Philosophy Compass 1/1 (2006): 95–113, 10.1111/j.1747-9991.2006.00011.x

The Paradox of Confirmation[1]

Branden Fitelson
University of California – Berkeley

Abstract

Hempel first introduced the paradox of confirmation in 1937. Since then, a very extensive literature on the paradox has evolved (Vranas 2004). Much of this literature can be seen as responding to Hempel's subsequent discussions and analyses of the paradox (Hempel 1945). Recently, it was noted that Hempel's intuitive (and plausible) resolution of the paradox was inconsistent with his official theory of confirmation (Fitelson and Hawthorne 2006). In this article, we will try to explain how this inconsistency affects the historical dialectic about the paradox and how it illuminates the nature of confirmation. In the end, we will argue that Hempel's intuitions about the paradox of confirmation were (basically) correct, and that it is his theory that should be rejected, in favor of a (broadly) Bayesian account of confirmation.

1. The Original Formulation of the Paradox

Informally and pre-theoretically, confirmation is a relation of "support" between statements or propositions. So, when we say that p confirms q, what we mean (roughly and intuitively) is that the truth of p provides (*some* degree of) support[2] for the truth of q. These are called *qualitative* confirmation claims. And, when we say that p confirms q more strongly than p confirms r, we mean (roughly and intuitively) that the truth of p provides *better* support for the truth of q than it does for the truth of r. These are called *comparative* confirmation claims. Confirmation *theory* aims to provide formal explications of both the qualitative and comparative informal "support" concepts involved in such claims. The paradox of confirmation is a paradox involving the qualitative relation of confirmation, but some of its contemporary resolutions appeal also to the comparative concept. We begin with the original formulation of the paradox.

Traditionally, the Paradox of Confirmation (as introduced in Hempel 1937) is based on the following two assumptions about the qualitative confirmation relation:

- **Nicod Condition (NC)**: For any object a and any properties F and G, the proposition that a has both F and G confirms the proposition that every F has G. A more formal version of (NC) is the following claim expressed in monadic predicate-logical symbolism:

For any individual term 'a' and any pair of predicates 'F' and 'G'
$(Fa \cdot Ga)$ confirms $(\forall x)(Fx \supset Gx)$.

In slogan form, (NC) might be expressed as "universal claims are confirmed by their positive instances." It is called the Nicod Condition because it was first endorsed by Nicod (1970). We will say much more about (NC) below.

- **Equivalence Condition (EC)**: For any propositions H_1, H_2, and E, if E confirms H_1 and H_1 is (classically) logically equivalent to H_2, then E confirms H_2.

The intuition behind (EC) is that if H_1 and H_2 are (classically) logically equivalent, then they make exactly the same predictions (indeed, they *say the same thing*), and so anything that counts as evidence for H_1 should also count as evidence for H_2.[3] From (NC) and (EC), we can deduce the following, "paradoxical conclusion":

- **Paradoxical Conclusion (PC)**: The proposition that a is both non-black and a non-raven, $(\sim Ba \cdot \sim Ra)$, confirms the proposition that every raven is black, $(\forall x)(Rx \supset Bx)$.

The canonical derivation of (PC) from (EC) and (NC) proceeds as follows:

1. By (NC), $(\sim Ba \cdot \sim Ra)$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$.
2. In classical logic, $(\forall x)(\sim Bx \supset \sim Rx)$ is equivalent to $(\forall x)(Rx \supset Bx)$.
3. By (1), (2), and (EC), $(\sim Ba \cdot \sim Ra)$ confirms $(\forall x)(Rx \supset Bx)$. *QED*.

The earliest analyses of this infamous paradox were offered by Hempel, Goodman, and Quine. Next, we will discuss how each of these philosophers attempted to resolve the paradox.

2. Early Analyses of the Paradox due to Hempel, Goodman, and Quine

2.1 The Analyses of Hempel and Goodman

Hempel (1945) and Goodman (1954) didn't view (PC) as paradoxical. Indeed, Hempel and Goodman viewed the argument above from (1) and (2) to (PC) as *sound*. So, as far as Hempel and Goodman are concerned, there is something misguided about whatever intuitions may have lead some philosophers to see "paradox" here. As Hempel explains (Goodman's discussion is very similar on this score), one might be misled into thinking that (PC) is false by conflating (PC) with a *different* claim – a claim that is *clearly* false. Hempel warns us that [our emphasis]

> in the seemingly paradoxical cases of confirmation, we are often not judging the relation of the given evidence E *alone* to the hypothesis H . . . instead, we tacitly introduce a comparison of H with a body of evidence which consists of E in conjunction with an additional amount of information we happen to have at our disposal.

More precisely, Hempel is warning us here not to conflate the following two claims:

- (PC) If one observes that an object a – *about which nothing is antecedently known* – is a non-black non-raven, then *this* observation confirms that all ravens are black.
- (PC★) If one observes that an object a – *which is already known to be a non-raven* – is non-black (hence, is a non-black non-raven), then *this* observation confirms that all ravens are black.

The distinction Hempel draws here is a crucial one, and it indicates that confirmation is really a *three-place* relation, between evidence (E), hypothesis (H), and a *background corpus* (K). This allows us to be a bit more precise still about Hempel's warning. The warning is not to conflate:

- (PC) $(\sim Ba \cdot \sim Ra)$ confirms $(\forall x)(Rx \supset Bx)$, relative to *tautological* background K_{T}.
- (PC★) $(\sim Ba \cdot \sim Ra)$ confirms $(\forall x)(Rx \supset Bx)$, relative to background $\sim Ra$.

Intuitively, it is pretty clear that (PC★) is false. After all, it seems clear that *observing a (known) non-raven cannot tell us anything about the color of ravens*. Or, to put things another way, if we already know that a is a non-raven, then we already know that a is neither a positive instance of nor a counterexample to the claim that all ravens are black. As such, observing its color does not (intuitively) provide any information relevant to whether or not all ravens are black. On the other hand, it is not at all clear that (PC) is false. If we know *nothing* about a, and then we observe it to be a non-black non-raven, this (intuitively) *can* tell us something relevant to whether or not all ravens are black, because such an observation can serve to *reduce the number of (possible) counterexamples to the claim that all ravens are black*. As such, this observation may well provide information that is relevant to whether or not all ravens are black (see Maher 1999 for a clear articulation of this line of argument). Hempel and Goodman provided an "independent argument" for (PC), which ran as follows:

> If the evidence E consists *only* of one object which . . . is a non-raven [$\sim Ra$], then E may reasonably be said to confirm that all objects are non-ravens [$(\forall x) \sim Rx$], and *a fortiori*, E supports the weaker assertion that all non-black objects are non-ravens [$(\forall x)(\sim Bx \supset \sim Rx)$], *i.e.*, that all ravens are black [$(\forall x)(Rx \supset Bx)$].

