Introductory Notes on The Normativity of Logic (Harman, Field, MacFarlane, and Me)

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Harman's four main worries about the normativity of deductive logic are (briefly):

1. Reasoning doesn't follow the pattern of logical consequence. When one has beliefs $A_1, \ldots, A_n$, and realizes that they together entail B, sometimes the best thing to do isn't to believe B but to drop one of the beliefs $A_1, \ldots, A_n$.
2. We shouldn't clutter up our minds with irrelevancies, but we'd have to if whenever we believed A and recognized that B was a consequence of it we believed B.
3. It is sometimes rational to have beliefs even while knowing they are jointly inconsistent, if one doesn't know how the inconsistency should be avoided.
4. No one can recognize all the consequences of his or her beliefs. Because of this, it is absurd to demand that one's beliefs be closed under consequence. For similar reasons, one can't always recognize inconsistencies in one's beliefs, so even putting aside point (3) it is absurd to demand that one's beliefs be consistent. [Harman also discusses an analogue of this worry, which applies to (*e.g.*, probabilistic) *degrees* of belief — see below.]

Most people in the literature concede to Harman that (1) is a good reason to reject:

(I) If one realizes that $A_1, \ldots, A_n$ together entail B, then if one believes $A_1, \ldots, A_n$, one ought to believe B.

Moreover, most people also seem to think that the problem with (I) is that its "ought" takes *narrow-scope*. And, most people seem to think that the following *wide-scope* version of (I) avoids worry (1):

(II) If one realizes that $A_1, \ldots, A_n$ entail B, then one ought not [believe $A_1, \ldots, A_n$, and not believe B].

It is important to contrast (II) with a different wide-scope norm:

(III) If one realizes that $A_1, \ldots, A_n$ entail B, then one ought not [believe $A_1, \ldots, A_n$, and believe $\sim B$].

I presume that most people would also endorse (III), *modulo* preface-y considerations (to which we will return when we discuss Harman's worry (3), below). OK, but *why* might these wide-scope norms be correct (*qua* norms)? Here's one (naïve) explanation of *why* a norm like (III) seems (*prima facie*) to be correct.

> We realize that believing that A is equivalent to believing that A is true. If one realizes that A entails B, then one realizes that the inference from A to B preserves truth. So, suppose you realize that A entails B, and you believe that A. Now, in such a situation, if you were to believe that $\sim B$, then your beliefs would be inconsistent, which is bad because inconsistency entails that some of your beliefs are false; and, *qua believers* we ought (*ceteris paribus*) to avoid being in a situation where we know *a priori* some of our beliefs are false. Why? Because truth and the avoidance of falsehood are *fundamental epistemic aims of belief*. Moreover, if you were to *withhold judgment* on B (in this situation), this would also be rather odd, since you would be aware that the truth of B must obtain if the truth of A (which you now believe) obtains. So, from the point of view of epistemic rationality, what grounds could you have for withholding belief?

I don't mean to defend this "explanation". But, it doesn't sound crazy to me (*i.e.*, *prima facie*, it sounds no less crazy to me than the norms themselves do). The important point here is that this explanation makes essential use of the assumption that deductive entailment is a matter of (necessary) *truth-preservation*. Thus, such an explanation is *not open* to anyone (*e.g.*, Field) who *rejects* the claim that realizing that A entails B is tantamount to realizing that the inference from A to B *preserves truth*. To my mind, the *truth-connection* is quite important in epistemology, and it's difficult to see how we're going to explain the correctness or goodness of these sorts of norms, without appealing to it (at some stage). Remember, *Harman accepts* the claim that entailment is (necessary) truth-preservation. So, such explanations *are* available to Harman.

Next: Harman's worry (3). Harman points out that it (sometimes) seems reasonable to have beliefs that one knows are inconsistent. Preface paradox cases are the most infamous examples of this phenomenon. My favorite is what I will call the *global preface case* (GPC). The general structure of the GPC is as follows:

- *A fortiori*, you believe each element of your current belief set $\mathcal{B}$.
- You know your belief set $\mathcal{B}$ is very large and complex.
- You know that all human beings are prone to epistemic error, and you ∴ (reasonably) believe that all sufficiently large and complex (human) belief sets contain some falsehoods (*viz.*, errors).

