Individual Coherence & Group Coherence

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- Today's presentation is about (i) formal, (ii) synchronic, (iii) epistemic (iv) coherence (v) requirements/norms.
  - (i) *Formal* coherence is to be distinguished from other sorts of coherence discussed in contemporary epistemology (*e.g.*, in some empirical, truth/knowledge-conducive sense [1]).
    - Ideally, whether a set of judgments is coherent (in the present sense) should (in principle) be determinable *a priori*.
  - (ii) *Synchronic* coherence has to do with the coherence of a set of judgments held by an agent S *at a single time* t.
    - So, we'll *not* be discussing any *diachronic* requirements.
  - (iii) *Epistemic* coherence is meant to involve some distinctively epistemic values (*e.g.*, *accuracy* and *evidential support*).
    - This is to be distinguished from *pragmatic* coherence (*e.g.*, immunity from arbitrage, dutch books, and the like).
  - (iv) *Coherence* should have something to do with how a set of judgments "hangs together" — in a wide/global sense.
  - (v) *Requirements/norms* are *evaluative*, and (merely) *necessary* (*i.e.*, violation is *ir*rational $\not\Rightarrow$ satisfaction is *rational*).

- Here is a — perhaps *the* — paradigm formal CR:
  - **The Consistency Norm for Belief** (CB). Agents should have *collections/sets* of beliefs that are *logically consistent*.
- (CB) follows from the following narrow/local norm:
  - **The Truth Norm for Belief** (TB). Agents should have beliefs that are *true* (*i.e.*, each *individual* belief should be true).
- Alethic norms [(CB)/(TB)] can conflict with evidential norms.
  - **The Evidential Norm for Belief** (EB). Agents should have beliefs that are *supported by the evidence*.
- In some cases (*e.g.*, preface cases), agents satisfy (EB) while violating (CB) — this generates an alethic/evidential *conflict*.
- Such alethic/evidential conflicts needn't give rise to states that receive an (overall) evaluation as *ir*rational (nor must they inevitably give rise to rational *dilemmas*) [4, 16, 11].
- We won't argue (much) for this claim today. We'll call this claim about the existence of some such cases *the datum*.

- Some philosophers construe *the datum* as reason to believe that ($\star$) *there are no coherence requirements for full belief*.
- Christensen [4] thinks (a) *credences* do have coherence requirements (*probabilism*); ($\star$) full beliefs do *not*; (b) what *seem* to be CRs for full belief can be explained *via* (a).
- Kolodny [16] agrees with ($\star$), but he disagrees with (a) and (b). He thinks (c) full belief is *explanatorily indispensible*; (d) there are *no* coherence requirements for *any* judgments; (e) what *seem* to be CRs for full belief can be explained *via* (EB).
- Christensen & Kolodny *agree* — *trivially*, *via* ($\star$) — that:
  - (†) *If* there are *any* coherence requirements for full belief, *then* (CB) is a coherence requirement for full belief.
- We [2, 8] agree with Christensen on (a) and Kolodny on (c), but we disagree with them on ($\star$), (d), (e), and (†). We'll explain how to ground "conflict-proof" CRs for full belief, by analogy with Joyce's [15, 13] argument(s) for probabilism.

- We won't rehearse Joyce's argument(s) here. We'll just give our analogous argument for our new (full belief) coherence requirement(s). First, background assumptions/notation.
  - $B(p) \stackrel{\text{def}}{=} S$ believes that p. $D(p) \stackrel{\text{def}}{=} S$ disbelieves that p.
- S makes judgments regarding propositions in a (finite) *agenda* ($\mathcal{A}$) of (classical, possible-worlds) propositions. We'll use "**B**" to denote the *set* of S's judgments on $\mathcal{A}$. [In other words, **B** is just a *finite binary vector* of Bs and Ds.]
- We'll assume the following about B/D on $\mathcal{A}$. The first two assumptions are integral to the framework. The last two assumptions are made for simplicity & can be relaxed [7].
  - **Accuracy conditions**. $B(p)$ [$D(p)$] is accurate iff p is T [F].
  - **Incompatibility**. $B(p) \Rightarrow \sim D(p)$.

  - **Opinionation**. $B(p) \vee D(p)$.
  - **(NC)** $D(p) \equiv B(\sim p)$.
- Note: in the J.A. literature, all four of these are presupposed.