This alternative argument for (PC) rests on four assumptions:

- **A Slight Modification of the Nicod Condition (NC★)**: Hempel's first premise here is that "$\sim Ra$ confirms $(\forall x) \sim Rx$". This assumes a general principle that is *close* to (NC), since $\sim Ra$ is equivalent to $\mathbf{T}a \cdot \sim Ra$, where $\mathbf{T}x$ is any predicate that tautologically applies to all objects (e.g., $\mathbf{T}x = Rx \vee \sim Rx$), and $(\forall x) \sim Rx$ is equivalent to $(\forall x)(\mathbf{T}x \supset \sim Rx)$. If one already accepts (NC) and (EC), then (NC★) should also be acceptable.
- **Monotonicity (M)**: If E confirms H, then $E \cdot X$ confirms H, for any X. This (or something akin to it – see note 4) is presupposed implicitly in the first step of the argument, which takes us from "$\sim Ra$ confirms $(\forall x) \sim Rx$" to "$\sim Ra \cdot \sim Ba$ confirms $(\forall x) \sim Rx$." As we will see below, assumption (M) – which is implied by Hempel's *theory* of confirmation – is rejected by contemporary confirmation theorists. We will also see that (M) is inconsistent with the (intuitive) Hempel-Goodman resolution of the paradox.
- **Special Consequence Condition (SCC)**: For all propositions H_1, H_2, and E, if E confirms H_1, and H_1 (classically) logically entails H_2, then E confirms H_2. This (or something akin to it) is assumed in the second step of the argument, which takes us from "$\sim Ra \cdot \sim Ba$ confirms $(\forall x) \sim Rx$" to "$\sim Ra \cdot \sim Ba$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$". (SCC) is another assumption which is implied by Hempel's theory of confirmation, but which is rejected by most contemporary confirmation theorists (see below).
- **The Equivalence Condition (EC)**: This is assumed at the very end of the argument, which takes us from "$\sim Ra \cdot \sim Ba$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$" to "$\sim Ra \cdot \sim Ba$ confirms $(\forall x)\ (Rx \supset Bx)$".

We mention this "independent argument" for (PC) not because it is more compelling than the original argument generating the paradox, but because it exposes several other key assumptions that Hempel and Goodman made about confirmation. These will become crucial, below.

Let's return now to (PC) *vs* (PC★). Even if you don't think that (PC) is clearly true, it should nonetheless be clear that (PC★) is false. Thus, Hempel and Goodman's explanation of the *appearance* of paradoxicality here – that it arises by conflating (PC) and (PC★) – is quite plausible and intuitive. Unfortunately, it is difficult to see how this resolution could be available to them, since it *contradicts* their *theory* of confirmation! This is because their theory of confirmation implies (M). To see why, note that Hempel and Goodman theoretically explicate "E confirms H relative to K" as "$E \cdot K$ entails Z", where Z is a proposition obtained from the syntax of H and E in a certain complex way, which Hempel (1943, 1945) specifies (the technical details of Hempel's theory of confirmation won't matter for present purposes). Of course, if E *by itself* (i.e., E $\cdot$ $\mathbf{T}$ for tautological $\mathbf{T}$) entails Z, then so does $E \cdot X$, for *any* X. As a result, in Hempel's confirmation theory, "E confirms H, relative to K_{T}" implies

"$E \cdot X$ confirms H, relative to K_T", for any proposition X. Thus, Hempel's theory implies (M).[4] Therefore, according to Hempel's theory of confirmation, *if (PC) is true, then (PC★) must also be true*. So, the intuitive suggestion made by Hempel and Goodman – that (PC) is true while (PC★) is false – *is logically incompatible with their theory of confirmation*. As far as we know, this logical inconsistency in Hempel and Goodman's discussions of the paradox of confirmation was not noted in the literature prior to (Fitelson and Hawthorne 2006). As we will see below, this inconsistency has some rather important consequences for the ensuing historical dialectic. Before tracing this subtle historical trajectory, we pause to consider Quine's radically different approach to the paradox of confirmation.

2.2 Quine on the Paradox of the Ravens

In his influential paper "Natural Kinds," Quine (1969) offers an analysis of the paradox of confirmation that deviates radically from the Hempel-Goodman (intuitive) line. Unlike Hempel and Goodman, Quine *rejects* the paradoxical conclusion (PC). Since Quine accepts classical logic, this forces him to reject either premise (1) or premise (2) of the (classically valid) canonical argument for (PC). Since Quine also accepts the (classical) equivalence condition (EC), he must accept premise (2). Thus, he is led, inevitably, to the rejection of premise (1). This means he must reject (NC) – and he does so. Indeed, according to Quine, not only does ($\sim Ba \cdot \sim Ra$) *fail to confirm* $(\forall x)(\sim Bx \supset \sim Rx)$, but also $\sim Ra$ *fails to confirm* $(\forall x) \sim Rx$. According to Quine, the failure of instantial confirmation in these cases stems from the fact that the predicates "non-black" [$\sim B$] and "non-raven" [$\sim R$] are not *natural kinds* – i.e., the objects falling under $\sim B$ and $\sim R$ are not "sufficiently similar" to undergird instantial confirmation of universal laws involving $\sim B$ or $\sim R$. Quine's suggestion is that only laws involving *natural kinds* will be confirmed by their positive instances. Thus, for Quine, (NC) is the source of the problem here. He suggests that the *unrestricted* version (NC) is false, and must be replaced by a *restricted* version that applies only to natural kinds:

> **Quine–Nicod Condition (QNC)**: For any object a and any *natural kinds* F and G, the proposition that a has both F and G confirms the proposition that every F has G. More formally, $(Fa \cdot Ga)$ confirms $(\forall x)(Fx \supset Gx)$, for any individual term a, provided that the predicates 'F' and 'G' refer to *natural kinds*.

To summarize, Quine thinks (PC) is false, and that the (valid) canonical argument for (PC) is unsound because (NC) is false. Furthermore, according to Quine, once (NC) is restricted in scope to *natural kinds*, the resulting restricted instantial confirmation principle (QNC) is true, but useless for deducing (PC).[5] Many contemporary commentators have taken the problems with (NC) to be much deeper than Quine seems to think (as we'll soon see). The real problems with (NC) and (QNC) only become clear when the paradox is cast in more precise Bayesian terms, in a way that will be explicated in the next part of this article. But, first, we will show how the Bayesian framework allows us to clarify the paradox and the historical debates surrounding it.

3. Contemporary Bayesian Clarifications of (NC) and (PC)

As we saw above, Hempel (1945) provided a cautionary remark about the paradox. He warned us not to conflate the paradoxical conclusion (PC) with a distinct (intuitively) *false* conclusion (PC*) that (intuitively) does *not* follow from (NC) and (EC). It is clear that Hempel was onto something important with his intuitive distinction between claims (PC) and (PC*), but (as we saw above) his confirmation theory just lacked the resources to properly spell out his intuitions. This is where contemporary Bayesian confirmation theory really comes in handy.

