

# Likelihoods, Counterfactuals, and Tracking

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- Today, I will be comparing likelihood vs counterfactual renditions of tracking *vis-à-vis* their *logical* properties.
- Specifically, I will mainly be focusing on the following counterfactual vs likelihood *components* of tracking:
  - **Likelihood Tracking** [4].  $S$  tracks<sub>*l*</sub>  $p$  just in case:
    - (L<sub>1</sub>)  $\Pr(Bp \mid p) > t$ .
    - (L<sub>2</sub>)  $\Pr(\sim B\sim p \mid p) > t$ .
    - (L<sub>3</sub>)  $\Pr(\sim Bp \mid \sim p) > t$ .
  - **Counterfactual Tracking** [3].  $S$  tracks<sub>*c*</sub>  $p$  just in case:
    - (C<sub>1</sub>)  $p \square \rightarrow Bp$ .
    - (C<sub>2</sub>)  $p \square \rightarrow \sim B\sim p$ .
    - (C<sub>3</sub>)  $\sim p \square \rightarrow \sim Bp$ .
- For simplicity, I will assume that we are talking about a specific agent  $S$  in a fixed (actual) belief context, relative to a fixed probability function  $\Pr$ , and a fixed threshold  $t > 1/2$ .
- Some of my comparisons will involve tracks<sub>*l*</sub> vs tracks<sub>*c*</sub>, and some will involve the underlying conditions (L<sub>*i*</sub>) vs (C<sub>*i*</sub>).

- The first issue involves the consequences for tracks<sub>*l*</sub> vs tracks<sub>*c*</sub> of assuming that  $S$ 's beliefs are consistent.
  - **Consistency.**  $Bp \Rightarrow \sim B\sim p$ .
- If we impose the constraint of Consistency, then conditions (L<sub>2</sub>) and (C<sub>2</sub>) become redundant and may be dropped.
- This relies on the following two properties of likelihoods and counterfactuals (for just about any theories thereof):
  - If  $\Pr(Bp \mid p) > t$  and  $Bp \Rightarrow \sim B\sim p$ , then  $\Pr(\sim B\sim p \mid p) > t$ . [In other words, (L<sub>1</sub>) and Consistency jointly entail (L<sub>2</sub>).]
  - If  $p \square \rightarrow Bp$  and  $Bp \Rightarrow \sim B\sim p$ , then  $p \square \rightarrow \sim B\sim p$ . [In other words, (C<sub>1</sub>) and Consistency jointly entail (C<sub>2</sub>).]
- This *similarity* leads to a simplification of both accounts for consistent agents. But, it also obscures a crucial *difference*:
  - $(p \square \rightarrow Bp \ \& \ p \square \rightarrow \sim B\sim p) \Rightarrow p \square \rightarrow (Bp \ \& \ \sim B\sim p)$ .
  - $[\Pr(Bp \mid p) > t \ \& \ \Pr(\sim B\sim p \mid p) > t] \not\Rightarrow \Pr(Bp \ \& \ \sim B\sim p \mid p) > t$ .
- As such, the counterfactual account does not need to add an extra condition to ensure that  $p \square \rightarrow (Bp \ \& \ \sim B\sim p)$ , but the likelihood account needs to add  $\Pr(Bp \ \& \ \sim B\sim p \mid p) > t$ .

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- Lewis's account of  $\Box \rightarrow$  implies the following:
  - **Centering.**  $p \ \& \ q \Rightarrow p \ \Box \rightarrow \ q$ .
- This is because Lewis's semantics for  $\Box \rightarrow$  assumes:
  - (i) The actual world is *the* closest world to itself.
  - (ii)  $p \ \Box \rightarrow \ q$  is (actually) true iff the closest ( $p \ \& \ q$ )-world is closer (to the actual world) than the closest ( $p \ \& \ \sim q$ )-world.
- This has unfortunate consequences for a (Lewisian) full counterfactual tracking relation, which also includes:
  - (C<sub>4</sub>)  $Bp$ .
  - (C<sub>5</sub>)  $p$ .
- Given (C<sub>4</sub>) and (C<sub>5</sub>), Centering implies (C<sub>1</sub>), which makes (C<sub>1</sub>) redundant assuming a Lewisian semantics for  $\Box \rightarrow$ .
- Nozick [3] and other tracking theorists [2] are well aware of this problem, which has led them to reject Centering.
- Nozick suggests revising (ii) so as to require only that  $q$  be true in some "sphere" of *close enough*  $p$ -worlds.
- While this avoids Centering and the resulting redundancy of (C<sub>1</sub>), it still suffers another sort of (Lewisian) redundancy.