- Now, we can explain how our new CRs were discovered, by analogy with Joyce's [15, 13] argument(s) for probabilism.
- Both arguments can be seen as involving three key steps.
- **Step 1**: Define $\mathring{\mathbf{B}}_w$ — the *vindicated (viz., alethically ideal* or *perfectly accurate)* judgment set, at world w.
  - $\mathring{\mathbf{B}}_w$ contains $B(p)$ [$D(p)$] iff p is true (false) at w.
    - Heuristically, we can think of $\mathring{\mathbf{B}}_w$ as the set of judgments that an omniscient agent would have (at w).
- **Step 2**: Define $d(\mathbf{B}, \mathring{\mathbf{B}}_w)$ — a measure of distance between **B** and $\mathring{\mathbf{B}}_w$. That is, a measure of **B**'s *distance from vindication*.
  - $d(\mathbf{B}, \mathring{\mathbf{B}}_w) \stackrel{\text{def}}{=}$ the number of inaccurate judgments in **B** at w.
    - *i.e.*, *Hamming distance* [6] between the binary vectors **B**, $\mathring{\mathbf{B}}_w$.
- **Step 3**: Adopt a *fundamental epistemic principle*, which uses $d(\mathbf{B}, \mathring{\mathbf{B}}_w)$ to ground a coherence requirement for **B**.
- This last step is the philosophically crucial one...

- Given our choices at Steps 1 and 2, there is ***a*** choice we can make at Step 3 that will yield (CB) as a requirement for **B**.

  Possible Vindication (PV). There exists some possible world w at which all of the judgments in **B** are accurate. Or, to put this more formally, in terms of d: $(\exists w)[d(\mathbf{B}, \mathring{\mathbf{B}}_w) = 0]$.
- Possible vindication is *one* way we could go here. But, our framework is much more general than the classical one. It allows for many other choices of fundamental principle.
- Like Joyce [15, 13] — who makes the analogous move with credences, to ground probabilism — we retreat from (PV) to the weaker: *avoidance of (weak) dominance in* $d(\mathbf{B}, \mathring{\mathbf{B}}_w)$.

  Weak Accuracy-Dominance Avoidance (WADA).

  There does *not* exist an alternative belief set $\mathbf{B}'$ such that:
  (i) $(\forall w)[d(\mathbf{B}', \mathring{\mathbf{B}}_w) \leq d(\mathbf{B}, \mathring{\mathbf{B}}_w)]$, and
  (ii) $(\exists w)[d(\mathbf{B}', \mathring{\mathbf{B}}_w) < d(\mathbf{B}, \mathring{\mathbf{B}}_w)]$.
- Completing Step 3 in this way reveals new CRs for **B**...

- Initially, it may seem undesirable for an account of epistemic rationality to allow for (kosher) doxastic states that cannot be perfectly accurate. But, as Foley [11] explains

  ...if the avoidance of recognizable inconsistency were an absolute prerequisite of rational belief, we could not rationally believe each member of a set of propositions and also rationally believe of this set that at least one of its members is false. But this in turn pressures us to be unduly cautious. It pressures us to believe only those propositions that are certain or at least close to certain for us, since otherwise we are likely to have reasons to believe that at least one of these propositions is false. At first glance, the requirement that we avoid recognizable inconsistency seems little enough to ask in the name of rationality. It asks only that we avoid certain error. It turns out, however, that this is far too much to ask.
- **Analogy**: consider the analogue of (PV) for credences. It requires all credences to be *extremal* (also too strong).
- In the full belief case, we need preface/lottery type cases to see the analogous point. But, Foley's point is a general one.

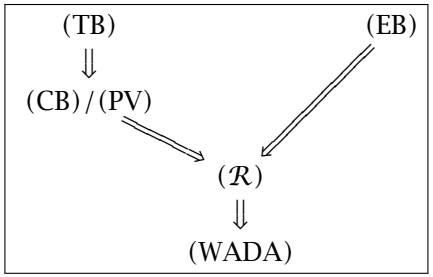
- Ideally, we want a coherence requirement that [like (CB)] can be motivated by considerations of *accuracy* (*viz.*, a CR that is *entailed by* alethic requirements such as TB/CB/PV).
- But, in light of (*e.g.*) preface cases, we also want a CR that is weaker than (CB). More precisely, we want a CR that is weaker that (CB) in such a way that it is also entailed by (EB).

☞ Such a CR would be alethic/evidential "conflict-proof".