In contrast to Hempelian confirmation theory (or, for that matter, other deductive theories of confirmation like hypothetico-deductivism), according to Bayesian confirmation theory, '*E* confirms *H*, given *K*', and '($E \cdot K$) confirms *H*, *unconditionally*' have quite different meanings. Essentially, this is possible because Bayesian explications of the confirmation relation do *not* entail *monotonicity* (M). Specifically, contemporary Bayesians offer the following account of conditional and unconditional confirmation – where hereafter, we will use the words "confirms" and "confirmation" in accordance with this Bayesian account:

- **Bayesian Confirmation**. *E* confirms *H*, *given K* (or *relative to K*), just in case $P[H \mid E \cdot K] > P[H \mid K]$. And, E confirms H, *unconditionally*, iff $P[H \mid E] > P[H]$, where P[•] is some suitable probability function.[6]

It is easy to see, on this account of (conditional and unconditional) confirmation, that there will be a natural distinction between (PC) and (PC*). From a Bayesian point of view this distinction becomes:

(PC) $P[(\forall x)(Rx \supset Bx) \mid {\sim}Ba{\cdot}{\sim}Ra] > P[(\forall x)(Rx \supset Bx)]$, *vs*
(PC*) $P[(\forall x)(Rx \supset Bx) \mid {\sim}Ba{\cdot}{\sim}Ra] > P[(\forall x)(Rx \supset Bx) \mid {\sim}Ra]$

Charitably, this is the sort of distinction Hempel had in mind when he articulated his *intuitions* about the paradox. And, we think this is crucial for understanding the ensuing historical dialectic regarding the paradox. The important point here is that Bayesian confirmation theory has the theoretical resources to distinguish conditional and unconditional confirmation, but traditional (classical) deductive accounts (like Hempel's theory and H–D) do not. As a result, Bayesian theory allows us to precisely articulate Hempel's intuition concerning why people might (falsely) believe that the paradoxical conclusion (PC) is false by conflating it with (PC*).

A key insight of Bayesian confirmation theory is that it represents confirmation as an *irreducibly three*-place relation between *evidence E*, *hypothesis H*, and *background corpus K*. From this perspective the traditional formulation of the paradox is *imprecise* in an important respect: *it leaves unclear which background corpus is presupposed* in the (NC) – and, as a result, also in the (PC). In other words, there is a *missing quantifier* in the traditional formulations of (NC) and (PC). Here are four possible precisifications of (NC) [the corresponding precisifications of (PC) should be obvious]:

- (NC_w) For any individual term '*a*' and any pair of predicates '*F*' and '*G*', there is *some* possible background K such that $(Fa \cdot Ga)$ confirms $(\forall x)(Fx \supset Gx)$, *given K*.
- (NC_α) Relative to our *actual* background corpus K_α, for any individual term '*a*' and any pair of predicates '*F*' and '*G*', $(Fa \cdot Ga)$ confirms $(\forall x)(Fx \supset Gx)$, *given* K_α.
- (NC_T) Relative to *tautological* (or *a priori*) background corpus K_T, for any individual term '*a*' and any pair of predicates '*F*' and '*G*', $(Fa \cdot Ga)$ confirms $(\forall x)(Fx \supset Gx)$, *given* K_T.
- (NC_s) Relative to *any possible* background corpus *K*, for any individual term '*a*' and any pair of predicates '*F*' and '*G*', $(Fa \cdot Ga)$ confirms $(\forall x)(Fx \supset Gx)$, *given K*.

Which rendition of (NC) is the one Hempel and Goodman had in mind? Well, (NC_w) seems *too weak* to be of much use. There is bound to be *some* corpus with respect to which non-black non-ravens confirm 'All non-black things are non-ravens', but this corpus may not be very interesting (*e.g.*, the corpus which contains '$(\sim Ba \cdot \sim Ra) \supset (\forall x)(\sim Bx \supset \sim Rx)$'!).

What about (NC_α)? Well, that depends. If we happen to (actually) *already know* that $\sim Ra$, then all bets are off as to whether $\sim Ba$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$, relative to K_α (as Hempel 1945 suggests, and Maher 1999, 2004 makes precise). So, only a suitably restricted version of (NC_α) would satisfy Hempel's constraint. (We'll return to this issue, below.)

How about (NC_s)? This rendition is clearly *too strong*. As we'll soon see, I. J. Good *demonstrated* that (NC_s) is *false* in a Bayesian framework.

What about (NC_T)? As Maher (1999, 2004) skillfully explains, Hempel and Goodman (and Quine) have something much closer to (NC_T) in mind. Originally, the question was whether learning *only* $(\sim Ba \cdot \sim Ra)$ *and nothing else* confirms that all ravens are black. And, it seems natural to understand this in terms of confirmation relative to "tautological (or *a priori*) background." We will return to the notion of "tautological confirmation," and the (NC_α) *vs* (NC_T) controversy, below. But, first, it is useful to discuss I. J. Good's knock-down counterexample to (NC_s), and his later (rather humorous but also insightful) attempt to formulate a counterexample to (NC_T).

4. Good's Counterexample to (NC_s) and his "Counterexample" to (NC_T)

Good (1967) asks us to consider the following example (we're paraphrasing here):

- Our background corpus *K* says that exactly one of the following hypotheses is true: (*H*) there are 100 black ravens, no non-black ravens, and 1 million other birds, or else (~*H*) there are 1,000 black ravens, 1 white raven, and 1 million other birds. And *K* also states that an object *a* is selected at random from all the birds. Given this background *K*,

$$\begin{aligned} &P[Ra \cdot Ba \mid (\forall x)(Rx \supset Bx) \cdot K] \\ &\quad = \frac{100}{1000100} < P[Ra \cdot Ba \mid \sim(\forall x)(Rx \supset Bx) \cdot K] = \frac{1000}{1001000} \end{aligned}$$

Hence, Good has described a background corpus *K* relative to which ($Ra \cdot Ba$) *dis*confirms $(\forall x)(Rx \supset Bx)$. This is sufficient to show that (NC_s) is false.

Hempel (1967) responded to Good by claiming that (NC_s) is *not* what he had in mind, since it smuggles too much "unkosher" (*a posteriori*) empirical knowledge into *K*. Hempel's challenge to Good was (again, charitably) to find a counterexample to (NC_T). However, In light of the inconsistency in Hempel's own thinking about the paradox, it is unclear how we should (now) understand his challenge to Good. On its face, Hempel's challenge seems *too strong*, since all Hempel (1945) *says* is that antecedently knowing $\sim Ra$ undermines the confirmation $\sim Ra \cdot \sim Ba$ provides for $(\forall x)(Rx \supset Bx)$. He does *not* say that *statistical information about the distribution of colors and species of objects in the universe* will undermine this confirmation relation. As such, it seems that Good should have responded by (a) pointing out that Hempel's theory of confirmation *contradicts his own caveat* about empirical background knowledge; and (b) even if we bracket that inconsistency, Hempel's caveat does not seem to rule-out the kinds of statistical background information Good presupposes in his counterexample to (NC_s). Be that as it may, Good (1968) *did* respond to Hempel's challenge by attempting to furnish a counterexample to (NC_T). Here is what he said [our brackets]:

> imagine an infinitely intelligent newborn baby having built-in neural circuits enabling him to deal with formal logic, English syntax, and subjective probability. He might now argue, after defining a [raven] in detail, that it is initially extremely unlikely that there are any [ravens], and therefore that it is extremely likely that all [ravens] are black. . . . On the other hand, if there are [ravens], then there is a reasonable chance that they are a variety of colours. Therefore, if we were to discover that even a black [raven] exists we would consider [$(\forall x)(Rx \supset Bx)$] to be less probable than it was initially.

