CHAPTER XVI

DESCRIPTIONS

167 We dealt in the preceding chapter with the words *all* and *some*; in this chapter we shall consider the word *the* in the singular, and in the next chapter we shall consider the word *the* in the plural. It may be thought excessive to devote two chapters to one word, but to the philosophical mathematician it is a word of very great importance: like Browning's Grammarian with the enclitic $\delta\epsilon$, I would give the doctrine of this word if I were "dead from the waist down" and not merely in a prison.

We have already had occasion to mention "descriptive functions," *i.e.* such expressions as "the father of x" or "the sine of x." These are to be defined by first defining "descriptions."

A "description" may be of two sorts, definite and indefinite (or ambiguous). An indefinite description is a phrase of the form "a so-and-so," and a definite description is a phrase of the form "the so-and-so" (in the singular). Let us begin with the former.

"Who did you meet?" "I met a man." "That is a very indefinite description." We are therefore not departing from usage in our terminology. Our question is: What do I really assert when I assert "I met a man"? Let us assume, for the moment, that my assertion is true, and that in fact I met Jones. It is clear that what I assert is *not* "I met Jones." I may say "I met a man, but it was not Jones"; in that case, though I lie, I do not contradict
168 myself, as I should do if when I say I met a | man I really mean that I met Jones. It is clear also that the person to whom I am

speaking can understand what I say, even if he is a foreigner and has never heard of Jones.

But we may go further: not only Jones, but no actual man, enters into my statement. This becomes obvious when the statement is false, since then there is no more reason why Jones should be supposed to enter into the proposition than why anyone else should. Indeed the statement would remain significant, though it could not possibly be true, even if there were no man at all. "I met a unicorn" or "I met a sea-serpent" is a perfectly significant assertion, if we know what it would be to be a unicorn or a sea-serpent, *i.e.* what is the definition of these fabulous monsters. Thus it is only what we may call the *concept* that enters into the proposition. In the case of "unicorn," for example, there is only the concept: there is not also, somewhere among the shades, something unreal which may be called "a unicorn." Therefore, since it is significant (though false) to say "I met a unicorn," it is clear that this proposition, rightly analysed, does not contain a constituent "a unicorn," though it does contain the concept "unicorn."

The question of "unreality," which confronts us at this point, is a very important one. Misled by grammar, the great majority of those logicians who have dealt with this question have dealt with it on mistaken lines. They have regarded grammatical form as a surer guide in analysis than, in fact, it is. And they have not known what differences in grammatical form are important. "I met Jones" and "I met a man" would count traditionally as propositions of the same form, but in actual fact they are of quite different forms: the first names an actual person, Jones; while the second involves a propositional function, and becomes, when made explicit: "The function 'I met x and x is human' is sometimes true." (It will be remembered that we adopted the convention of using "sometimes" as not implying more than once.) This proposition is obviously not of the form "I met x," which accounts | for the existence of the 169
proposition "I met a unicorn" in spite of the fact that there is

no such thing as "a unicorn."

For want of the apparatus of propositional functions, many logicians have been driven to the conclusion that there are unreal objects. It is argued, *e.g.* by Meinong,[1] that we can speak about "the golden mountain," "the round square," and so on; we can make true propositions of which these are the subjects; hence they must have some kind of logical being, since otherwise the propositions in which they occur would be meaningless. In such theories, it seems to me, there is a failure of that feeling for reality which ought to be preserved even in the most abstract studies. Logic, I should maintain, must no more admit a unicorn than zoology can; for logic is concerned with the real world just as truly as zoology, though with its more abstract and general features. To say that unicorns have an existence in heraldry, or in literature, or in imagination, is a most pitiful and paltry evasion. What exists in heraldry is not an animal, made of flesh and blood, moving and breathing of its own initiative. What exists is a picture, or a description in words. Similarly, to maintain that Hamlet, for example, exists in his own world, namely, in the world of Shakespeare's imagination, just as truly as (say) Napoleon existed in the ordinary world, is to say something deliberately confusing, or else confused to a degree which is scarcely credible. There is only one world, the "real" world: Shakespeare's imagination is part of it, and the thoughts that he had in writing Hamlet are real. So are the thoughts that we have in reading the play. But it is of the very essence of fiction that only the thoughts, feelings, etc., in Shakespeare and his readers are real, and that there is not, in addition to them, an objective Hamlet. When you have taken account of all the feelings roused by Napoleon in writers and readers of history, you have not touched the actual man; but in the case of Hamlet you have come to the end of him. If no one
170 thought about Hamlet, there would be nothing left of him; if

[1] *Untersuchungen zur Gegenstandstheorie und Psychologie*, 1904.

no one had thought about Napoleon, he would have soon seen to it that some one did. The sense of reality is vital in logic, and whoever juggles with it by pretending that Hamlet has another kind of reality is doing a disservice to thought. A robust sense of reality is very necessary in framing a correct analysis of propositions about unicorns, golden mountains, round squares, and other such pseudo-objects.