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- Assume we change Lewis's semantics by revising (ii) as:
  - (ii')  $p \ \Box \rightarrow \ q$  is true iff all *close enough*  $p$ -worlds are  $q$ -worlds.
- This avoids Centering because some *close enough*  $p$ -worlds can be  $\sim q$ -worlds, *even if* the actual world is a ( $p \ \& \ q$ )-world.
- But, (C<sub>5</sub>) – the truth of  $p$  – remains redundant in such a (full) counterfactual tracking account, because of (C<sub>3</sub>) and (C<sub>4</sub>).
- To see why this is the case, note that (i) and (ii') share the following property with the Lewisian semantics for  $\Box \rightarrow$ 
  - (†)  $p \ \Box \rightarrow \ q \Rightarrow p \ \supset \ q$ .
- Assuming (†), it's easy to derive (C<sub>5</sub>) from (C<sub>3</sub>) and (C<sub>4</sub>).
  - **Proof.** By (†), we have  $\sim p \ \Box \rightarrow \ \sim Bp \Rightarrow \sim p \ \supset \ \sim Bp$ . So, by (C<sub>3</sub>), we may infer  $\sim p \ \supset \ \sim Bp$ , *i.e.*,  $Bp \ \supset \ p$ . Then, by (C<sub>4</sub>) and *modus ponens* for  $\supset$ , we may infer  $p$ , *i.e.*, (C<sub>5</sub>).    $\square$
- So, any such semantics for  $\Box \rightarrow$  must allow for the actual world *not* to be included in its own "sphere" in order to avoid this redundancy. Thus, (i) must be *radically* revised.
- This seems odd. Here, we seem to have an advantage of likelihood-tracking, which suffers neither redundancy.

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- At this point, one might be tempted to try *probabilistic* constraints on "closeness". Gunderson [2] combines (ii) and
  - (iii) The closest ( $p \ \& \ q$ )-world is closer than the closest ( $p \ \& \ \sim q$ )-world iff  $\Pr(q \mid p) > 1/2$  and  $\Pr(q \mid p) \gg \Pr(q \mid \sim p)$ .
- Depending on how one interprets " $\gg$ ", one gets different semantical consequences for  $\Box \rightarrow$ . Two possibilities are:
  - (LD)  $\Pr(q \mid p) \gg \Pr(q \mid \sim p)$  iff  $\Pr(q \mid p) - \Pr(q \mid \sim p) > d$ .
  - (LR)  $\Pr(q \mid p) \gg \Pr(q \mid \sim p)$  iff  $\Pr(q \mid p) / \Pr(q \mid \sim p) > r$ .
- If we use (LD), then (assuming Consistency), the resulting account of tracking<sub>c</sub> *reduces to (is equivalent to)* tracks<sub>l</sub>.
- If we use (LR), then (assuming Consistency), the resulting tracking<sub>c</sub> relation is *strictly logically stronger than* tracks<sub>l</sub>.
- There are compelling independent reasons for Gunderson *not* to use (LD). *E.g.*, (LD) implies  $p \ \Box \rightarrow \ q \Rightarrow \sim p \ \Box \rightarrow \ \sim q$  [1].
- Moreover, even if Gunderson goes with (LR), the resulting  $\Box \rightarrow$  still doesn't behave very much like a *conditional*. And, the notion of "closeness" one gets from (iii) is rather bizarre.
- Some concluding remarks about tracking,  $\Box \rightarrow$ , and  $\Pr(\cdot \mid \cdot)$ .

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- It is interesting that Gunderson was led to a semantics for  $\Box \rightarrow$  that makes his tracking<sub>c</sub> so similar to Sherri's tracks<sub>l</sub>.
- Unfortunately, Gunderson's " $\Box \rightarrow$ " doesn't really seem to be a *conditional* anymore — especially on the (LD) reading.
- This makes me wonder why we're bothering to "reverse engineer" a semantics for " $\Box \rightarrow$ " to suit a non-redundant account of tracks<sub>c</sub>. Why not cut out the middle-man here?
- That is, why not just work directly with likelihoods from the outset, and dispense with counterfactual talk altogether?
- Moral: the energy spent trying to come up with semantics for " $\Box \rightarrow$ " suitable for non-redundant tracking<sub>c</sub> is better spent doing what Sherri does so well in her book [4].

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[1] Eells, E. and Fitelson, B., 2002, "Symmetries and Asymmetries in Evidential Support", *Philosophical Studies* 107: 129-142.

[2] L.B. Gunderson, 2004, "Outline of a New Semantics for Counterfactuals", *Pacific Philosophical Quarterly*, 85: 1-20.

[3] R. Nozick, 1981, *Philosophical Explanations*, OUP.

[4] S. Roush, 2006, *Tracking Truth: Knowledge, Evidence, and Science*, OUP.

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