This "counterexample" to (NC_T) is far from conclusive, as stated [see Maher (1999) for a trenchant analysis of this passage]. Ultimately, the problem here is that in order to give a rigorous and compelling counterexample to (NC_T), one needs a *theory* of "tautological confirmation" – i.e., of "confirmation relative to tautological background". Good *doesn't have* such a theory (nor do most contemporary probabilists), which explains the lack of rigor and persuasiveness of "Good's Baby." However, Patrick Maher *does* have such an account; and he has applied it in his recent, neo-Carnapian, Bayesian analysis of the paradox of the ravens.

5. Maher's Neo-Carnapian Analysis of the Ravens Paradox

Carnap (1950, 1952, 1971, 1980) proposed various theories of "tautological confirmation" in terms of "logical probability". Recently Patrick Maher (1999, 2004) has brought a Carnapian approach to bear on the ravens paradox, with some very enlightening results. For our purposes it is useful to emphasize two consequences of Maher's neo-Carnapian, Bayesian analysis of the paradox. First, Maher shows that (PC★) is *false* on a neo-Carnapian theory of (Bayesian) confirmation. That is, if we take a suitable class of Carnapian *logical* (or *a priori*) probability functions $P_c[\bullet \mid \bullet]$ – e.g., either those of Maher (1999) or Maher (2004) – as our "probabilities relative to tautological background", then we get the following result (see Maher (1999))

$$P_c[(\forall x)(Rx \supset Bx) \mid {\sim}Ba \cdot {\sim}Ra] = P_c[(\forall x)(Rx \supset Bx) \mid {\sim}Ra]$$

Intuitively, this says that observing the color of (known) non-ravens tells us nothing about the color of ravens, relative to tautological background corpus. This is a theoretical vindication of Hempel's intuitive claim that (PC★) is false – a vindication that is difficult (*impossible*, given what Hempel says about (PC)) to make out in Hempel's deductive theory of confirmation. But, all is not beer and skittles for such a Bayesian reconstruction of Hempel's intuitions about the paradox.

More recently, Maher (2004) has convincingly argued (contrary to what he had previously suggested (1999)) that, within a proper neo-Carnapian Bayesian framework, Hempel's (NC_T) is *false*, and so is its Quinean "restriction" (QNC_T). That is, Maher (2004) has shown that (from a Carnapian/Bayesian point of view) *pace* Hempel, Goodman, *and* Quine, *even relative to tautological background, positive instances do not necessarily confirm universal generalizations – not even for generalizations that involve only natural kinds*! The technical details of Maher's counterexample to (QNC_T) [hence, to (NC_T) as well] would take us too far afield.[7] But, we mention it here because it shows that probabilistic approaches to confirmation are much richer and more powerful than traditional, deductive approaches. And, we think, Maher's work *finally* answers Hempel's challenge to Good – a challenge that went unanswered for nearly forty years.

Moreover, Maher's results also suggest that Quine's analysis in "Natural Kinds" was off the mark. Contrary to what Quine suggests, the problem with (NC) is not merely that it needs to be restricted in scope to certain kinds of properties. The problems with (NC) run much deeper than that. Even the most promising Hempelian precisification of (NC) is false, and a restriction to "natural kinds" does not help, since Maher-style, neo-Carnapian counterexamples can be generated employing only "natural kinds" in Quine's sense.[8]

While Maher's neo-Carnapian analysis is quite illuminating, it is by no means in the mainstream of the contemporary Bayesian literature. Most contemporary Bayesians reject the Carnapian approach to logical probabilities and the Carnapian assumption that there is any such thing as "degree of confirmation relative to tautological background." Because contemporary Bayesians have largely rejected this Carnapian project, they take a rather different tack to handle the paradox of confirmation.

6. The Canonical Contemporary Bayesian Approaches to the Paradox

Perhaps somewhat surprisingly, almost all contemporary Bayesians implicitly assume that the paradoxical conclusion is *true*. And, they aim only to "soften the impact" of (PC) by trying to establish certain *comparative* and/or *quantitative* confirmational claims. Specifically, Bayesians typically aim to show (at least) that the observation of a black raven, $(Ba \cdot Ra)$, confirms that all ravens are black *more strongly than* the observation of a non-black non-raven, $({\sim}Ba \cdot {\sim}Ra)$ does, relative to our *actual* background corpus K_α, which is assumed to contain no "unkosher" information about a in particular (although it will contain *statistical* information reflecting our beliefs about the distributions of things in the actual world, and so it will not be *tautological*). Specifically, most contemporary Bayesians aim to show (at least) that relative to some measure c of how strongly evidence supports a hypothesis, the following COMParative claim holds:[9]

$$(\mathrm{COMP}_c)\quad c[(\forall x)(Rx \supset Bx), (Ra \cdot Ba) \mid K_\alpha] > c[(\forall x)(Rx \supset Bx), ({\sim}Ba \cdot {\sim}Ra) \mid K_\alpha].$$

Here $c(H, E \mid K)$ is some *Bayesian* measure of the *degree* to which E confirms H, relative to background corpus K. Note: unlike Carnap, most contemporary Bayesians do not think there is any such thing as logical (or *a priori*) probability $\mathrm{P}_c[\bullet \mid K_\mathrm{T}]$. So, for most contemporary Bayesians, the salient probability is $\mathrm{P}[p \mid K_\alpha]$, which is to be interpreted as the rational epistemic probability of p, given an (actual) background knowledge corpus K_α (which is not tautological). The typical Bayesian strategy is to isolate constraints on K_α that are as minimal as possible (hopefully, even ones that Hempel would have seen as kosher), so as to guarantee that (COMP_c) obtains, with respect to $\mathrm{P}[\bullet \mid K_\alpha]$.

As it stands, (COMP_c) is somewhat unclear. There are *many* Bayesian relevance measures c that have been proposed and defended in the contemporary literature on Bayesian confirmation. The four most popular of these measures are the following (see Fitelson (1999) and Fitelson (2001) for historical surveys).[10]

- The Difference: $d[H, E \mid K] = \mathrm{P}[H \mid E \cdot K] - \mathrm{P}[H \mid K]$
- The Log-Ratio: $r[H, E \mid K] = \log(\mathrm{P}[H \mid E \cdot K] \mathbin{/} \mathrm{P}[H \mid K])$
- The Log-Likelihood-Ratio: $l[H, E \mid K] = \log(\mathrm{P}[E \mid H \cdot K] \mathbin{/} \mathrm{P}[E \mid {\sim}H \cdot K])$
- The Normalized Difference: $s[H, E \mid K] = \mathrm{P}[H \mid E \cdot K] - \mathrm{P}[H \mid {\sim}E \cdot K]$

Measures d, r, and l all satisfy the following *desideratum*, for all H, E_1, E_2, and K:

(†) If $\mathrm{P}[H \mid E_1 \cdot K] > \mathrm{P}[H \mid E_2 \cdot K]$, then $c[H, E_1 \mid K] > c[H, E_2 \mid K]$.

