Judgment aggregation

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For references:
philpapers.org/archive/LISTTO.pdf

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Introduction

- The aim of this talk is to introduce the theory of judgment aggregation.
- To give you a flavour of this theory, my focus will be on:
  - explaining the key problems, concepts and ideas,
  - presenting a few illustrative theorems.
- I will set aside as many technicalities as possible.

For a more technical survey, see: philpapers.org/archive/LISTTO.pdf

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Introduction

- **Key question:** How can a group of individuals form collective judgments (true/false, acceptance/rejection) on some propositions, based on the group members' individual judgments on these propositions?
- **Key characteristic:** The propositions are interconnected, i.e., the judgments on some propositions logically constrain the judgments on others.

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Introduction

- **Relevance:** Judgment aggregation problems arise in many different settings:
  - political (e.g., legislatures, committees, referenda)
  - epistemic (e.g., expert panels, groups of scientists, juries and courts)
  - computational (e.g., merging multiple information sets or data streams, combining the 'judgments' of different search engines etc.)

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Overview

- Two paradoxes of judgment aggregation
- The judgment aggregation model
- An impossibility result and its interpretation
- Escape routes
- A map of logical space

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The doctrinal paradox

(Kornhauser/Sager 1986, cf. Vacca 1921)

- A three-member court has to decide whether a defendant is liable for breach of contract.
- Three propositions are under consideration:

  p : The defendant did action X.

  q : The defendant was contractually obliged not to do X.

  r : The defendant is liable.
- According to legal doctrine, p and q are jointly necessary and sufficient for r. In short, $r \leftrightarrow (p \wedge q)$.

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The doctrinal paradox

	p (action)	q (obligation)	r (liable)
Individual 1	True	True	True
Individual 2	True	False	False
Individual 3	False	True	False
Majority	True	True	False

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The doctrinal paradox

	p (action)	*q* (obligation)	*r* (liable)
Individual 1	True	True	(True)
Individual 2	True	False	(False)
Individual 3	False	True	(False)
Premise-based rule	True	True =>	True

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The doctrinal paradox

	p (action)	*q* (obligation)	*r* (liable)
Individual 1	(True)	(True)	True
Individual 2	(True)	(False)	False
Individual 3	(False	True)	False
Conclusion-based rule	-	-	False

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The doctrinal paradox

- The **doctrinal paradox** consists in the fact that:

  In decisions on a conclusion based on multiple premises, premise-based and conclusion-based rules may lead to opposite outcomes.

- But we can make second observation from the example:

  Relative to the connection rule ("legal doctrine") $r \leftrightarrow (p \wedge q)$, the majority judgments on p, q, and r are inconsistent, i.e.,

  $\{p, q, \neg r\} \cup \{r \leftrightarrow (p \wedge q)\}$ is an inconsistent set.

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A more general problem

- This observation suggests a more general problem, not dependent on the presence of a legal doctrine or exogenously given connection rule, nor on any distinction between premises and conclusions.
- The more general problem consists in the fact that:

  Majority voting on multiple, logically connected propositions may generate inconsistent majority judgments.

- Pettit (2001) has called this problem the **discursive dilemma**.

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An illustration

- An expert panel seeks to make collective judgments on the following propositions:

  p : Atmospheric greenhouse gases are above threshold X.

  $p \rightarrow q$: If greenhouse gases are above threshold X, the Arctic ice-shield will disappear.

  q : The Arctic ice-shield will disappear.

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An illustration

	p	$p \rightarrow q$	q
Individual 1	True	True	True
Individual 2	False	True	False
Individual 3	True	False	False
Majority	True	True	False

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Condorcet's paradox revisited

	$x > y$	$y > z$	$x > z$
Individual 1 ($x > y > z$)	True	True	True
Individual 2 ($y > z > x$)	False	True	False
Individual 3 ($z > x > y$)	True	False	False
Majority	True	True	False

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The earlier example

	p	$p \rightarrow q$	q
Individual 1	True	True	True
Individual 2	False	True	False
Individual 3	True	False	False
Majority	True	True	False

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Where we are so far

- The present examples
  - show that majority voting on interconnected propositions may generate inconsistent majority judgments,
  - but leave open whether other aggregation rules are immune to this problem and, if so, which ones.
- I will now consider this question more generally.