- On this basis, you (reasonably) believe that $\mathcal{B}$ contains some falsehoods.
- But, now, $\mathcal{B}$ is *inconsistent*, and you know this *a priori*. Thus, you now know *a priori* that some of your beliefs are false (though you know not *which*), and you also know *a priori* that *every* proposition *follows* (in a classical, truth-preservation sense) *from* your current belief set $\mathcal{B}$.
- But, presumably, there are still *some* propositions p that you should *not* believe — *e.g.*, there will (presumably) still be some p's *that you know are false*, and you shouldn't believe *those* p's! Indeed, it even seems plausible to suggest that you should believe that $\sim p$ *if you know that* p *is false.*
- If all of this reasoning makes sense, then we seem to have examples in which is it "OK" to have inconsistent beliefs, and also examples in which both (II) and (III) seem to be false.

In light of preface-type cases, Field suggests the following alternatives to (II) and (III):

(II*) If one realizes that $A_1, \ldots, A_n$ entail B, then one ought not [believe $A_1 \mathbin{\&} \cdots \mathbin{\&} A_n$, and not believe B].

(III*) If one realizes that $A_1, \ldots, A_n$ entail B, then one ought not [believe $A_1 \mathbin{\&} \cdots \mathbin{\&} A_n$, and believe $\sim B$].

Even the preface paradox cases (GPC) are (intuitively) *not* counterexamples to (II*) or (III*). Let's think about how (II*)/(III*) avoid such preface (GPC) counterexamples. Nobody (Harman included) thinks it's OK for you to have *particular* beliefs which you *know* to be *false*. Thus, the key to the GPC is that you have no idea *which* of your beliefs $A_1, \ldots, A_n$ is false, and this is why you are under no rational pressure to *revise* any of your *particular* beliefs. But, *if* you believed $\mathbb{A} \overset{\text{def}}{=} A_1 \mathbin{\&} \cdots \mathbin{\&} A_n$ in a GPC, *then* you *would* be under rational pressure to revise *that* belief, since you'd *know (a priori) that* $\mathbb{A}$ *is (necessarily) false.* This is how (II*)/(III*) avoid preface-type counterexamples. Next, a brief digression concerning GPC's *vs argumentative contexts.*

There is something highly artificial about GPC's. And, even *non-global* preface cases share a similar artificiality. The artificiality is that we're talking about *such* large and complex sets of beliefs. As a result, it's not even clear we can *grasp* or *make sense* of the conjunctive propositions $A_1 \mathbin{\&} \cdots \mathbin{\&} A_n$ alluded to in (II*) and (III*). Consequently, it seems that (II*) and (III*) are *trivially* true in such contexts. If that's right, then (insofar as they are *motivated by* preface-type cases) they are not very interesting principles. Having said that, I do think these principles can be non-trivially applied in other contexts. Allow me to explain. In a non-global preface case, we're not talking about your entire belief set $\mathcal{B}$. Rather, we're talking about a (relatively small-ish) proper subset of $\mathcal{B}$ — say, a book $\mathcal{B}' \subset \mathcal{B}$ that you have written. As long as the book is a *sufficiently* large and complex subset of your beliefs, the same "preface reasoning" will lead us to the conclusion that it's "OK" for you to believe that $\mathcal{B}'$ contains some falsehoods — even while you believe *each element* of $\mathcal{B}'$. And, that's sufficient to ensure the desired "preface paradox structure". However, even in such NGPC's, the *conjunction* of the elements of $\mathcal{B}'$ is likely to be beyond our grasp (and so not a reasonable object of belief, for completely independent reasons). Thus, even in NGPC's, principles like (II*) and (III*) seem otiose. But, let's zoom-in even further. Let's talk about *a particular argument* $\mathcal{A} \subset \mathcal{B}'$ in your book. In this argument, there is a *conclusion* Q and a set of *premises* $\mathcal{P} = \{P_1, \ldots, P_n\}$. And, let us suppose that the argument $\mathcal{A}$ constitutes *the reasons why* (or *the basis upon which*) you believe Q. They are also being offered by you as *providing some reason* for *others* to believe Q. In these sorts of *argumentative contexts*, I'm inclined to think that principles (II*) and (III*) *can* be non-trivially applied. In fact, I think I'd be inclined to accept something like the following principle for argumentative contexts (as an evaluative ideal):