But, interestingly, measure s does *not* satisfy (†). So, if one uses either d, r, or l to measure confirmation, then one can establish the desired comparative claim *simply* by demonstrating that:

$$(\mathrm{COMP_P}) \quad \mathrm{P}[(\forall x)(Rx \supset Bx) \mid Ra \cdot Ba \cdot K_\alpha] > \mathrm{P}[(\forall x)(Rx \supset Bx) \mid {\sim}Ba \cdot {\sim}Ra \cdot K_\alpha]$$

On the other hand, if one uses s, then one has a bit more work to do to establish the desired comparative conclusion, because ($\mathrm{COMP_P}$) does not entail (COMP_s).[11]

Some Bayesians go farther than this by trying to establish not only the *comparative* claim ($\mathrm{COMP}c$), but also the *quantitative* claim that the observation of a non-black non-raven confirms that "All ravens are black" to a "minute" degree. That is, in addition to the comparative claim, some Bayesians also go for the following QUANTative claim:

$$(\mathrm{QUANT}_c) \quad c[(\forall x)(Rx \supset Bx), ({\sim}Ba \cdot {\sim}Ra) \mid K_\alpha] > 0, \text{ but } \textit{very nearly } 0.$$

Let's begin by discussing the canonical contemporary Bayesian *comparative* analysis of the paradox. In essence, almost all such accounts trade on the following three assumptions about K_α, where it is assumed that the object a is sampled at random from the universe:[12]

(1) $\mathrm{P}[{\sim}Ba \mid K_\alpha] > \mathrm{P}[Ra \mid K_\alpha]$

(2) $\mathrm{P}[Ra \mid (\forall x)(Rx \supset Bx) \cdot K_\alpha] = \mathrm{P}[Ra \mid K_\alpha]$

(3) $\mathrm{P}[{\sim}Ba \mid (\forall x)(Rx \supset Bx) \cdot K_\alpha] = \mathrm{P}[{\sim}Ba \mid K_\alpha]$

Basically, assumption (1) relies on our knowledge that (according to K_α) there are more non-black objects in the universe than there are ravens. This seems like a very plausible distributional constraint on K_α, since – as far as we *actually know* – it is true. Assumptions (2) and (3) are more controversial. We will say more about them shortly. First, we note an important and pretty well-known theorem (see Vranas 2004 for a proof).

THEOREM. (1)–(3) jointly entail (COMP_{P}). [Therefore, since d, r, and l each satisfy (†), it follows that (1)–(3) entail (COMP_d), (COMP_r), and (COMP_l).]

In fact, (1)–(3) entail *much more* than (COMP_{P}), as the following theorem illustrates:

THEOREM. (1)–(3) also entail the following:

(4) $\text{P}[(\forall x)(Rx \supset Bx) \mid {\sim}Ba \cdot {\sim}Ra \cdot \text{K}_\alpha] > \text{P}[(\forall x)(Rx \supset Bx) \mid K_\alpha]$

(5) $s[(\forall x)(Rx \supset Bx), (Ra \cdot Ba) \mid K_\alpha] > s[(\forall x)(Rx \supset Bx), ({\sim}Ba \cdot {\sim}Ra) \mid K_\alpha]$

In other words, (4) tells us that assumptions (1)–(3) entail that the observation of a non-black non-raven confirms that all ravens are black – i.e., that the paradoxical conclusion (PC) is true. And, (5) tells us that even according to s (a measure that *violates* (†)) the observation of a black raven confirms that all ravens are black more strongly than the observation of a non-black non-ravens does.

The fact that (1)–(3) entail (4) and (5) indicates that the canonical Bayesian assumptions go far beyond the comparative claim most Bayesians want. Why, for instance, should a Bayesian be committed to the *qualitative* paradoxical conclusion (PC)? After all, as Patrick Maher and I. J. Good have made so clear, probabilists don't *have* to be committed to qualitative claims like (NC) and (PC). It would be nice if there were assumptions weaker than (1)–(3) that sufficed to establish (*just*) the *comparative* claim (COMP_{P}), while implying no commitment to *qualitative* claims like (PC). Happily, there are such weaker conditions. But, before we turn to them, we first need to briefly discuss the *quantitative* Bayesian approaches.

Various Bayesians go farther than (COMP_c) in their analysis of the ravens paradox. They seek to identify (stronger) constraints, stronger background knowledge K_α, that entails *both* (COMP_c) *and* (QUANT_c). The most common strategy along these lines is simply to strengthen (1), as follows:

(1′) $\text{P}[{\sim}Ba \mid K_\alpha] >> \text{P}[Ra \mid K_\alpha]$ – i.e., there are *far* fewer ravens than non-black things in the universe.

Recently, Peter Vranas (2004) has provided a very detailed analysis of quantitative Bayesian approaches to the ravens paradox along these lines. We won't dwell too much on the details of these approaches. Vranas does an excellent job of analyzing them. However, some brief remarks on a result Vranas proves and uses in his analysis are worth considering here.

Vranas shows that assumptions (1′) and (3) (without (2)) are *sufficient* for (QUANT_c) to hold (i.e. for $(\forall x)(Rx \supset Bx)$ to be positively confirmed by $(\sim Ba \cdot \sim Ra)$, given K_α, but only by a very small amount) for all four measures of confirmation *d*, *r*, *l*, and *s*. Moreover, he argues that in the presence of (1′), (3) is "approximately necessary" for (QUANT_c). What he *proves* is that, given (1′), and supposing that $P[H \mid K_\alpha]$ is not too small, the following approximate claim is *necessary* for (QUANT_c):

$$(3')\quad P[\sim Ba \mid (\forall x)(Rx \supset Bx) \cdot K_\alpha] \approx P[\sim Ba \mid K_\alpha].$$

Vranas then argues that Bayesians have given no good reason for this necessary (and sufficient) condition. Thus, he concludes, Bayesian resolutions of the paradox that claim non-black non-ravens confirm by a tiny bit, due to assumption (1′), have failed to establish a condition they must employ to establish this claim – they have failed to establish (3′).[13]

Vranas's claim that (3) is "approximately necessary" for (QUANT_c) may be somewhat misleading. It makes it sound as if (3) has a certain property. But, in fact, nothing about (3) itself follows from Vranas's results. It is more accurate to say (as Bob Dylan might) that "approximately (3)" [i.e., (3′)] is *necessary* for (QUANT_c). To see the point, note that (3) is a rather strong *independence* assumption, which entails many other identities, including:

$$(3.1)\quad P[(\forall x)(Rx \supset Bx) \mid Ba \cdot K_\alpha] = P[(\forall x)(Rx \supset Bx) \mid K_\alpha], \text{ and}$$

$$(3.2)\quad P[(\forall x)(Rx \supset Bx) \mid Ba \cdot K_\alpha] = P[(\forall x)(Rx \supset Bx) \mid \sim Ba \cdot K_\alpha]$$

But, (3′) is *not* an independence assumption. Indeed, (3′) is far weaker than an independence assumption, and it does *not* entail the parallel *approximates*:

$$(3'.1)\quad P[(\forall x)(Rx \supset Bx) \mid Ba \cdot K_\alpha] \approx P[(\forall x)(Rx \supset Bx) \mid K_\alpha], \text{ or}$$

$$(3'.2)\quad P[(\forall x)(Rx \supset Bx) \mid Ba \cdot K_\alpha] \approx P[(\forall x)(Rx \supset Bx) \mid \sim Ba \cdot K_\alpha]$$

Vranas argues convincingly that strong independence assumptions like (3) (and (2)) have not been well motivated by Bayesians who endorse the quantitative approach to the ravens paradox. He rightly claims that this is a lacuna in the canonical quantitative Bayesian analyses of the paradox. But, what he ultimately shows is somewhat weaker than appearances might suggest. In the next section we will describe (*pace* Vranas and most other commentators) considerably weaker sets of assumptions for the comparative Bayesian approaches (analogous results for the quantitative approaches are proved in Fitelson and Hawthorne 2006).