- We can show that our new CRs [*e.g.*, (WADA)] fit the bill, *if* we assume the following "probabilistic-evidentialist" necessary condition for the satisfaction of (EB).
  - **Necessary Condition for Satisfying (EB).** S satisfies (EB), *i.e.*, S's judgments are (all) *justified*, ***only if*** **($\mathcal{R}$)** there exists some probability function $\Pr(\cdot)$ which probabilifies each of her beliefs and dis-probabilifies each of her disbeliefs.
- "Probabilistic-evidentialists" will disagree about *which* $\Pr(\cdot)$ undergirds (EB) [3, 20, 12, 14]; but, they agree on (EB) $\Rightarrow$ ($\mathcal{R}$).

- Here are the logical relationships between key norms:

Truth Norm for Belief:	(TB)
	$\Downarrow \not\Uparrow$
Consistency Norm for Belief (*viz.*, PV):	(CB)/(PV)
	$\Downarrow \not\Uparrow$
☞ Weak Accuracy-Dominance Avoidance:	(WADA)
	$\Uparrow \not\Downarrow$
Evidential Norm for Belief:	(EB)



- See Extras slide 17 for a more detailed map with 10 norms.

- Suppose we have a panel of three judges (J_1, J_2, J_3). This panel will vote on an *agenda*, which stems from:

  Question. In the reunited Germany, should the German parliament and the seat of government move to Berlin or stay in Bonn?
- Suppose the panel votes on these two (atomic) *premises*:
  - $P \stackrel{\text{def}}{=}$ the parliament should move.
  - $G \stackrel{\text{def}}{=}$ the seat of government should move.
- There is also the following "conclusion" the truth-value of which is determined by the truth-values of the premises:
  - $P \,\&\, G \stackrel{\text{def}}{=}$ both the parliament and the seat of government should move.
- Suppose the judges render the following judgments (votes):

	P?	G?	$P \,\&\, G$?
J_1	B	D	$\Rightarrow D$
J_2	D	B	$\Rightarrow D$
J_3	B	B	$\Rightarrow B$

- For each judge, the conclusion judgment is determined by ($\Rightarrow$) the premise judgments (*i.e.*, assuming judges are *cogent*).

- This is the simplest example of the *doctrinal paradox* ([17], [19]).

Doctrinal Paradox

	P?	G?	$P \,\&\, G$?
J_1	B	D	$\Rightarrow D$
J_2	D	B	$\Rightarrow D$
J_3	B	B	$\Rightarrow B$
Majority:	B	B	D

☞ Naïve majority aggregation can yield inconsistent aggregations of consistent individual judgment sets.

- Various modifications of naïve majority rule have been proposed, so as to "restore consistency." Example:
  - **Premise-Based Majority Procedure.** Use majority rule on premises, and then *enforce* cogency to yield conclusion.
- The premise-based procedure can make sense (esp. if the premises constitute the agenda that is explicitly voted on).

Premise-Based Majority Procedure

	P?	G?	P & G?
J_1	B	D	$\Rightarrow D$
J_2	D	B	$\Rightarrow D$
J_3	B	B	$\Rightarrow B$
Majority:	B	B	D (ignored)
Premise-majority + cogency:			$\Rightarrow \boldsymbol{B}$

- Another "consistent" variant of majority rule is:
  - **Conclusion-Based Majority Procedure**. Use majority rule on the conclusion (as implied by the cogency of the individual judges), and then *remain silent* on the premises.
- Procedures that are silent on some members of some agendas are called *incomplete*. For more on these, see [18].
- Question: how does *coherence* interact with aggregation rules?

- In our paper [2], we address the following three questions:
  - (Q1) If judges are *consistent*, must (naïve) majority be *coherent*?
  - (Q2) If judges are *coherent*, must (naïve) majority be coherent?
  - (Q3) Do *any* "good" procedures *always* preserve coherence?
- Re (Q1), the answer is YES. *Possibility Theorem* [2, Thm. 3].
  - Let $\Pr(p) \stackrel{\text{def}}{=}$ the proportion of judges who believe that p. Theorem 3 then follows as a corollary of $(\mathcal{R}) \Rightarrow$ (WADA).
- Regarding (Q2), the (short) answer is NO [2, Theorem 6].
  - There are (complex) examples in which (naïve) majority rule does *not* preserve coherence (for instance, [2, Appendix]).
- Regarding (Q2), the (long) answer is NO, but…[2, Theorem 4].
  - Virtually all agendas in the literature ("truth-functional" $\mathcal{A}$s) are such that (naïve) majority rule *does* preserve coherence.
- Re (Q3), the answer is NO. *Impossibility Theorem* [2, Theorem 6].
- This is just one application of our new CRs. We encourage you to substitute "coherence" for "consistency" and *explore*!

