A Bayesian Account of Independent Evidence with Applications

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Overview

Preliminaries I: Some Bayesian Background

- E confirms H (given K) iff E is correlated with H under $\Pr(\cdot \mid K)$.
- *i.e.*, E confirms H (given K) iff $\Pr(H \mid E \,\&\, K) > \Pr(H \mid K)$.
- There are *many* logically equivalent ways to say this, *e.g.*,
  - E confirms H (given K) iff $\Pr(E \mid H \,\&\, K) > \Pr(E \mid \bar{H} \,\&\, K)$.
  - E confirms H (given K) iff $\Pr(H \,\&\, E \mid K) > \Pr(H \mid K) \cdot \Pr(E \mid K)$.
- This leads to a *plethora* of possible (Bayesian) *relevance measures* of the *degree* to which E confirms H (given K).
- Various differences, ratios, *etc.*, of the left/right sides can be used.
- This plurality of measures of confirmation is known to have consequences for many arguments in the literature (Fitelson 1999).
- We'll return to the plurality issue later on ...

Preliminaries II: Four Relevance Measures

- The following four measures have been proposed & defended:
  - The *Difference*: $d(H, E \mid K) =_{df} \Pr(H \mid E \,\&\, K) - \Pr(H \mid K)$
  - The Log-*Ratio*: $r(H, E \mid K) =_{df} \log\left[\frac{\Pr(H \mid E \,\&\, K)}{\Pr(H \mid K)}\right]$
  - The Log-*Likelihood-Ratio*: $l(H, E \mid K) =_{df} \log\left[\frac{\Pr(E \mid H \,\&\, K)}{\Pr(E \mid \bar{H} \,\&\, K)}\right]$
  - Christensen's (1999) "Normalized" Difference:

$$s(H, E \mid K) =_{df} \Pr(H \mid E \,\&\, K) - \Pr(H \mid \bar{E} \,\&\, K)$$

$$= \frac{1}{\Pr(\bar{E} \mid K)} \cdot d(H, E \mid K).^{\text{a}}$$

- These four relevance measures are known to be ordinally non-equivalent, and to disagree in some important respects ...

[a] This equality holds provided, of course, that $\Pr(\bar{E} \mid K) \neq 0$.

Preliminaries III: Notation & Background Evidence

- "$\mathfrak{c}(H, E_1 \mid E_2)$" reads "the degree to which E_1 confirms H according to $\mathfrak{c}$, conditional on E_2 being part of our background evidence."
- "$\mathfrak{c}(H, E_1)$" reads "the degree to which E_1 confirms H according to $\mathfrak{c}$, *not* conditional on E_2 being part of our background evidence."
- There may things other than E_1, E_2 in the background evidence, but I'll assume these are *held fixed* in the comparisons I'll be doing.
- So, I will not explicitly write down all the members of K. I will just focus on the salient parts (E_1, E_2, *etc.*) of K for our purposes.
- It is important to keep in mind that "$\mathfrak{c}$" is a *variable* which ranges over individual measures of confirmation. With these conventions in hand, we're ready to discuss confirmational independence ...

Confirmational Independence I: The Basic Ideas

- Let H be the hypothesis that something is wrong with a computer, and E be the evidence that nothing happens when the computer's power switch is moved to the "on" position.
- If the background evidence K includes facts such as that the computer is plugged in, *etc.*, then E will confirm H.
- If K specifies that the computer is not plugged in and that it needs to be plugged in to work, then E will not confirm H.
- So, for any adequate measure of confirmation $\mathfrak{c}$, there will inevitably be cases in which $\mathfrak{c}(H, E_1 \mid E_2) \neq \mathfrak{c}(H, E_1)$.
- When this happens, we say that E_1 *is confirmationally dependent on* E_2 *regarding* H *according to* $\mathfrak{c}$ (4-place relation).