7. A New Bayesian Approach to the Paradox

In the comparative case, the primary aim is to establish (COMP_P). As we have seen, Bayesians typically make two quite strong independence assumptions in order to achieve this goal. Happily, a perfectly satisfactory analysis of the *ravens* may be given that employs no independence assumptions at all.

In this section, we offer a solution to the raven paradox that is more general than other solutions we know of. It draws on much weaker assumptions. It *solves* the paradox in that it supplies quite plausible sufficient conditions for the observation of a black raven to confirm "All ravens are black" *more strongly* than observation of a non-black non-raven would confirm it (see Fitelson and Hawthorne 2006 for *necessary and* sufficient conditions). These conditions do not draw on probabilistic independence (they are *strictly weaker* than the standard independence assumptions). And they in no way depend on whether Nicod's Condition (NC) is satisfied. Our conditions can be satisfied both in cases were a positive instance confirms that all ravens are black and in cases where a positive instance disconfirms that all ravens are black (as in Good's counterexample to NC_s).

Let 'H' abbreviate "All ravens are black" – i.e., '$(\forall x)(Rx \supset Bx)$'. Let '$K$' be a statement of whatever background knowledge you may think relevant – e.g. K might imply, among other things, that ravens exist and that non-black things exist, $((\exists x)Rx \cdot (\exists x){\sim}Bx)$. One object, call it '$a$' will be observed for color and to see whether it is a raven. The idea is to assess, in advance of observing it, whether a's turning out to be a black raven, $(Ra \cdot Ba)$, would make H more strongly supported than would a's turning out to be a non-black non-raven, $({\sim}Ra \cdot {\sim}Ba)$. That is, we want to find plausible sufficient conditions (in Fitelson and Hawthorne 2006, we report *necessary and* sufficient conditions) for $P[H \mid Ba \cdot Ra \cdot K] > P[H \mid {\sim}Ba \cdot {\sim}Ra \cdot K]$. We assume throughout *only this*:

Background Assumptions about *K*: $0 < P[H \mid Ba \cdot Ra \cdot K] < 1$, $0 < P[H \mid {\sim}Ba \cdot {\sim}Ra \cdot K] < 1$, $P[{\sim}Ba \cdot Ra \mid K] > 0$, and $P[{\sim}Ba \mid K] > P[Ra \mid K]$.

That is, we assume only that finding a to be a black raven neither absolutely *proves* nor absolutely *falsifies* "All ravens are black"; and the same goes if a is found to be a non-black non-raven. In addition we assume that it is at least possible, given only background K, that a will turn out to be a non-black raven. And, finally, we assume that there are more non-black objects in the universe than there are ravens (which is the *uncontroversial* assumption (1) that appears in the orthodox Bayesian treatment of the paradox). Given these background assumptions about K, the following Theorem can be proven.

THEOREM. If $P[H \mid Ra \cdot K] \geq P[H \mid {\sim}Ba \cdot K]$, then $P[H \mid Ba \cdot Ra \cdot K] > P[H \mid {\sim}Ba \cdot {\sim}Ra \cdot K]$.

Thus, *pace* Vranas, we have a much weaker sufficient condition for (COMP_P), which does *not* presuppose any kind of probabilistic independence (or even "approximate independence"). Moreover, unlike the strong independence assumptions in the traditional Bayesian accounts, the preconditions of our theorem do *not* imply (PC).[14] So, we have discovered a *purely comparative* Bayesian approach to the paradox of confirmation, which avoids the troubling independence assumptions that appear in traditional Bayesian accounts, and which does not require commitment to the paradoxical conclusion (PC). Intuitively, what our sufficient condition says is that the observation that *a* is a raven does not confirm that all ravens are black any less strongly than the observation that *a* is non-black. We think this assumption is quite plausible (it is certainly far more plausible than the independence assumptions which Vranas rightly criticizes). Other (more general) recent theorems for both the qualitative and quantitative cases are reported and discussed in detail in (Fitelson and Hawthorne 2006).

8. Coda: Hempel meets Bayes?

As we have seen, most contemporary Bayesians (Maher 2004 is a notable exception here) *accept* (PC). In this sense, modern Bayesians are rather "Hempelian" at heart. Moreover, in his plausible and intuitive resolution of the paradox, Hempel appeals to "tautological" *vs* "nontautological" confirmation, which is a very Bayesian-friendly idea. Unfortunately, Hempel's Bayesian-friendly *intuitions contradict* his monotonic, deductive, non-Bayesian *theory* of confirmation. Putting these things together, we can formulate a two-pronged "Hempel-Bayes resolution" of the paradox. The first prong is to distinguish (PC) and (PC★) in their Bayesian forms: (PC_α) and ($PC^\star_\alpha$). Plausibly, relative to our actual background knowledge, (PC★) is false (*i.e.*, ($PC^\star_\alpha$) is false). On the other hand, (PC_α)'s truth-value will depend on the actual (known) statistical distribution of objects in the universe. This is enough to make the Hempel-Bayes point that conflating (PC_α) and ($PC^\star_\alpha$) can generate the *appearance* of a paradox (*e.g.*, if (PC_α) turns out to be true). The second prong is that *even if* (PC_α) should turn out to be true, only very weak conditions need to be satisfied in order to ensure that the observation of a non-black non-raven confirms that all ravens are black *less strongly than* the observation of a black raven does. This approach harnesses the power and richness of contemporary Bayesian confirmation theory, without abandoning Hempel's original intuitions about the paradox of confirmation, and without saddling the contemporary confirmation theorist with commitments to either (NC) or (PC).

Notes

[1] Much of the material in this article (especially, the theorems in the last sections) is drawn from Fitelson and Hawthorne (2006). As such, Jim Hawthorne is equally responsible for many of the results and analyses reported here. This also explains the use of "we" rather than "I" throughout the article.

[2] We will remain neutral at this point as to whether this relation of support is logical or epistemic in nature. Later, we will see both epistemic and logical readings of "confirms."

[3] There are some people who have rejected (EC) in response to the paradox of confirmation. See, for instance, Scheffler (1963), Sylvan and Nola (1991), and Gemes (1999). We will not discuss such responses to the paradox in this article.

[4] Strictly speaking, this isn't exactly correct. What's true (strictly speaking) is that, according to Hempel's (1943, 1945) theory, if E confirms X, then so does $E \cdot K$, for any K – *provided that* K *doesn't mention any individuals not already mentioned in* E *and* H. But, in the case at hand, this caveat is satisfied, since $\sim Ra$ (K) only mentions the individual a, which is the only individual mentioned in E and H. Hence, no loss of generality results (for our present purposes) by stating the monotonicity condition (M) in this simplified way.

