

Judgment Under Uncertainty Revisited: Probability vs Confirmation

Branden Fitelson

Department of Philosophy
&
Center for Cognitive Science (RuCCS)
Rutgers University

branden@fitelson.org
http://fitelson.org/

- 1 Overview
- 2 Historical Background: Sources of Contemporary Confusion
 - The Carnapian Ambiguity
 - The Bayesian Ambiguity
- 3 Some Normative Principles for (Bayesian) Confirmation
 - Two Symmetry Principles
 - A Logical Principle
- 4 Some Psychological Data on Probability & Confirmation
 - Kahneman and Tversky (et al) on Probability Judgments
 - Crupi et al on Probability, Confirmation, and “Linda”
 - A Pioneering Recent Study of Osherson et al
 - Armchair Suggestions for Further Psychological Research
- 5 References

- Carnap [1] aims to provide a formal explication of an informal concept (relation) he calls “confirmation”.
- He clarifies “ E confirms H ” in various ways, including:
 - (*) E provides some positive evidential support for H .
- His formal explication of “ E confirms H ” (in [1]) is:
 - (1) E confirms H iff $\Pr(H | E) > r$, where \Pr is a suitable (“logical”) probability function, and r is a threshold value.
- Unfortunately, Carnap [1] is not entirely consistent in his formal analyses and applications of confirmation.
- Popper [13] points out that in some parts of [1], Carnap has a *different* explication of confirmation in mind, namely:
 - (2) E confirms H iff $\Pr(H | E) > \Pr(H)$, where \Pr is a suitable (“logical”) probability function. [*i.e.*, *correlation* under \Pr]
- In response to Popper, Carnap [2] postulated an *ambiguity* in the concept of confirmation [(1)- vs (2)-confirmation].
- To some modern readers (*e.g.*, *me*), this seems inadequate, since (2) seems to be a *better explication* of the informal concept (*) that Carnap aimed to explicate in the first place.

- To see why (2) is more similar to (*) than (1) is, note that (1) can be satisfied *even if E lowers the probability of H* .
- Example: Let H be the hypothesis that John does *not* have HIV, and let E be a *positive* test result for HIV from a highly reliable test. Plausibly, in such cases, we could have both:
 - $\Pr(H | E) > r$, for just about any threshold value r , but
 - $\Pr(H | E) < \Pr(H)$, since E *lowers* the probability of H .
- So, if we adopt Carnap’s (1)-explication, then we must say that E confirms H in such cases. But, in (*)-terms, this implies E provides some *positive evidential support for H* !
- I take it we don’t want to say *that*. Intuitively, what we want to say here is that, while H is (still) *highly probable given E* , (nonetheless) E provides (strong?) evidence *against H* .
- Rather than *ambiguity*, I’d say this reflects *confusion* about the nature of the concept (*) Carnap was trying to explicate.
- Even Carnap [2] says (2) is “more interesting” than (1).
- Contemporary (Bayesian) confirmation theorists seem to agree. They no longer think of confirmation in (1)-terms ...

Overview Historical Background Philosophical Considerations Psychological Considerations References

- Bayesianism assumes that the *epistemic* degrees of belief (that is, the *credences*) of rational agents are *probabilities*.
- Let $\Pr(H)$ be the degree of belief that a rational agent a assigns to H at some time t (call this a 's "prior" for H).
- Let $\Pr(H | E)$ be the degree of belief that a would assign to H (just after t) were a to learn E at t (a 's "posterior" for H).
- Toy Example: Let H be the proposition that a card sampled from some deck is a ♠, and E assert that the card is black.
- Making the standard assumptions about sampling from 52-card decks, $\Pr(H) = \frac{1}{4}$ and $\Pr(H | E) = \frac{1}{2}$. So, (learning that) E (or supposing that E) *raises the probability of H* .
- Following Popper [13], Bayesians define confirmation in a way that is *formally* very similar to Carnap's (2)-explication.
- For Bayesians, E confirms H for an agent a at a time t iff $\Pr(H | E) > \Pr(H)$, where \Pr captures a 's credences at t .
- While this is *formally* very similar to Carnap's (2), it does not assume that there are objective, "logical" probabilities.