In obedience to the feeling of reality, we shall insist that, in the analysis of propositions, nothing "unreal" is to be admitted. But, after all, if there *is* nothing unreal, how, it may be asked, *could* we admit anything unreal? The reply is that, in dealing with propositions, we are dealing in the first instance with symbols, and if we attribute significance to groups of symbols which have no significance, we shall fall into the error of admitting unrealities, in the only sense in which this is possible, namely, as objects described. In the proposition "I met a unicorn," the whole four words together make a significant proposition, and the word "unicorn" by itself is significant, in just the same sense as the word "man." But the *two* words "a unicorn" do not form a subordinate group having a meaning of its own. Thus if we falsely attribute meaning to these two words, we find ourselves saddled with "a unicorn," and with the problem how there can be such a thing in a world where there are no unicorns. "A unicorn" is an indefinite description which describes nothing. It is not an indefinite description which describes something unreal. Such a proposition as "x is unreal" only has meaning when "x" is a description, definite or indefinite; in that case the proposition will be true if "x" is a description which describes nothing. But whether the description "x" describes something or describes nothing, it is in any case not a constituent of the proposition in which it occurs; like "a unicorn" just now, it is not a subordinate group having a meaning of its own. All this results from the fact that, when "x" is a description, "x is unreal" or "x does not exist" is not nonsense, but is always significant and sometimes true.

171 We may now proceed to define generally the meaning of propositions which contain ambiguous descriptions. Suppose we wish to make some statement about "a so-and-so," where "so-and-so's" are those objects that have a certain property ϕ, *i.e.* those objects x for which the propositional function ϕx is true. (*E.g.* if we take "a man" as our instance of "a so-and-so," ϕx will be "x is human.") Let us now wish to assert the property ψ of "a so-and-so," *i.e.* we wish to assert that "a so-and-so" has that property which x has when ψx is true. (*E.g.* in the case of "I met a man," ψx will be "I met x.") Now the proposition that "a so-and-so" has the property ψ is *not* a proposition of the form "ψx." If it were, "a so-and-so" would have to be identical with x for a suitable x; and although (in a sense) this may be true in some cases, it is certainly not true in such a case as "a unicorn." It is just this fact, that the statement that a so-and-so has the property ψ is not of the form ψx, which makes it possible for "a so-and-so" to be, in a certain clearly definable sense, "unreal." The definition is as follows:—

> The statement that "an object having the property ϕ has the property ψ"

means:

> "The joint assertion of ϕx and ψx is not always false."

So far as logic goes, this is the same proposition as might be expressed by "some ϕ's are ψ's"; but rhetorically there is a difference, because in the one case there is a suggestion of singularity, and in the other case of plurality. This, however, is not the important point. The important point is that, when rightly analysed, propositions verbally about "a so-and-so" are found to contain no constituent represented by this phrase. And that is why such propositions can be significant even when there is no such thing as a so-and-so.

The definition of *existence*, as applied to ambiguous descriptions, results from what was said at the end of the preceding

chapter. We say that "men exist" or "a man exists" if the propositional function "x is human" is sometimes true; and gen- 172
erally "a so-and-so" exists if "x is so-and-so" is sometimes true. We may put this in other language. The proposition "Socrates is a man" is no doubt *equivalent* to "Socrates is human," but it is not the very same proposition. The *is* of "Socrates is human" expresses the relation of subject and predicate; the *is* of "Socrates is a man" expresses identity. It is a disgrace to the human race that it has chosen to employ the same word "is" for these two entirely different ideas—a disgrace which a symbolic logical language of course remedies. The identity in "Socrates is a man" is identity between an object named (accepting "Socrates" as a name, subject to qualifications explained later) and an object ambiguously described. An object ambiguously described will "exist" when at least one such proposition is true, *i.e.* when there is at least one true proposition of the form "x is a so-and-so," where "x" is a name. It is characteristic of ambiguous (as opposed to definite) descriptions that there may be any number of true propositions of the above form—Socrates is a man, Plato is a man, etc. Thus "a man exists" follows from Socrates, or Plato, or anyone else. With definite descriptions, on the other hand, the corresponding form of proposition, namely, "x is the so-and-so" (where "x" is a name), can only be true for one value of x at most. This brings us to the subject of definite descriptions, which are to be defined in a way analogous to that employed for ambiguous descriptions, but rather more complicated.