Confirmational Independence II: More Basics

- If $\mathfrak{c}(H, E_1 \mid E_2) = \mathfrak{c}(H, E_1)$, then we say that E_1 *is confirmationally independent of* E_2 *regarding* H *according to* $\mathfrak{c}$.
- If $\mathfrak{c}(H, E_1 \mid E_2) = \mathfrak{c}(H, E_1)$, *and* $\mathfrak{c}(H, E_2 \mid E_1) = \mathfrak{c}(H, E_2)$, then E_1 and E_2 are said to be *mutually confirmationally independent (or, simply,* ***independent****) regarding* H *according to* $\mathfrak{c}$.
- Consider the following quote from Peirce (1878):

  > ... two arguments which are entirely independent, neither weakening nor strengthening the other, ought, when they concur, to produce a[n intensity of] belief equal to the sum of the intensities of belief which either would produce separately.

- Peirce's insight about the additivity (or linearity) of mutually confirmationally independent evidence is fundamental ...

Peirce's Additivity/Linearity Desideratum

- We can express Peirce's additivity desideratum as follows:

($\mathcal{A}$) If E_1 and E_2 are mutually confirmationally independent regarding H according to $\mathfrak{c}$, then $\mathfrak{c}(H, E_1 \,\&\, E_2) = \mathfrak{c}(H, E_1) + \mathfrak{c}(H, E_2)$.

- The important thing here is not so much that $\mathfrak{c}(H, E_1 \,\&\, E_2)$ be the *sum* of $\mathfrak{c}(H, E_1)$ and $\mathfrak{c}(H, E_2)$, but that $\mathfrak{c}(H, E_1 \,\&\, E_2)$ be *some* function (which is *linear*, in some sense) of $\mathfrak{c}(H, E_1)$ and $\mathfrak{c}(H, E_2)$.
- That is, $\mathfrak{c}(H, E_1 \,\&\, E_2)$ for two independent pieces of evidence should depend *only* (and, in some sense, *linearly*) on $\mathfrak{c}(H, E_1)$ and $\mathfrak{c}(H, E_2)$ — there should be no "interaction" terms.
- As it turns out, this weaker desideratum ($\mathcal{A}'$) is satisfied by all four of our candidate measures — *except* s. This is bad news for s.[a]

[a] See Eells & Fitelson (2000a, 2000b) for more bad news about s.

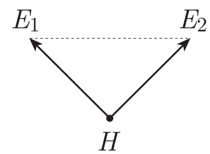
A Negation Symmetry Desideratum

- Intuitively, if two pieces of evidence E_1 and E_2 are confirmationally independent regarding H according to $\mathfrak{c}$, then they should also be confirmationally independent regarding $\bar{H}$ according to $\mathfrak{c}$.
- This negation symmetry desideratum ($\mathcal{N}$) is satisfied by all four of our candidate measures — *except* r. This is bad news for r.[a]
- We have been able to narrow the field of four candidate measures down to two (d and l) using only very weak, high-level desiderata.
- Adjudicating between d and l is more difficult, and will require an appeal to stronger, low-level, probabilistic considerations . . .

[a]This is related to the fact that each of our four candidate measures, *except* r, satisfies the following *hypothesis symmetry* condition: (HS) $\mathfrak{c}(H, E \mid K) = -\mathfrak{c}(\bar{H}, E \mid K)$. See Fitelson (1999) and Eells & Fitelson (2000b) for more bad news about r.

Screening-Off and Confirmational Independence I

- Let E_1 be a newspaper report of the outcome H of a baseball game, and E_2 be a (causally) independently derived radio report of the (same) outcome of the same baseball game.
- As Sober (1989) explains, this is (intuitively) a case in which we have two pieces of independent evidence regarding a common cause.
- What probabilistic feature of this example undergirds our intuition that E_1 and E_2 provide *independent* support for H?



- Reichenbach's (1956) Theorem implies that E_1 and E_2 *cannot* be *unconditionally* (stochastically) independent in this kind of example.
- It must be some *other* probabilistic feature ...

Screening-Off and Confirmational Independence II

- I suggest (as did Sober) that *the* probabilistic fact which explains why we take E_1 and E_2 to be mutually confirmationally independent regarding H is the fact that H *screens-off* E_1 *from* E_2.
- This suggests an intuitive probabilistic sufficient[a] condition for confirmational independence (and a new *desideratum* for $\mathfrak{c}$):

($\mathcal{S}$) If H screens-off E_1 from E_2, then E_1 and E_2 should be mutually confirmationally independent regarding H according to $\mathfrak{c}$.