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(TB) S ought believe p just in case p is true.

(PV) $(\exists w)[d(\mathbf{B}, \mathring{\mathbf{B}}_w) = 0]$. That is, $\mathbf{B}$ is *deductively consistent*.

(SADA) $\nexists\, \mathbf{B}'$ such that: $(\forall w)[d(\mathbf{B}', \mathring{\mathbf{B}}_w) < d(\mathbf{B}, \mathring{\mathbf{B}}_w)]$.

$\sim$**($\exists_{\beta_1}$)** $\nexists\, \beta \subseteq \mathbf{B}$ s.t.: $(\forall w)\left[> \frac{1}{2} \textit{ of the members of } \beta \textit{ are inaccurate at } w\right]$.

$(\mathcal{R})$ $\exists$ a probability function $\Pr(\cdot)$ such that, $\forall p \in \mathcal{A}$:

$$B(p) \textit{ iff } \Pr(p) > \tfrac{1}{2}, \text{ and } D(p) \textit{ iff } \Pr(p) < \tfrac{1}{2}.$$

(EB) S ought believe p just in case p is supported by S's evidence.

Note: this assumes *only* $(\exists \Pr)(\forall p)\left[\Pr(p) > \frac{1}{2} \textit{ iff } B(p)\right]$.

$\sim$**($\exists_{\beta_2}$)** $\nexists\, \beta \subseteq \mathbf{B}$ s.t.: $\begin{array}{c} (\forall w)\left[\geq \frac{1}{2} \textit{ of the members of } \beta \textit{ are inaccurate at } w\right] \\ \& \\ (\exists w)\left[> \frac{1}{2} \textit{ of the members of } \beta \textit{ are inaccurate at } w\right] \end{array}$.

(WADA) $\nexists\, \mathbf{B}'$ s.t.: $(\forall w)[d(\mathbf{B}', \mathring{\mathbf{B}}_w) \leq d(\mathbf{B}, \mathring{\mathbf{B}}_w)]$ & $(\exists w)[d(\mathbf{B}', \mathring{\mathbf{B}}_w) < d(\mathbf{B}, \mathring{\mathbf{B}}_w)]$.

$\sim$**($\exists_{\beta_3}$)** $\nexists\, \beta \subseteq \mathbf{B}$ s.t.: $(\forall w)\left[\geq \frac{1}{2} \textit{ of the members of } \beta \textit{ are inaccurate at } w\right]$.

(NC) S disbelieves p iff S believes $\sim p$ [*i.e.*, $D(p) \equiv B(\sim p)$].

- Here is what the logical relations look like, among all of the 10 norms for (opinionated) **B**. [Double (single) arrows represent *known* (*conjectured*) entailments. And, if there is no path, then we believe (or conjecture) that there is no entailment.]

$$
\begin{tikzcd}
 & (\mathrm{TB}) \arrow[d, Rightarrow] & & (\mathrm{EB}) \arrow[dl, Rightarrow] & \\
 & (\mathrm{CB})/(\mathrm{PV}) \arrow[dr, Rightarrow] & & & \\
\sim(\exists_{\beta_3}) \arrow[d, Rightarrow] & & (\mathcal{R}) \arrow[ll, Rightarrow] \arrow[rr, Rightarrow] \arrow[d, Rightarrow] & & (\mathrm{WADA}) + (\mathrm{NC}) \\
\sim(\exists_{\beta_2}) \arrow[d, Rightarrow] \arrow[rr, Leftrightarrow] & & (\mathrm{WADA}) \arrow[d, Rightarrow] & & \\
\sim(\exists_{\beta_1}) \arrow[rr, Leftrightarrow] & & (\mathrm{SADA}) & &
\end{tikzcd}
$$