(†) One should offer (or accept) a deductive argument (*i.e.*, an argument one realizes is deductively valid) $P_1, \ldots, P_n \therefore Q$ ***for*** *a claim* Q *only if one believes that said argument is sound* (*viz.*, not *merely* valid).

Let's call contexts in which (†) holds *argumentative contexts*. If there are such contexts, they will be different from *preface* contexts (where the A_i's are not *premises*). And, they will give rise to stronger norms than those discussed by Field. I will return to such norms, below, when I get to constraints on *degrees* of belief.

Moving on from Harman's worries (1) and (3), I will discuss (some of) Field's responses to worry (4) next. Field breaks (4) down into two component worries. Worry (4a) is what he calls the "computational aspect" of (4). Worry (4b) is deeper and more subtle. There is *prima facie* reason to *strengthen* (II*) and (III*), as:

(II*$_s$) If $A_1, \ldots, A_n$ entail B, then one ought not [believe $A_1 \mathbin{\&} \cdots \mathbin{\&} A_n$, and not believe B].

(III*$_s$) If $A_1, \ldots, A_n$ entail B, then one ought not [believe $A_1 \,\&\, \cdots \,\&\, A_n$, and believe $\sim B$].

A reason for wanting to remove the "one realizes that" caveat from (II*)/(III*) is discussed by MacFarlane:

> [one natural reading of (II*)/(III*) suggests that] the more ignorant we are of what follows logically from what, the freer we are to believe whatever we please however logically incoherent it is. But this looks backward. We seek logical knowledge so that we know how we ought to revise our beliefs: not just how we *will be* obligated to revise them when we acquire this logical knowledge, but how we *are* obligated to revise them even now, in our state of ignorance.

But, if we move to (II*$_s$)/(III*$_s$), this seems to be an *over*-reaction to MacFarlane's worry about (II*)/(III*), since it is *not humanly possible* to believe all the logical consequences of one's beliefs (even in the *wide-scope* sense implied by II*$_s$/III*$_s$). And, this seems to drain the resulting "norms" of their *normative force.* Or, to put it more positively, it seems quite clear (but not for Williamson?) that it can be reasonable to believe things which *turn out* to be inconsistent, for some very subtle reason that has eluded all investigators for many years.[1] Field proposes the following "obviousness cheat" as a "middle-way" of amending (II*) and (III*):

(II*$_o$) If $A_1, \ldots, A_n$ *obviously* entail B, then one ought not [believe $A_1 \,\&\, \cdots \,\&\, A_n$, and not believe B].

(III*$_o$) If $A_1, \ldots, A_n$ *obviously* entail B, then one ought not [believe $A_1 \,\&\, \cdots \,\&\, A_n$, and believe $\sim B$].

I don't think that Field's "obviousness" maneuver is too much different from Harman's requirement that the agent "realize" that the entailment hold. It seems to me that "obvious" is an *epistemic* term, and it doesn't really address MacFarlane's worry about "the true logic" *vs* "the logic someone (reasonably, but perhaps falsely) accepts". Finally, Field proposes laying down a *list* of "simple" rules, single applications of which are to count as "the obvious entailments", and then restricting the domain of application of the principles:

(II*$_r$) If B follows from $A_1, \ldots, A_n$ *by a single application of a rule on the list*, then one ought not [believe $A_1 \,\&\, \cdots \,\&\, A_n$, and not believe B].