[5] Interestingly, while Hempel and Goodman are completely unsympathetic to Quine's strategy *here*, they are much more sympathetic to such maneuvers in the context of the *Grue* Paradox. In this sense, Quine's approach to the paradoxes is more unified and systematic than Hempel's or Goodman's, since they give "special treatment" to Grue-predicates, while Quine views the problem – in *both* paradoxes of confirmation – to be rooted in the "non-naturalness" of the referents of the predicates involved. For what it's worth, we think a unified and systematic approach to the paradoxes is to be preferred. But, we think a unified *Bayesian* approach is preferable to Quine's instantial approach. However, our preferred Bayesian treatment of Grue will have to wait for another occasion.

[6] For simplicity (and following modern Bayesian tradition), we will just assume that P[•] is some rational credence function, and that it behaves in accordance with the standard (Kolmogorov 1956) axiomatization of the (classical) probability calculus. If we were to follow Hájek (2003) and Joyce (1999) and assume that P[• | •] is a *Popper function*, then the story would become much more complicated and less unified (formally). See Fitelson (2003) for a discussion of the disunifying effect the adoption of Popper functions would have on Bayesian confirmation theory.

[7] In informal and intuitive terms, Maher (2004) characterizes his counterexample to (QNC) (hence, also to (NC)) as follows: "According to standard logic, 'All unicorns are white' is true if there are no unicorns. Given what we know, it is almost certain that there are no unicorns and hence 'All unicorns are white' is almost certainly true. But now imagine that we discover a white unicorn; this astounding discovery would make it no longer so incredible that a non-white unicorn exists and hence would disconfirm 'All unicorns are white.'" As such, Maher's example can be seen as a precisification of "Good's Baby."

[8] Metaphysically, there may be a problem with "non-natural kinds" (in Quine's sense – e.g., disjunctive and negative properties) participating in certain kinds of causal or other law-like relations. This sort of problem has been suggested in the contemporary literature by Armstrong (1978), Shoemaker (1980), and others. But, we think this metaphysical fact (if it is a fact) has few (if any) *confirmational* consequences. Confirmation is a logical or epistemic relation, which may or may not align neatly with metaphysical relations like causation or law-likeness. A proper defense of this position on natural kinds and confirmation would require providing an adequate Bayesian analysis of the Grue paradox, along similar lines to the resolution of the Raven paradox, presented below. While we think such a resolution *can* be provided (contrary to what most people in the contemporary literature seem to think), that it beyond the scope of this article.

[9] As Chihara (1981) points out, "there is no such thing as *the* Bayesian solution. There are many different 'solutions' that Bayesians have put forward using Bayesian techniques." That said, we present here what we take to be the most standard assumptions Bayesians tend to make in their handling of the paradox – assumptions that are *sufficient* for the desired comparative and quantitative confirmation-theoretic claims. On this score, we follow Vranas (2004). However, not all Bayesians make precisely these assumptions. To get a sense of the variety of Bayesian approaches, see, e.g.: Alexander (1958), Chihara (1981), Earman (1992), Eells (1982), Gaifman (1979), Gibson (1969), Good (1960, 1961), Hesse (1974), Hooker and Stove (1968), Horwich (1982), Hosiasson-Lindenbaum (1940), Howson and Urbach (1993), Jardine (1965), Mackie (1963), Nerlich (1964), Suppes (1966), Swinburne (1971), Wilson (1964), Woodward (1985), Hintikka (1969), Humburg (1986), Maher (1999, 2004), and Vranas (2004).

[10] We take *logarithms* of the ratio measures just to ensure that they are positive in cases of confirmation, negative in cases of disconfirmation, and zero in cases of neutrality of irrelevance.

This is a useful convention for present purposes, but since logs don't alter the ordinal structure of the measures, it is a *mere* convention.

[11] This has led some defenders of s to abandon it as a measure of incremental confirmation. See Joyce (2003, fn. 11). See, also, Eells and Fitelson (2000, 2002) and Fitelson (2001) for further peculiarities of the measure s. For a rigorous counterexample to the implication (COMP_P) $\Rightarrow$ (COMP_s), the details of which are beyond the scope of this article, see the Appendix of Fitelson (2001).

[12] Often, Bayesians use a *two-stage sampling* model in which *two* objects a and b are sampled at random from the universe, where K_α entails ($Ra \cdot {\sim}Bb$) (e.g., Earman (1992)). On that model we still have (2), but (3) is replaced with $P[{\sim}Bb \mid H \cdot K_\alpha] = P[{\sim}Bb \mid K_\alpha]$, and ($\mathrm{COMP}_P$) is replaced by ($\mathrm{COMP}_P'$) $P[H \mid Ra \cdot Ba \cdot K_\alpha]\ P[H \mid {\sim}Bb \cdot {\sim}Rb \cdot K_\alpha]$. So, no real loss of generality comes from restricting our treatment to "one-stage sampling" – i.e., to the selection of a single object a, which K_α doesn't specify to be either an R or a ${\sim}B$ (Vranas 2004, fns. 10 and 18). We prefer a *one*-stage sampling approach because we think it is closer in spirit to what Hempel and Goodman took the original paradox to be about – where K_α is assumed *not* to have any implications about the color or species of the objects sampled, and where a single object is observed "simultaneously" for its color and species.

[13] However, Vranas does not argue that (3′) is false or implausible – only that no good argument for its plausibility has been given. So, it is consistent with his result that one might be able to find some plausible condition X that, together with (1′), implies (QUANT_c). Vranas' result would then show that condition X (together with (1′)) also implies (3′) – and so in effect would provide a plausibility argument for (3′). Some of the results proved in the last two sections of Fitelson and Hawthorne (2006) provide such plausible conditions, X.

[14] Our assumption $P[H \mid Ra \cdot K] \geq P[H \mid {\sim}Ba \cdot K]$ is *strictly weaker* than the independence assumptions (2) and (3), since together those imply $P[H \mid Ra \cdot K] = P[H \mid {\sim}Ba \cdot K] = P[H]$. Interestingly, this new, weaker sufficient condition for (COMP_P) discovered by Fitelson and Hawthorne (2006) does *not* entail (COMP_c) if one uses the measure s to gauge degree of confirmation. This is further evidence of the inadequacy of s as a confirmation measure.

Bibliography

Alexander, H. G. 1958. "The paradoxes of confirmation," *British Journal for the Philosophy of Science*, 9, 227–233.

Armstrong, D. 1978. *A Theory of Universals*. Cambridge: Cambridge University Press.

Carnap, R. 1950. *Logical Foundations of Probability*. Chicago: Chicago University Press.

Carnap, R. 1952. *The Continuum of Inductive Methods*. Chicago: University of Chicago Press.

Carnap, R. 1971. "A Basic System of Inductive Logic, Part I," in *Studies in Inductive Logic and Probability*, ed. R. Carnap and R. C. Jeffrey, vol. 1. Berkeley: University of California Press.

Carnap, R. 1980. "A Basic System of Inductive Logic, Part II," in *Studies in Inductive Logic and Probability*, ed. R. C. Jeffrey, vol. 2. Berkeley: University of California Press.

Chihara, C. 1981. "Quine and the confirmational paradoxes," in *Midwest Studies in Philosophy. Vol. 6: The Foundations of Analytic Philosophy*, ed. P. A. French, T. E. Uehling Jr., and H. K. Wettstein. Minneapolis: University of Minnesota Press, 425–52.