- Proof of the claim that $\sim(\exists_{\beta_2}) \Leftrightarrow$ (WADA).
  - ($\Leftarrow$) We'll prove the contrapositive. Suppose that some $\mathbf{S} \subseteq \mathbf{B}$ is a (β_2) witnessing set. Let $\mathbf{B}'$ agree with $\mathbf{B}$ on all judgments outside $\mathbf{S}$ and disagree with $\mathbf{B}$ on all judgments in $\mathbf{S}$. By the (β_2) definition of a witnessing set, $\mathbf{B}'$ weakly dominates $\mathbf{B}$ in distance from vindication $[d(\mathbf{B}, \mathring{\mathbf{B}}_w)]$. Thus, $\mathbf{B}$ is incoherent.
  - ($\Rightarrow$) [Contrapositive again.] Suppose $\mathbf{B}$ is incoherent, *i.e.*, that there is some $\mathbf{B}'$ that weakly dominates $\mathbf{B}$ in distance from vindication $[d(\mathbf{B}, \mathring{\mathbf{B}}_w)]$. Let $\mathbf{S}$ be the set of judgments on which $\mathbf{B}$ and $\mathbf{B}'$ disagree. Then, $\mathbf{S}$ is a (β_2) witnessing set.
- A similar proof can be given for: $\sim(\exists_{\beta_1}) \Leftrightarrow$ (SADA).
- The following two claims are conjectured to be true.
  - $\sim(\exists_{\beta_3}) \rightarrow (\mathcal{R})$.
  - $(\mathcal{R}) \leftarrow$ (WADA) & (NC).
- These two questions remain open. We know there are no small counterexamples, but we see no proof strategy (help!).

- Proof of the claim that $(\mathcal{R}) \Rightarrow$ (WADA).

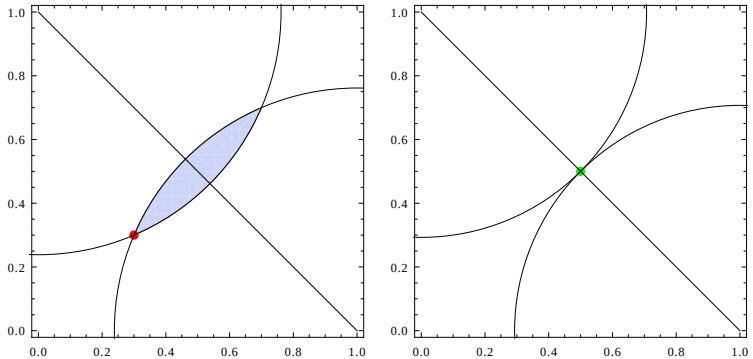
  Let Pr be a probability function that represents **B** in sense of $(\mathcal{R})$. Consider the expected distance from vindication of a belief set — the sum of $\Pr(w)d(\mathbf{B}, \mathring{\mathbf{B}}_w)$. Since $d(\mathbf{B}, \mathring{\mathbf{B}}_w)$ is a sum of components for each proposition (1 if **B** disagrees with w on the proposition and 0 if they agree), and since expectations are linear, the expected distance from vindication is the sum of the expectation of these components. The expectation of the component for disbelieving p is $\Pr(p)$ while the expectation of the component for believing p is $1 - \Pr(p)$. Thus, if $\Pr(p) > 1/2$ then believing p is the attitude that uniquely minimizes the expectation, while if $\Pr(p) < 1/2$ then disbelieving p is the attitude that uniquely minimizes the expectation. Thus, since Pr represents **B**, this means that **B** has strictly lower expected distance from vindication than any other belief set with respect to Pr. Suppose, for *reductio*, that some $\mathbf{B}'$ (weakly) dominates **B**. Then, $\mathbf{B}'$ must be no farther from vindication than **B** in any world, and thus $\mathbf{B}'$ must have expected distance from vindication no greater than that of **B**. But **B** has strictly lower expected distance from vindication than any other belief set. Contradiction. $\therefore$ no $\mathbf{B}'$ can dominate **B**, and so **B** must be coherent.

- To appreciate the significance of $(\mathcal{R}) \Rightarrow$ (WADA), it is helpful to think about a standard lottery example.
- Consider a fair lottery with n tickets & exactly one winner.
- For each $j \leqslant n$ (for $n \geqslant 3$), let p_j be the proposition that the jth ticket is *not* the winning ticket. And, let q be the proposition that *some* ticket *is* the winner.
- Finally, let LOTTERY be the following judgment set:

$$\{B(p_j) \mid 1 \leqslant j \leqslant n\} \cup \{B(q)\}$$

- It follows from $(\mathcal{R}) \Rightarrow$ (WADA) that LOTTERY is *coherent*.
  - Consider the probability function $\Pr(\cdot)$ that assigns each ticket an equal probability $\left(\frac{1}{n}\right)$ of winning. This function *represents* LOTTERY in precisely the sense required by $(\mathcal{R})$.
- But, of course, LOTTERY is logically *inconsistent*. This nicely explains *why* (WADA) is *strictly weaker* than (CB)/(PV).