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview Historical Background Philosophical Considerations Psychological Considerations References

- There are *many logically equivalent* (but *syntactically distinct*) ways of saying E confirms H , in the Bayesian sense.
- Here are the three most common ways:
 - E confirms H iff $\Pr(H | E) > \Pr(H)$. [$\frac{1}{2} > \frac{1}{4}$]
 - E confirms H iff $\Pr(E | H) > \Pr(E | \sim H)$. [$1 > \frac{1}{3}$]
 - E confirms H iff $\Pr(H | E) > \Pr(H | \sim E)$. [$\frac{1}{2} > 0$]
- By taking differences or ratios of the left/right sides of such inequalities, various confirmation *measures* $c(H, E)$ emerge.
- A plethora of such confirmation measures have been used in the literature of Bayesian confirmation theory. See my thesis [5] for a survey. Here are the four most popular c 's:
 - $d(H, E) \stackrel{\text{def}}{=} \Pr(H | E) - \Pr(H)$
 - $r(H, E) \stackrel{\text{def}}{=} \log \left[\frac{\Pr(H | E)}{\Pr(H)} \right] \doteq \frac{\Pr(H | E) - \Pr(H)}{\Pr(H | E) + \Pr(H)}$
 - $l(H, E) \stackrel{\text{def}}{=} \log \left[\frac{\Pr(E | H)}{\Pr(E | \sim H)} \right] \doteq \frac{\Pr(E | H) - \Pr(E | \sim H)}{\Pr(E | H) + \Pr(E | \sim H)}$
 - $s(H, E) \stackrel{\text{def}}{=} \Pr(H | E) - \Pr(H | \sim E)$

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview Historical Background Philosophical Considerations Psychological Considerations References

- Question: do these (and other) measures disagree only *conventionally*, or do they disagree in substantive ways?
- Note: mere *numerical* differences between measures are not important, since they need not affect *ordinal* judgments of what is more/less well confirmed than what (by what).
- If two measures c_1 and c_2 agree on *all comparisons*, then we say that c_1 and c_2 are *ordinally equivalent* ($c_1 \doteq c_2$). That is:

$$c_1 \doteq c_2 \stackrel{\text{def}}{=} c_1(H, E) \geq c_1(H', E') \text{ iff } c_2(H, E) \geq c_2(H', E')$$
- **Fact.** No two of $\{d, r, l, s\}$ are ordinally equivalent.
- OK, but do they disagree on *important* applications or in *important* cases? Unfortunately, they disagree *radically*.
- **Fact.** *Almost every* argument/application in the literature is valid for *only some* choices of d, r, l, s . I have called this *the problem of measure sensitivity*. See my [5] for a survey.
- We need some *normative principles* to narrow the field ...

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview Historical Background Philosophical Considerations Psychological Considerations References

- Consider the following two propositions concerning a card c , drawn at random from a standard deck of playing cards:

E : c is the ace of spades. H : c is *some* spade.
- I take it as intuitively clear and uncontroversial that:
 - The degree to which E confirms $H \neq$ the degree to which H confirms E , since $E \models H$, but $H \not\models E$. [$c(H, E) \neq c(E, H)$]
 - The degree to which E confirms $H \neq$ the degree to which $\sim E$ disconfirms H , since $E \models H$, $\sim E \not\models \sim H$. [$c(H, E) \neq -c(H, \sim E)$]
- Therefore, *no adequate measure of confirmation c should be such that either $c(H, E) = c(E, H)$ or $c(H, E) = -c(H, \sim E)$ for all E and H and for all probability functions \Pr* . I'll call these two symmetry desiderata S_1 and S_2 , respectively.
- Note: for all H, E , and for all \Pr , $r(H, E) = r(E, H)$ and $s(H, E) = -s(H, \sim E)$. That is, r violates S_1 and s violates S_2 .
- *Both d and l satisfy these S -desiderata*. This narrows the field to d and l [4]. We can narrow the field further still ...