Earman, J. 1992. *Bayes or bust? A Critical Examination of Bayesian Confirmation Theory*. Cambridge, MA: MIT Press.

Eells, E. 1982. *Rational Decision and Causality*. New York: Cambridge University Press.

Eells, E. and Fitelson, B. 2000. "Measuring Confirmation and Evidence," *Journal of Philosophy*, 97, 663–72.

Eells, E. and Fitelson, B. 2002. "Symmetries and Asymmetries in Evidential Support," *Philosophical Studies*, 107, 129–42.

Fitelson, B. 1999. "The Plurality of Bayesian Measures of Confirmation and the Problem of Measure Sensitivity," *Philosophy of Science*, 66, S362–S378.

Fitelson, B. 2001. "Studies in Bayesian Confirmation Theory," Ph.D. Dissertation, University of Wisconsin.

Fitelson, B. 2003. "Review of Jim Joyce's *The Foundations of Causal Decision Theory*," *Mind*, 112, 545–51.

Fitelson, B. Forthcoming. "Inductive Logic," in *The Philosophy of Science: An Encyclopedia*, ed. J. Pfeifer and S. Sarkar. New York: Routledge Press.
Fitelson, B., and Hawthorne, J. 2006. "How Bayesian Confirmation Theory Handles the Paradox of the Ravens," in *Probability in Science*, ed. E. Eells and J. Fetzer. Chicago: Open Court.
Gaifman, H. 1979. "Subjective Probability, Natural Predicates and Hempel's Ravens," *Erkenntnis*, 14, 105–47.
Gemes, K. 1999. "Carnap-Confirmation, Content-Cutting, and Real Confirmation," unpublished essay.
Gibson, L. 1969. "On 'Ravens and Relevance' and a Likelihood Solution of the Paradox of Confirmation," *British Journal for the Philosophy of Science*, 20, 75–80.
Good, I. J. 1960. "The Paradox of Confirmation," *British Journal for the Philosophy of Science*, 11, 145–9.
Good, I. J. 1961. "The Paradox of Confirmation (II)," *British Journal for the Philosophy of Science*, 12, 63–4.
Good, I. J. 1967. "The White Shoe is a Red Herring," *British Journal for the Philosophy of Science*, 17, 322.
Good, I. J. 1968. "The White Shoe qua Herring is Pink," *British Journal for the Philosophy of Science*, 19, 156–7.
Goodman, N. 1954. *Fact, Fiction, and Forecast*. London: Athlone Press.
Hájek, A. 2003. "What Conditional Probability Could not be," *Synthese*, 13, 273–323.
Hempel, C. G. 1937. "Le problème de la vérité," *Theoria*, 3, 206–46.
Hempel, C. G. 1943. "A Purely Syntactical Definition of Confirmation," *Journal of Symbolic Logic*, 8, 122–43.
Hempel, C. G. 1945. "Studies in the Logic of Confirmation I & II," *Mind*, 54, 1–26 & 97–121.
Hempel, C. G. 1967. "The White Shoe: No Red Herring," *British Journal for the Philosophy of Science*, 18, 239–40.
Hesse, M. 1974. *The Structure of Scientific Inference*. London: MacMillan.
Hintikka, J. 1969. "Inductive Independence and the Paradoxes of Confirmation," in *Essays in honor of Carl G. Hempel: A tribute on the occasion of his sixty-fifth birthday*, ed. N. Rescher. Dordrecht: Reidel, 24–46.
Hawthorne, J. 2005. "Degree-of-belief and Degree-of-support: Why Bayesians Need both Notions," *Mind*, 114, 277–320.
Hooker, C. A., and Stove, D. 1968. "Relevance and the Ravens," *British Journal for the Philosophy of Science*, 18, 305–15.
Horwich, P. 1982. *Probability and Evidence*. New York: Cambridge University Press.
Hosiasson-Lindenbaum, J. 1940. "On Confirmation," *Journal of Symbolic Logic*, 5, 133–48.
Howson, C., and Urbach P. 1993. *Scientific Rreasoning: The Bayesian Approach*, 2nd ed. Chicago: Open Court.
Humburg, J. 1986. "The Solution of Hempel's Raven Paradox in Rudolf Carnap's System of Inductive Logic," *Erkenntnis*, 24, 57–72.
Jardine, R. 1965. "The Resolution of the Confirmation Paradox," *Australasian Journal of Philosophy*, 43, 359–68.
Joyce, J. 1999. *The Foundations of Causal Decision Theory*. Cambridge: Cambridge University Press.
Joyce, J. 2003. "Bayes's Theorem," *The Stanford Encyclopedia of Philosophy*, ed. E. N. Zalta. http://plato.stanford.edu/archives/win2003/entries/bayes-theorem/.
Kolmogorov, A. 1956. *Foundations of Probability*, 2nd English ed. Providence, Rhode Island: AMS Chelsea Publishing.
Mackie, J. L. 1963. "The Paradox of Confirmation," *British Journal for the Philosophy of Science*, 13, 265–77.
Maher, P. 1999. "Inductive Llogic and the Ravens Paradox," *Philosophy of Science*, 66, 50–70.
Maher, P. 2004. "Probability Captures the Logic of Scientific Confirmation," in *Contemporary Debates in the Philosophy of Science*, ed. C. Hitchcock. Oxford: Blackwell, 69–93.
Nerlich, G. 1964. "Mr. Wilson on the Paradox of Confirmation," *Australasian Journal of Philosophy*, 42, 401–5.

Nicod, J. 1970. *Geometry and Induction*. Berkeley and Los Angeles: University of California Press. English translation of works originally published in French in 1923 and 1924.

Quine, W. V. O. 1969. "Natural Kinds," in *Ontological Relativity and other Essays*. New York: Columbia University Press, 114–38.

Scheffler, I. 1963. *The Anatomy of Inquiry: Philosophical Studies in the Theory of Science*. New York: Alfred A. Knopf, 1963.

Shoemaker, S, 1980. "Causality and Properties," reprinted in *Identity, Cause and Mind*. Cambridge: Cambridge University Press.

Suppes, P. 1966. "A Bayesian Approach to the Paradoxes of Confirmation", in *Aspects of Inductive Logic*, ed. J. Hintikka and P. Suppes. Amsterdam: North-Holland, 198–207.

Swinburne, R. 1971. "The Paradoxes of Confirmation – A Survey," *American Philosophical Quarterly*, 8, 318–30.

Sylvan, R., and Nola, R. 1991. "Confirmation Without Paradoxes," in *Advances in Scientific Philosophy: Essays in Honour of Paul Weingartner*, ed. G. Schurz and G. J. W. Dorn. Amsterdam: Rodopi, 5–44.

Vranas, P. 2004. "Hempel's Raven Paradox: A Lacuna in the Standard Bayesian Solution," *British Journal for the Philosophy of Science*, 55, 545–60.

Wilson, P. R. 1964. "A New Approach to the Confirmation Paradox," *Australasian Journal of Philosophy*, 42, 393–401.

Woodward, J. 1985. "Critical Review: Horwich on the Ravens, Projectability and Induction," *Philosophical Studies*, 47, 409–28.