We come now to the main subject of the present chapter, namely, the definition of the word *the* (in the singular). One very important point about the definition of "a so-and-so" applies equally to "the so-and-so"; the definition to be sought is a definition of propositions in which this phrase occurs, not a definition of the phrase itself in isolation. In the case of "a so-and-so," this is fairly obvious: no one could suppose that "a man" was a definite object, which could be defined by itself.

173 Socrates is a man, Plato is a man, Aristotle is a man, but we cannot infer that "a man" means the same as "Socrates" means and also the same as "Plato" means and also the same as "Aristotle" means, since these three names have different meanings. Nevertheless, when we have enumerated all the men in the world, there is nothing left of which we can say, "This is a man, and not only so, but it is *the* 'a man,' the quintessential entity that is just an indefinite man without being anybody in particular." It is of course quite clear that whatever there is in the world is definite: if it is a man it is one definite man and not any other. Thus there cannot be such an entity as "a man" to be found in the world, as opposed to specific men. And accordingly it is natural that we do not define "a man" itself, but only the propositions in which it occurs.

In the case of "the so-and-so" this is equally true, though at first sight less obvious. We may demonstrate that this must be the case, by a consideration of the difference between a *name* and a *definite description*. Take the proposition, "Scott is the author of *Waverley*." We have here a name, "Scott," and a description, "the author of *Waverley*," which are asserted to apply to the same person. The distinction between a name and all other symbols may be explained as follows:—

A name is a simple symbol whose meaning is something that can only occur as subject, *i.e.* something of the kind that, in Chapter XIII., we defined as an "individual" or a "particular." And a "simple" symbol is one which has no parts that are symbols. Thus "Scott" is a simple symbol, because, though it has parts (namely, separate letters), these parts are not symbols. On the other hand, "the author of *Waverley*" is not a simple symbol, because the separate words that compose the phrase are parts which are symbols. If, as may be the case, whatever *seems* to be an "individual" is really capable of further analysis, we shall have to content ourselves with what may be called "relative individuals," which will be terms that, throughout the context in
174 question, are never analysed and never occur | otherwise than

as subjects. And in that case we shall have correspondingly to content ourselves with "relative names." From the standpoint of our present problem, namely, the definition of descriptions, this problem, whether these are absolute names or only relative names, may be ignored, since it concerns different stages in the hierarchy of "types," whereas we have to compare such couples as "Scott" and "the author of *Waverley*," which both apply to the same object, and do not raise the problem of types. We may, therefore, for the moment, treat names as capable of being absolute; nothing that we shall have to say will depend upon this assumption, but the wording may be a little shortened by it.

We have, then, two things to compare: (1) a *name*, which is a simple symbol, directly designating an individual which is its meaning, and having this meaning in its own right, independently of the meanings of all other words; (2) a *description*, which consists of several words, whose meanings are already fixed, and from which results whatever is to be taken as the "meaning" of the description.

A proposition containing a description is not identical with what that proposition becomes when a name is substituted, even if the name names the same object as the description describes. "Scott is the author of *Waverley*" is obviously a different proposition from "Scott is Scott": the first is a fact in literary history, the second a trivial truism. And if we put anyone other than Scott in place of "the author of *Waverley*," our proposition would become false, and would therefore certainly no longer be the same proposition. But, it may be said, our proposition is essentially of the same form as (say) "Scott is Sir Walter," in which two names are said to apply to the same person. The reply is that, if "Scott is Sir Walter" really means "the person named 'Scott' is the person named 'Sir Walter,'" then the names are being used as descriptions: *i.e.* the individual, instead of being named, is being described as the person having that name.
This is a way in which names are frequently used | in practice, 175