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The judgment aggregation model

individual judgments

aggregation rule

collective judgments

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The model: propositions and the agenda

- The **propositions** are represented in **propositional logic**.
- There are:
  - **atomic propositions**, e.g., p, q, r,… (basic propositions);
  - **compound propositions** with the connectives $\neg$ (not), $\wedge$ (and), $\vee$ (or), $\rightarrow$ (if… then …), $\leftrightarrow$ (if and only if),
    e.g., $p \rightarrow q$, $p \wedge q$, $r \leftrightarrow (p \wedge q)$ (capturing interconnections).
- The **agenda**, denoted X, is the set of propositions (and their negations) on which judgments are to be made.
- In the expert-panel example above,
  $X = \{p, \neg p, p \rightarrow q, \neg(p \rightarrow q), q, \neg q\}$.

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The model: individuals and their judgments

- The **individuals** are denoted 1, 2, …, n ($n \geq 2$).
- Each individual i's **judgment set** is the set J_i of all propositions in X that individual i accepts, formally $J_i \subseteq X$.
- A judgment set J_i is
  - **consistent** if it is a logically consistent set;
  - **complete** (in X) if it contains a member of each pair $p, \neg p \in X$.
- A **profile of individual judgment sets** is an n-tuple $(J_1, J_2, \ldots, J_n)$.

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The model: aggregation rules

- An **aggregation rule** is a function F which maps each profile of individual judgment sets in some domain to a corresponding collective judgment set.
- Examples of aggregation rules:
  - majority voting;
  - supermajority or unanimity rules;
  - dictatorship of one individual;
  - inverse dictatorship of one individual;
  - premise- and conclusion-based voting.

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The key question

- Can we find aggregation rules that generate consistent collective judgments and satisfy some other desirable conditions?
- The answer to this question depends

  (i) on the conditions we impose on the aggregation rule, and

  (ii) on the nature of the logical interconnections between the propositions in the agenda.

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Conditions on an aggregation rule

- **Universal domain.** The aggregation rule *F* accepts as admissible inputs all profiles of consistent and complete individual judgment sets.
- **Collective rationality.** *F* generates consistent and complete collective judgment sets.
- **Anonymity.** *F* is invariant under permutations of the individuals in a profile.
- **Systematicity.** The collective judgment on each proposition depends only on individual judgments on that proposition (**"independence"**), and the pattern of dependence is the same for all propositions (**"neutrality"**).
- **Unanimity preservation.** Whenever all individuals submit the same judgment set, this becomes the collective one.
- Note: Majority voting satisfies all except collective rationality (and here it violates the consistency part).

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An impossibility result

- Suppose the agenda exhibits some **non-trivial interconnections** between propositions:

  It contains at least
  - two atomic propositions, p, q, and
  - their conjunction ($p \wedge q$) or disjunction ($p \vee q$) or material implication ($p \rightarrow q$).
- **Theorem 1.** (List/Pettit 2002)

  There exists no aggregation rule satisfying universal domain, collective rationality, anonymity, and systematicity.

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An impossibility result

- This theorem has been strengthened and extended in several ways.
- One result has become particularly salient, because it can be viewed as the judgment-aggregation analogue (and in fact generalization of) a classic impossibility theorem in social choice theory, namely Arrow's theorem.

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Arrow's theorem generalized

- **Theorem 2.**
  (Dietrich/List 2007, Dokow/Holzman 2010, drawing on Nehring/Puppe 2002)
  For any "strongly connected" agenda, any aggregation rule satisfying universal domain, collective rationality, independence, and unanimity preservation is a dictatorship.
- Some remarks:
  - This result applies when the agenda of propositions (X) is somewhat richer in logical connections than assumed for the previous theorem.
  - An example of a "strongly connected" agenda is one with at least two atomic propositions and both their conjunction and their disjunction.

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How should we interpret the present results?