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	●	○○○○○	

- If we think of inductive logic as a *quantitative generalization* of deductive logic, then the following *logical desideratum* seems natural (it's also implicit in the previous example):
 - (†) **Quantitative Rendition.** $c(H, E)$ should be *maximal* when $E \models H$ and $c(H, E)$ should be *minimal* when $E \models \sim H$.
 - (†) **Comparative Rendition.** If $E \models H$ but $E' \not\models H'$, then the following inequality should hold: $c(H, E) \geq c(H', E')$.
- The measure d violates these desiderata. For, when $E \models H$:

$$d(H, E) = \Pr(H | E) - \Pr(H) = 1 - \Pr(H) = \Pr(\sim H)$$
- So, if the prior probability of H is sufficiently high, then (according to d) E will confirm H *very weakly, even if* $E \models H$.
- From an inductive-logical point of view, this is absurd, since *the logical strength of a valid argument should not depend on how probable its conclusion is* (or on its truth-value).
- Indeed, of all the Bayesian measures of confirmation that have been used in the literature, only l (or its ordinal equivalents) satisfy all three of our desiderata: $S_1, S_2, (\dagger)$.

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	○○	●○○○	

- Kahneman and Tversky [9] amassed lots of data, which they claimed indicated *violations* of normative principles for probability judgments (*i.e.*, violations of the Pr-axioms).
- If Carnap was confused (along with many others) about the probability/confirmation distinction, could this confusion also underlie some of these erroneous Pr judgments?
- Two examples from K&T come to mind. First, their experiments on the neglect of “base rate” information.
- When people are asked to assess the probability that John has AIDS, given that he tested positive for AIDS according to a very reliable test protocol, they often report high values.
- This *seems* to violate Bayes’s Theorem, since AIDS has such a low base rate (prior?) in the population (and they know this). This does *seem* to be a poor probability judgment [10].
- But, could this also reflect a *good* underlying *confirmation* or *evidential support* judgment? Note: $l(H, E)$ is very close to the value reported by experts in these examples [7].

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	○○	●○○○	

- A second example from K&T that’s worth thinking about in this connection is the so-called “conjunction fallacy”.
 - (E) Linda is 31, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice and she also participated in antinuclear demonstrations.
- Is it more probable, given E , that Linda is (H_1) a bank teller, or ($H_1 \& H_2$) a bank teller & active in the feminist movement?
- Most people answer that $H_1 \& H_2$ is more probable (given E) than H_1 is. This violates Pr-theory, since $H_1 \& H_2 \models H_1$.
- Note: it is possible to have $l(H_1 \& H_2, E) > l(H_1, E)$. Thus, E could constitute *better evidence for* $H_1 \& H_2$ than for H_1 [11]. Indeed, we [3] have recently proven the following:

Theorem. For almost all confirmation measures c , if

 - (i) $c(H_2, E | H_1) > 0$ and
 - (ii) $c(H_1, E) \leq 0$,
 then $c(H_1 \& H_2, E) > c(H_1, E)$.
- And, conditions (i) and (ii) do seem to hold in the Linda case.

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	○○	●○○○	

- Interestingly, until recently there have been almost no psychological studies on how people *actually* make confirmation judgments (in the present, Bayesian sense).
- This was surprising to me, mainly for the following reasons:
 - Because of the long-standing confusion about probability vs confirmation in the philosophical literature, I thought that this should be a ripe area for psychological research.
 - I've suspected that confirmation judgments should be more robust than Pr-judgments, since they are (normatively!) less sensitive to subjective factors (in particular, “priors” [6]).
- I am happy to report that this now seems to be evolving into a ripe area for psychological research. Dan Osherson and his colleagues are largely responsible for this change.
- One thing we’d like to know is whether people tend to make *quantitative* judgments of confirmational strength that accord with normatively adequate measures like l .
- A recent study [12] was designed to answer this question ...

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	○○	○○●○○	

- As far as I know, the forthcoming study by Osherson et al [12] is the first designed explicitly to test Bayesian measures of confirmation against each other for descriptive accuracy.
- Their study involved 24 undergraduates (U. of Trento). They were (individually) faced with the following scenario.
 - They were shown two opaque urns (A, B), where A contains 30/10 black/white balls, and B contains 15/25 B/W balls.
 - A fair coin was tossed, and an urn selected at random.
 - Then, 10 balls were drawn (at random) without replacement.
 - After each draw, they were asked to rank the *evidential impact* of that draw on the hypotheses (a) that A was chosen, and (b) that B was chosen, on a scale with 7 “ticks”.
 - Tick 1: “weakens my conviction extremely”, tick 7: “strengthens my conviction extremely”. Tick 4: “no effect”.
 - Then, the subject was asked to estimate *probabilities* $\Pr(A | E)$ and $\Pr(B | E)$ and *likelihoods* $\Pr(E | A)$ and $\Pr(E | B)$.
 - Finally, these subjective estimates of probabilities and likelihoods were plugged-in to the various measures of confirmation. And, correlation statistics were calculated.