and there will, as a rule, be nothing in the phraseology to show whether they are being used in this way or *as* names. When a name is used directly, merely to indicate what we are speaking about, it is no part of the *fact* asserted, or of the falsehood if our assertion happens to be false: it is merely part of the symbolism by which we express our thought. What we want to express is something which might (for example) be translated into a foreign language; it is something for which the actual words are a vehicle, but of which they are no part. On the other hand, when we make a proposition about "the person called 'Scott,'" the actual name "Scott" enters into what we are asserting, and not merely into the language used in making the assertion. Our proposition will now be a different one if we substitute "the person called 'Sir Walter.'" But so long as we are using names *as* names, whether we say "Scott" or whether we say "Sir Walter" is as irrelevant to what we are asserting as whether we speak English or French. Thus so long as names are used *as* names, "Scott is Sir Walter" is the same trivial proposition as "Scott is Scott." This completes the proof that "Scott is the author of *Waverley*" is not the same proposition as results from substituting a name for "the author of *Waverley*," no matter what name may be substituted.

When we use a variable, and speak of a propositional function, ϕx say, the process of applying general statements about ϕx to particular cases will consist in substituting a name for the letter "x," assuming that ϕ is a function which has individuals for its arguments. Suppose, for example, that ϕx is "always true"; let it be, say, the "law of identity," $x = x$. Then we may substitute for "x" any name we choose, and we shall obtain a true proposition. Assuming for the moment that "Socrates," "Plato," and "Aristotle" are names (a very rash assumption), we can infer from the law of identity that Socrates is Socrates, Plato is Plato, and Aristotle is Aristotle. But we shall commit a fallacy if we attempt to infer, without further premisses, that the author of *Waverley* is the author of *Waverley*. This results

from what we have just proved, that, if we substitute a name 176
for "the author of *Waverley*" in a proposition, the proposition we obtain is a different one. That is to say, applying the result to our present case: If "x" is a name, "$x = x$" is not the same proposition as "the author of *Waverley* is the author of *Waverley*," no matter what name "x" may be. Thus from the fact that all propositions of the form "$x = x$" are true we cannot infer, without more ado, that the author of *Waverley* is the author of *Waverley*. In fact, propositions of the form "the so-and-so is the so-and-so" are not always true: it is necessary that the so-and-so should *exist* (a term which will be explained shortly). It is false that the present King of France is the present King of France, or that the round square is the round square. When we substitute a description for a name, propositional functions which are "always true" may become false, if the description describes nothing. There is no mystery in this as soon as we realise (what was proved in the preceding paragraph) that when we substitute a description the result is not a value of the propositional function in question.

We are now in a position to define propositions in which a definite description occurs. The only thing that distinguishes "the so-and-so" from "a so-and-so" is the implication of uniqueness. We cannot speak of "*the* inhabitant of London," because inhabiting London is an attribute which is not unique. We cannot speak about "the present King of France," because there is none; but we can speak about "the present King of England." Thus propositions about "the so-and-so" always imply the corresponding propositions about "a so-and-so," with the addendum that there is not more than one so-and-so. Such a proposition as "Scott is the author of *Waverley*" could not be true if *Waverley* had never been written, or if several people had written it; and no more could any other proposition resulting from a propositional function ϕx by the substitution of "the author of *Waverley*" for "x." We may say that "the author of
Waverley" means "the value of x for which 'x wrote *Waverley*' 177

is true." Thus the proposition "the author of *Waverley* was Scotch," for example, involves:

(1) "x wrote *Waverley*" is not always false;
(2) "if x and y wrote *Waverley*, x and y are identical" is always true;
(3) "if x wrote *Waverley*, x was Scotch" is always true.

These three propositions, translated into ordinary language, state:

(1) at least one person wrote *Waverley*;
(2) at most one person wrote *Waverley*;
(3) whoever wrote *Waverley* was Scotch.

All these three are implied by "the author of *Waverley* was Scotch." Conversely, the three together (but no two of them) imply that the author of *Waverley* was Scotch. Hence the three together may be taken as defining what is meant by the proposition "the author of *Waverley* was Scotch."

We may somewhat simplify these three propositions. The first and second together are equivalent to: "There is a term c such that 'x wrote *Waverley*' is true when x is c and is false when x is not c." In other words, "There is a term c such that 'x wrote *Waverley*' is always equivalent to 'x is c.'" (Two propositions are "equivalent" when both are true or both are false.) We have here, to begin with, two functions of x, "x wrote *Waverley*" and "x is c," and we form a function of c by considering the equivalence of these two functions of x for all values of x; we then proceed to assert that the resulting function of c is "sometimes true," *i.e.* that it is true for at least one value of c. (It obviously cannot be true for more than one value of c.) These two conditions together are defined as giving the meaning of "the author of *Waverley* exists."