- de Finetti [5] proved the following result, which is a simple (formal) precursor to Joycean arguments for probabilism.
  - **Theorem** (de Finetti). A credal set **b** is dominated in (Euclidean) "distance from vindication" by some alternative credal set **b**′ *just in case* **b** is *non-probabilistic*.
- Here is a "geometric proof" of the simplest case of de Finetti's theorem — involving a single, contingent claim p.



- There are many advantages to adopting ($\mathcal{R}$), rather than (WADA), as our (ultimate) CR for full belief. Here are a few:
  - First, (WADA) is (intutively) *too weak* to serve as our (ultimate) CR — $\{B(p), B(\sim p)\}$ may be *non-dominated*.

	P	$\sim P$	$B(P)$	$B(\sim P)$	$B(P)$	$D(\sim P)$	$D(P)$	$B(\sim P)$
w_1	**F**	**T**	−	+	−	−	+	+
w_2	**T**	**F**	+	−	+	+	−	−

($\mathcal{R}$) entails (NC), and $\therefore$ *rules-out* $\{B(p), B(\sim p)\}$.

  - We conjecture that ($\mathcal{R}$) $\Leftrightarrow$ (WADA) + (NC). Indeed, we conjecture that ($\mathcal{R}$) is the strongest CR (uncontroversially) entailed by both alethic and evidential considerations.
  - ($\mathcal{R}$) entails (WADA$_d$), for *any additive distance measure* d. In this sense, ($\mathcal{R}$) is *robust* across choices of d [recall, (WADA) is defined *via* the "equal p-weight" Hamming distance].
  - (WADA) only makes sense for *finite* agendas, whereas ($\mathcal{R}$) is easily applicable to *infinite* agendas (if there be such).

- David McCarthy and B.F. [10] have figured out how to apply our framework to *comparative confidence* judgments.
- Let ⌜$p \succeq q$⌝ be interpreted as ⌜S is at least as confident in the truth of p as she is in the truth of q (at time t)⌝.
- Let $\mathfrak{C}$ be the set of all of S's $\succeq$-judgments (over $\mathcal{A} \times \mathcal{A}$).
  - **Step 1**. Define "*the vindicated set at* w" ($\mathring{\mathfrak{C}}_w$).
    - $\mathring{\mathfrak{C}}_w$ is the set containing $p \succ q$ ($p \sim q$) iff p is true at w and q is false at w (p and q have the same truth-value at w).
  - **Step 2**. Define "distance from $\mathfrak{C}$ to $\mathring{\mathfrak{C}}_w$" [$\delta(\mathfrak{C}, \mathring{\mathfrak{C}}_w)$]
    - $\delta(\mathfrak{C}, \mathring{\mathfrak{C}}_w) \stackrel{\text{def}}{=}$ *Kemeny distance* [6, §10.2] between $\mathfrak{C}$ and $\mathring{\mathfrak{C}}_w$.
  - **Step 3**. Adopt an *epistemic principle*, which uses $\delta(\mathfrak{C}, \mathring{\mathfrak{C}}_w)$ to ground a CR for $\mathfrak{C}$ — *weak* δ*-dominance avoidance* (WADA$_\delta$).

☞ **Conjecture**. Provided that $\mathfrak{C}$ is total and $\sim$-transitive, $\mathfrak{C}$ is *not* weakly δ-dominated by *any* (total) relation $\mathfrak{C}'$ ***iff*** $\mathfrak{C}$ is representable by some numerical probability function.

- Kenny is writing a paper [7] that explains how to relax the assumption of opinionation in our framework.
- Our present approach is equivalent to assigning (in)accurate judgments an "accuracy score" of $(-\mathfrak{w})$ $+\mathfrak{r}$ (where $\mathfrak{w} \geq \mathfrak{r} > 0$), and calculating the "overall accuracy score" for **B** (at w) as the *sum* of "accuracy scores" over all $p \in \mathcal{A}$ (at w).
- Kenny's idea: allow S to *suspend on* p [$S(p)$], and then "score" suspensions with a *neutral* "accuracy score" of *zero*.
- On this neutral accuracy scheme for suspensions, we get a nice generalization of our representation Theorem (II).

(III) **Theorem**. An agent S will avoid (strict) dominance in "total accuracy score" ***if*** their **B** can be represented as follows:

There exists a probability function $\Pr(\cdot)$ such that, $\forall p \in \mathcal{A}$:

$$B(p) \textit{ iff } \Pr(p) > \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}},$$

$$D(p) \textit{ iff } \Pr(p) < 1 - \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}},$$

$$S(p) \textit{ iff } \Pr(p) \in \left[1 - \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}, \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}\right].$$