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	○○	○○●○○	

- The experimenters also plugged-in *objective* probabilities and likelihoods, to see what predictions *those* yielded.
- The results were (to me) somewhat (pleasantly!) surprising:
 - Of the measures d, r , and l , the measure l was significantly better at predicting confirmation judgments, both using the subjective and the objective probabilities and likelihoods.
 - Note: their protocol was unable to test the accuracy of s .
 - Several additional measures from the literature were tested, and l was significantly better than *all* of the other measures, when *objective* probabilities/likelihoods were used.
 - l was not significantly worse than any other measure tested, when *subjective* probabilities/likelihoods were used.
 - The posterior probabilities (either objective or subjective) were *very poor* predictors. This indicates that the subjects distinguished confirmation & probability [Carnap’s (1) & (2)].
- This (plus subj \neq obj) confirms what I have long suspected: people are better at making confirmation judgments than probability judgments. Of course, more studies are needed.
- Now, for some research suggestions from the armchair ...

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	○○	○○○○●	

- First, I would suggest looking at *comparative/relational* confirmation judgments, rather than *quantitative* ones. I suspect these will be even more robust and objective [6].
- Second, I would suggest controlling for certain other pragmatic factors that may confound (or create) differences between measures. Jim Joyce has discussed such factors [8].
- Third, the protocol of Osherson *et al* was unable to test the descriptive accuracy of the measure s . It would be nice to generalize their protocol to include s (and others like it).
- Finally, I would also like to see some experiments designed *explicitly* to distinguish *qualitative* confirmation judgments from probability-threshold judgments [Carnapian (1) vs (2)].
- I suspect that people’s judgments about “what confirms what” come apart *sharply* from their judgments of what is “probable”. But, it would be nice to have more data on this.
- E.g.*: I bet jurors who learn their (guilty) verdict was false will retract “probable” claims, *not* “supported-by- E ” claims.

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org

Overview	Historical Background	Philosophical Considerations	Psychological Considerations	References
	○○○○	○○	○○○○○	

- R. Carnap, (1950), *Logical foundations of probability*, 1st ed., U. Chicago Press.
- R. Carnap, (1962), *Logical foundations of probability*, 2nd ed., U. Chicago Press.
- V. Crupi, B. Fitelson and K. Tentori (2008), *Probability, Confirmation, and the Conjunction Fallacy, Thinking & Reasoning*, URL: <http://bit.ly/hMn Pau>.
- E. Eells and B. Fitelson, (2002), Symmetries and asymmetries in evidential support, *Philosophical Studies*, URL: <http://bit.ly/g2UJXu>.
- B. Fitelson, (2001), *Studies in Bayesian confirmation theory*, PhD. thesis, University of Wisconsin, URL: <http://fitelson.org/thesis.pdf>.
- B. Fitelson, (2007), Likelihoodism, Bayesiansim, and relational confirmation, *Synthese*, URL: <http://fitelson.org/synthese.pdf>.
- G. Gigerenzer, (2000), *Adaptive thinking*, Oxford University Press.
- J. Joyce, forthcoming, On the plurality of probabilist measures of evidential relevance, (presented at *PSA 2004*, and forthcoming in *Philosophical Review*).
- D. Kahneman, P. Slovic, and A. Tversky (eds.), (1982), *Judgment under uncertainty: heuristics and biases*, Cambridge University Press.
- J.J. Koehler, (1996), The base rate fallacy reconsidered: normative, descriptive and methodological challenges, *Behavioral and Brain Sciences* 19: 1-53.
- I. Levi, (1985), “Illusions about Uncertainty”, *British J. for the Philosophy of Sci.*
- D. Osherson, K. Tentori, V. Crupi, and N. Bonini, (2008), Comparison of confirmation measures, *Cognition*, URL: <http://bit.ly/hnaR8Z>.
- K. Popper, (1954), Degree of confirmation, *British Journal of Phil. Sci.*, 5:143-149.

Branden Fitelson Judgment Under Uncertainty Revisited: Probability vs Confirmation fitelson.org