I'm not sure how this addresses MacFarlane's worry, since (presumably) one could still be ignorant of even the correct "simple" logical rules. Indeed, Field continues to be worried about this. He says:

> On what is probably the most natural interpretation of (II*$_r$), the "simple rules" it talks about are simple rules of *the correct logic*. …But there is a case to be made that this consequence of (II*$_r$) is incorrect. Suppose that classical logic is in fact correct, but that Bob has made a very substantial case for weakening it (we may even suppose that no advocate of classical logic has yet to give an adequate answer to his case). Suppose that usually Bob reasons in accordance with the non-classical logic he advocates, but that occasionally he slips into classical reasoning that is not licensed by his own theory. Isn't it when he *slips and reasons classically* that he is violating rational norms? But (II*$_r$) (on the natural interpretation) says that it is the other occasions, when he follows the logic he believes in, that he is violating norms.

This is *hard* (see *fn.* 1). Moreover, there is something strange about this passage. What does it *mean* for a logic to be "correct"? Field believes that the "correct" logic is *not* going to be a (generally) *truth-preserving* logic (call this thesis NTP). He does not believe NTP on "epistemic" grounds (*i.e.*, his arguments for NTP do not appeal to *epistemic* principles relating to the structure of "good inferences"). Rather, Field believes NTP because he believes that the best formal theory of truth contains a deducibility relation $\vdash$ that does not preserve truth (in a specific, technical sense). And, he thinks that the "true entailment relation" should be co-extensional with the $\vdash$-relation in our best formal theory of truth. I won't get into Field's meta-logical arguments for these claims. But, they do raise the *possibility* of providing non-epistemic (and non-question-begging) arguments about the nature of "the correct logic". I think Harman is unfair to Field on this point. But, I don't have the space here to get into this subtle part of Field's work. Next: Field on "clutter avoidance".

The way Field handles Harman's "clutter avoidance" problem (2) is by making an *implicit/explicit* belief distinction. Specifically, Field proposes the following principle for explicit/implicit full belief:

(**) If $A_1, \ldots, A_n$ together obviously entail B (or if B follows from $A_1, \ldots, A_n$ *by a single application of a rule on the list*), then one shouldn't *explicitly* believe $A_1, \ldots, A_n$ without (at least) *implicitly* believing B.

Field also talks (as does Milne) about analogous "bridge principles" involving *degrees* of belief. He proposes the following norm relating (obvious/simple) entailment and (implicit constrains on?) degree of belief:

(D) If $A_1, \ldots, A_n$ together obviously entail B, then this imposes the following (implicit?) constraint on one's degrees of belief in $A_1, \ldots, A_n$ and B [which he denotes $P(A_1), \ldots, P(A_n), P(B)$]: $P(B) \geq P(A_1) + \cdots + P(A_n) - (n-1)$.

[1] Field gives the example of the discovery of continuous functions mapping the unit interval onto the unit square, which everyone assumed were impossible, until Peano *demonstrated* such functions. This also happens in *logic* (think Russell, Gödel, *etc.*). Here, we're bumping up against an *internalism/externalism* debate about justifications concerning beliefs involving logic/math, *etc.*

In the special case where $n = 1$, this reduces to the following "bridge principle" for degrees of belief:

(D_1) If A obviously entails B, then this imposes the following (implicit?) constraint on one's d.o.b's: $P(A) \le P(B)$.

There is a technical fact about *probabilities* — embodied in the following Theorem — that undergirds (D):

> **Theorem**. If $A_1, \ldots, A_n \vDash B$, then $\Pr(B) \ge \Pr(A_1) + \cdots + \Pr(A_n) - (n-1)$.
> [Note: provided $\Pr(A_1) + \cdots + \Pr(A_n) - (n-1) > 0$, this lower-bound is *the tightest bound imposed by logic*.]

So, *if* degrees of belief are *probabilities* (more on *that* issue later in the semester), then one can see how one might end-up with a norm like (D_1). Here is one possible "explanation" of the correctness of (D_1), which is open to those who think of entailment as *truth-preservation*, *and* $P(\cdot)$ as a measure of *probability* of truth:

> If A obviously entails B, then it is obviously the case that all situations/possible worlds in which A is true are also situations/possible worlds in which B is true. And, if it's obvious to you that whenever A is true, B must also be true, then you should not assign a degree of belief to A that is greater than the degree of belief you assign to B. Why? Because, in such a situation, it is obvious to you that there are *strictly more possibilities* in which B is true than there are possibilities in which A is true. And, this implies that the *probability* of B's truth *cannot be less than* the *probability* of A's truth (*whatever* Pr-*measure* one has in mind here). Finally, degrees of belief just *are* probabilities of truth. *Ergo*, the reasonableness of (D_1).