We may now define "the term satisfying the function ϕx exists." This is the general form of which the above is a particular case. "The author of *Waverley*" is "the term satisfying the

function 'x wrote *Waverley*.'" And "the so-and-so" will | always 178
involve reference to some propositional function, namely, that which defines the property that makes a thing a so-and-so. Our definition is as follows:—

"The term satisfying the function ϕx exists" means:

"There is a term c such that ϕx is always equivalent to 'x is c.'"

In order to define "the author of *Waverley* was Scotch," we have still to take account of the third of our three propositions, namely, "Whoever wrote *Waverley* was Scotch." This will be satisfied by merely adding that the c in question is to be Scotch. Thus "the author of *Waverley* was Scotch" is:

> "There is a term c such that (1) 'x wrote *Waverley*' is always equivalent to 'x is c,' (2) c is Scotch."

And generally: "the term satisfying ϕx satisfies ψx" is defined as meaning:

> "There is a term c such that (1) ϕx is always equivalent to 'x is c,' (2) ψc is true."

This is the definition of propositions in which descriptions occur.

It is possible to have much knowledge concerning a term described, *i.e.* to know many propositions concerning "the so-and-so," without actually knowing what the so-and-so is, *i.e.* without knowing any proposition of the form "x is the so-and-so," where "x" is a name. In a detective story propositions about "the man who did the deed" are accumulated, in the hope that ultimately they will suffice to demonstrate that it was A who did the deed. We may even go so far as to say that, in all such knowledge as can be expressed in words—with the exception of "this" and "that" and a few other words of which the meaning varies on different occasions—no names, in the strict sense, occur, but what seem like names are really

descriptions. We may inquire significantly whether Homer existed, which we could not do if "Homer" were a name. The proposition "the so-and-so exists" is significant, whether true or false; but if *a* is the so-and-so (where "*a*" is a name), the words
179 "*a* exists" are meaningless. It is only of descriptions —definite
or indefinite—that existence can be significantly asserted; for, if "*a*" is a name, it *must* name something: what does not name anything is not a name, and therefore, if intended to be a name, is a symbol devoid of meaning, whereas a description, like "the present King of France," does not become incapable of occurring significantly merely on the ground that it describes nothing, the reason being that it is a *complex* symbol, of which the meaning is derived from that of its constituent symbols. And so, when we ask whether Homer existed, we are using the word "Homer" as an abbreviated description: we may replace it by (say) "the author of the *Iliad* and the *Odyssey*." The same considerations apply to almost all uses of what look like proper names.

When descriptions occur in propositions, it is necessary to distinguish what may be called "primary" and "secondary" occurrences. The abstract distinction is as follows. A description has a "primary" occurrence when the proposition in which it occurs results from substituting the description for "x" in some propositional function ϕx; a description has a "secondary" occurrence when the result of substituting the description for x in ϕx gives only *part* of the proposition concerned. An instance will make this clearer. Consider "the present King of France is bald." Here "the present King of France" has a primary occurrence, and the proposition is false. Every proposition in which a description which describes nothing has a primary occurrence is false. But now consider "the present King of France is not bald." This is ambiguous. If we are first to take "x is bald," then substitute "the present King of France" for "x," and then deny the result, the occurrence of "the present King of France" is secondary and our proposition is true; but if we are to take "x is not bald" and substitute "the present King of France" for "x," then "the present King of France" has a primary occurrence and the proposition is false. Confusion of primary and secondary occurrences is a ready source of fallacies where descriptions are concerned.

Descriptions occur in mathematics chiefly in the form of 180
descriptive functions, *i.e.* "the term having the relation R to y," or "the R of y" as we may say, on the analogy of "the father of y" and similar phrases. To say "the father of y is rich," for example, is to say that the following propositional function of c: "c is rich, and 'x begat y' is always equivalent to 'x is c,'" is "sometimes true," *i.e.* is true for at least one value of c. It obviously cannot be true for more than one value.

The theory of descriptions, briefly outlined in the present chapter, is of the utmost importance both in logic and in theory of knowledge. But for purposes of mathematics, the more philosophical parts of the theory are not essential, and have therefore been omitted in the above account, which has confined itself to the barest mathematical requisites.