- This gives us an interesting way to adjudicate between d and l, since *only* l (up to order-preserving transformation) satisfies $\mathcal{S}$.
- This completes application #1 of our account of confirmational independence — to the problem of the plurality of measures.

[a]I do *not* think screening-off should be *necessary* for confirmational independence.

An Application to Evidential Diversity

- Here's an interesting consequence of Peirce's linearity condition $\mathcal{A}'$:

($\mathcal{D}$) If each of E_1 and E_2 individually confirms H, and if E_1 and E_2 are mutually confirmationally independent regarding H according to $\mathfrak{c}$, then $\mathfrak{c}(H \mid E_1 \,\&\, E_2) > \mathfrak{c}(H \mid E_1)$ and $\mathfrak{c}(H \mid E_1 \,\&\, E_2) > \mathfrak{c}(H \mid E_2)$.

- $\mathcal{D}$ identifies a sufficient (but *not* necessary) condition for increased confirmational power. And, $\mathcal{D}$ is not strongly sensitive to choice of measure of confirmation (like $\mathcal{A}'$, $\mathcal{D}$ is satisfied by d, r, and l).
- I suggest that $\mathcal{D}$ can be used to give a (partial) Bayesian account of of the confirmational significance of evidential diversity (CSED).
- If 'diverse' new (confirmatory) data are *confirmationally independent of* old (confirmatory) data, then they will combine with the old data to form a more confirmationally powerful whole.
- This may explain (in *some* cases) why 'diversity' can be valuable.

Comparison with Howson & Urbach's "Correlation" Approach

- Howson & Urbach (1989) use the following to argue that *unconditionally dependent* data are confirmationally *dependent*:

($\mathcal{H}$) *If* the following probabilistic '*ceteris paribus* clause is satisfied:
$(CP)\ \Pr(E_1 \mid H) = \Pr(E_2 \mid H) = \Pr(E_1 \,\&\, E_2 \mid H) = 1$,
then if $\Pr(E_2 \mid E_1) > \Pr(E_2)$, then $\mathfrak{c}(H, E_2 \mid E_1) < \mathfrak{c}(H, E_2)$.

- $\mathcal{H}$ incorrectly appeals only to *unconditional* (in)dependence of the data, and not to *conditional* (in)dependence of the data.
- As a result, $\mathcal{H}$ applies only to a narrow class of (deductive) cases (similar criticisms are made, independently, by Myrvold 1996).
- $\mathcal{H}$ is true *only if* $\mathfrak{c} = r$. $\mathcal{H}$ is *false* if $\mathfrak{c} = d, s$, or l. So, Howson & Urbach's argument is strongly sensitive to their choice (r) of measure of confirmation (and we have argued that r is inadequate).

Tabular Summary of Key Results

	Is $\mathcal{P}$ satisfied by the measure:			
Name of Property $\mathcal{P}$	d?	r?	l?	s?
Peirce's Linearity Property $\mathcal{A}'$	YES	YES	YES	NO
Negation Symmetry Property $\mathcal{N}$	YES	NO	YES	YES
Screening-Off Property $\mathcal{S}$	NO	NO	YES	NO
Howson & Urbach's Property $\mathcal{H}$	NO	YES	NO	NO

Concluding Remarks

- The intuitive notion of confirmational independence described by Peirce is comfortably at home in the modern Bayesian framework.
- By appealing only to high-level, intuitive aspects of independent evidence, we can drastically narrow the field of relevance measures.
- Further considerations show that only the measure l forges the appropriate general connection between probabilistic *screening-off* (*viz.*, *causal* independence) and *confirmational* independence.
- We can also apply the notion of confirmational independence to the problem of CSED, yielding a robust, intuitive (partial) account.
- Our account of CSED seems more appealing than one alternative "correlation" account, which ignores the importance of *screening-off*, and is strongly sensitive to choice of measure.

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