Once again, I'm not defending this "explanation", but (a) it seems no less crazy than the norm (D_1) itself, (b) it is not open to Field (or anyone who thinks entailment does *not* generally preserve truth — NTP), and (c) I can't think of any *alternative* explanation of the correctness of (D_1), which would be open to NTPers like Field, and which would make more sense. That's not an argument, it's a challenge to NTPers who wish to defend these sorts of norms. In closing, one last digression about *argumentative contexts* and *probabilities*.

One of the "odd" things about probabilities (and, hence, probabilistic degrees of belief) is that substantial amounts of probability can be "lost" in the course of (classically) valid deductions. Interestingly, though, this can only happen when the deductive arguments in question have *multiple* premises. Let's go back to our Theorem (above). When an argument has only one premise ($n = 1$ case of Theorem), its conclusion cannot be less probable than its premise. But, suppose $n \ge 2$. Then, the probability of the conclusion *can* be less than the probability of *any individual premise* (though precise *constraints* will be imposed by our Theorem).

Suppose that each premise P_i of an argument $\mathcal{A}$ has a probability (or, if you prefer, a rational degree of belief) greater than $1 - \epsilon$, for some small ϵ. Then, the following is a Corollary of our Theorem:

> **Corollary**. If, for all i, $\Pr(P_i) > 1 - \epsilon$, then $\Pr(P_1 \,\&\, \cdots \,\&\, P_n) > 1 - n\epsilon$.

When $n = 1$, this Corollary is *trivial*, since in that case the conjunction of the premises just *is* the sole premise of the argument itself. But, when $n > 1$, this Corollary constrains how much probability *can* be "lost" in an n-premise application of *conjunction introduction*. And, this relates back to our previous digression concerning *argumentative contexts*. Recall, I proposed the following principle for such contexts:

(†) One should offer (or accept) a deductive argument (*i.e.*, an argument one realizes is deductively valid) $P_1, \ldots, P_n \therefore Q$ ***for** a claim Q only if one believes that said argument is sound* (*viz.*, not *merely* valid).

I also think the following principle is pretty plausible:

(‡) If S believes p, then S's degree of belief in p should *not* be *very low* (*i.e.*, if S believes p, then $P(p)$ should be greater than some number, say, $1 - \delta$, where δ may depend on context, *etc.*).

Putting (†) and (‡) together yields the following (stronger-than-Corollary) constraint on degrees of belief:

($\star$) If S offers (or accepts) a deductive argument (*i.e.*, an argument that S realizes is deductively valid) $P_1, \ldots, P_n \therefore Q$ ***for** a claim Q*, then S's degree of belief in *the conjunction* of its premises ($P_1 \,\&\, \cdots \,\&\, P_n$) should be greater than $1 - \delta$ (where δ may depend on context, *etc.*).

In contexts where (†) makes sense (if there be such argumentative contexts), we will also have a stronger constraint on one's degrees of belief in the *conclusion* Q. This is because, in such contexts, we have (effectively) "collapsed" the argument in question to a *single-premise* argument, *from* $P_1 \,\&\, \cdots \,\&\, P_n$ *to* Q. Hence:

($\star\star$) If S offers (or accepts) a deductive argument (*i.e.*, an argument that S realizes is deductively valid) $P_1, \ldots, P_n \therefore Q$ ***for** a claim Q*, then S's degree of belief in Q should be greater than $1 - \delta$.

So, while Field only ends-up with the *weak* $P(Q) \ge P(P_1 \,\&\, \cdots \,\&\, P_n) > 1 - n\epsilon$ constraint, I would argue that *proper deductive argumentative practice* imposes the *stronger* constraint: $P(Q) \ge P(P_1 \,\&\, \cdots \,\&\, P_n) > 1 - \delta$.