IF (i) we are faced with judgment aggregation problems on agendas with non-trivial logical connections, and

(ii) we regard the specified conditions on an aggregation rule as indispensable,

THEN (non-dictatorial) judgment aggregation is impossible.

To avoid this conclusion, we must deny either (i) or (ii).

- Unless we can somehow avoid non-trivial judgment aggregation problems altogether, denying (i) does not seem to be an option.
- So what options do we have?

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Escape routes

We can:

- Accept a dictatorship
- Relax unanimity preservation
- Relax universal domain
- Relax collective rationality
- Relax systematicity/independence

} implausible

} more plausible

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Relaxing systematicity/independence

- If we relax this condition, there exist some attractive aggregation rules satisfying the other conditions, e.g.,
  - premise-based rules (as illustrated above),
  - conclusion-based rules (also illustrated above), which also require relaxing completeness,
  - sequential priority rules (List 2004),
  - distance-based rules (Pigozzi 2006).
- The cost of this route is manipulability by
  (i) agenda setting and
  (ii) strategic voting.

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Escape routes

We can:

- Accept a dictatorship
- Relax unanimity preservation
- Relax universal domain
- Relax collective rationality
- Relax systematicity/independence
- A final escape route:

  Give up the binary (true/false, accept/reject) format of judgments and admit judgments that come in degrees (e.g., credences)

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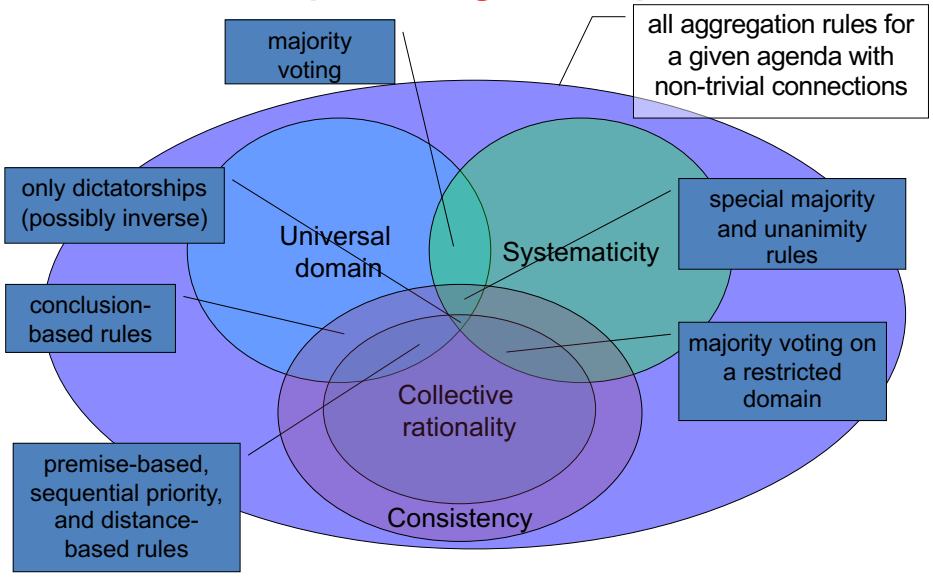
A map of logical space



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Literature (selection)

- Initial spark: "doctrinal paradox" (Kornhauser/Sager 1986,1993; Chapman 1998; cf. Vacca 1921), re-interpreted and generalized as a "discursive dilemma" (Pettit 2001; also Brennan 2001)
- Axiomatic model of judgment aggregation, combining social choice theory and logic (List/Pettit 2002, 2004)
- Technical work, on impossibility and possibility results (List 2003, Pauly/van Hees 2006, Dietrich 2006, 2007, Bovens/Rabinowicz 2006, Nehring/Puppe 2010, Gärdenfors 2006, Pigozzi 2006, Dietrich/List 2007, Dokow/Holzman 2010, Dietrich 2014, and others)
- Related to a large literature on social choice (e.g. Arrow 1951, Wilson 1975, Nehring/Puppe 2002)
- Precursors: Condorcet (18th century), Guilbaud (1966)
- For an overview, see my review article in *Synthese* (2014); see also List/Puppe (2009)

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