Gunderson *not* to use (LD). *E.g.*, (LD) implies $p \ \Box \rightarrow \ q \Rightarrow \sim p \ \Box \rightarrow \ \sim q$ [1].
- Moreover, even if Gunderson goes with (LR), the resulting $\Box \rightarrow$ still doesn't behave very much like a *conditional*. And, the notion of "closeness" one gets from (iii) is rather bizarre.
- Some concluding remarks about tracking, $\Box \rightarrow$, and $\Pr(\cdot \mid \cdot)$.

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- It is interesting that Gunderson was led to a semantics for $\Box \rightarrow$ that makes his tracking_c so similar to Sherri's tracks_l.
- Unfortunately, Gunderson's " $\Box \rightarrow$ " doesn't really seem to be a *conditional* anymore — especially on the (LD) reading.
- This makes me wonder why we're bothering to "reverse engineer" a semantics for " $\Box \rightarrow$ " to suit a non-redundant account of tracks_c. Why not cut out the middle-man here?
- That is, why not just work directly with likelihoods from the outset, and dispense with counterfactual talk altogether?
- Moral: the energy spent trying to come up with semantics for " $\Box \rightarrow$ " suitable for non-redundant tracking_c is better spent doing what Sherri does so well in her book [4].

[1] Eells, E. and Fitelson, B., 2002, "Symmetries and Asymmetries in Evidential Support", *Philosophical Studies* 107: 129-142.

[2] L.B. Gunderson, 2004, "Outline of a New Semantics for Counterfactuals", *Pacific Philosophical Quarterly*, 85: 1-20.

[3] R. Nozick, 1981, *Philosophical Explanations*, OUP.

[4] S. Roush, 2006, *Tracking Truth: Knowledge, Evidence, and Science*, OUP.

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