CHAPTER I

THE ELIMINATION OF METAPHYSICS

THE traditional disputes of philosophers are, for the most part, as unwarranted as they are unfruitful. The surest way to end them is to establish beyond question what should be the purpose and method of a philosophical inquiry. And this is by no means so difficult a task as the history of philosophy would lead one to suppose. For if there are any questions which science leaves it to philosophy to answer, a straightforward process of elimination must lead to their discovery.

We may begin by criticizing the metaphysical thesis that philosophy affords us knowledge of a reality transcending the world of science and common sense. Later on, when we come to define metaphysics and account for its existence, we shall find that it is possible to be a metaphysician without believing in a transcendent reality; for we shall see that many metaphysical utterances are due to the commission of logical errors, rather than to a conscious desire on the part of their authors to go beyond the limits of experience. But it is convenient for us to take the case of those who believe that it is possible to have knowledge of a transcendent reality as a starting-point for our discussion. The arguments which we use to refute them will subsequently be found to apply to the whole of metaphysics.

One way of attacking a metaphysician who claimed to have knowledge of a reality which transcended the phenomenal world would be to inquire from what premises his propositions were deduced. Must he not begin, as other men do, with the evidence of his senses? And if so, what valid process of reasoning can possibly lead him to the

conception of a transcendent reality? Surely from empirical premises nothing whatsoever concerning the properties, or even the existence, of anything super-empirical can legitimately be inferred. But this objection would be met by a denial on the part of the metaphysician that his assertions were ultimately based on the evidence of his senses. He would say that he was endowed with a faculty of intellectual intuition which enabled him to know facts that could not be known through sense-experience. And even if it could be shown that he was relying on empirical premises, and that his venture into a non-empirical world was therefore logically unjustified, it would not follow that the assertions which he made concerning this non-empirical world could not be true. For the fact that a conclusion does not follow from its putative premise is not sufficient to show that it is false. Consequently one cannot overthrow a system of transcendent metaphysics merely by criticizing the way in which it comes into being. What is required is rather a criticism of the nature of the actual statements which comprise it. And this is the line of argument which we shall, in fact, pursue. For we shall maintain that no statement which refers to a 'reality' transcending the limits of all possible sense-experience can possibly have any literal significance; from which it must follow that the labours of those who have striven to describe such a reality have all been devoted to the production of nonsense.

It may be suggested that this is a proposition which has already been proved by Kant. But although Kant also condemned transcendent metaphysics, he did so on different grounds. For he said that the human understanding was so constituted that it lost itself in contradictions when it ventured out beyond the limits of possible experience and attempted to deal with things in themselves. And thus he made the impossibility of a transcendent metaphysic not,

as we do, a matter of logic, but a matter of fact. He asserted, not that our minds could not conceivably have had the power of penetrating beyond the phenomenal world, but merely that they were in fact devoid of it. And this leads the critic to ask how, if it is possible to know only what lies within the bounds of sense-experience, the author can be justified in asserting that real things do exist beyond, and how he can tell what are the boundaries beyond which the human understanding may not venture, unless he succeeds in passing them himself. As Wittgenstein says, 'in order to draw a limit to thinking, we should have to think both sides of this limit',[1] a truth to which Bradley gives a special twist in maintaining that the man who is ready to prove that metaphysics is impossible is a brother metaphysician with a rival theory of his own.[2]

Whatever force these objections may have against the Kantian doctrine, they have none whatsoever against the thesis that I am about to set forth. It cannot here be said that the author is himself overstepping the barrier he maintains to be impassable. For the fruitlessness of attempting to transcend the limits of possible sense-experience will be deduced, not from a psychological hypothesis concerning the actual constitution of the human mind, but from the rule which determines the literal significance of language. Our charge against the metaphysician is not that he attempts to employ the understanding in a field where it cannot profitably venture, but that he produces sentences which fail to conform to the conditions under which alone a sentence can be literally significant. Nor are we ourselves obliged to talk nonsense in order to show that all sentences of a certain type are necessarily devoid of literal significance. We need only formulate the criterion which enables us to test whether a sentence

1. *Tractatus Logico-Philosophicus*, Preface.
2. Bradley, *Appearance and Reality*, 2nd ed., p. 1.

expresses a genuine proposition about a matter of fact, and then point out that the sentences under consideration fail to satisfy it. And this we shall now proceed to do. We shall first of all formulate the criterion in somewhat vague terms, and then give the explanations which are necessary to render it precise.

The criterion which we use to test the genuineness of apparent statements of fact is the criterion of verifiability. We say that a sentence is factually significant to any given person, if, and only if, he knows how to verify the proposition which it purports to express – that is, if he knows what observations would lead him, under certain conditions, to accept the proposition as being true, or reject it as being false. If, on the other hand, the putative proposition is of such a character that the assumption of its truth, or falsehood, is consistent with any assumption whatsoever concerning the nature of his future experience, then, as far as he is concerned, it is, if not a tautology, a mere pseudo-proposition. The sentence expressing it may be emotionally significant to him; but it is not literally significant. And with regard to questions the procedure is the same. We inquire in every case what observations would lead us to answer the question, one way or the other; and, if none can be discovered, we must conclude that the sentence under consideration does not, as far as we are concerned, express a genuine question, however strongly its grammatical appearance may suggest that it does.

As the adoption of this procedure is an essential factor in the argument of this book, it needs to be examined in detail.

In the first place, it is necessary to draw a distinction between practical verifiability, and verifiability in principle. Plainly we all understand, in many cases believe, propositions which we have not in fact taken steps to verify. Many of these are propositions which we could

verify if we took enough trouble. But there remain a number of significant propositions, concerning matters of fact, which we could not verify even if we chose; simply because we lack the practical means of placing ourselves in the situation where the relevant observations could be made. A simple and familiar example of such a proposition is the proposition that there are mountains on the farther side of the moon.[3] No rocket has yet been invented which would enable me to go and look at the farther side of the moon, so that I am unable to decide the matter by actual observation. But I do know what observations would decide it for me, if, as is theoretically conceivable, I were once in a position to make them. And therefore I say that the proposition is verifiable in principle, if not in practice, and is accordingly significant. On the other hand, such a metaphysical pseudo-proposition as 'the Absolute enters into, but is itself incapable of, evolution and progress',[4] is not even in principle verifiable. For one cannot conceive of an observation which would enable one to determine whether the Absolute did, or did not, enter into evolution and progress. Of course it is possible that the author of such a remark is using English words in a way in which they are not commonly used by English-speaking people, and that he does, in fact, intend to assert something which could be empirically verified. But until he makes us understand how the proposition that he wishes to express would be verified, he fails to communicate anything to us. And if he admits, as I think the author of the remark in question would have admitted, that his words were not intended to express either a tautology or a proposition which was capable, at least in principle, of being

3. This example has been used by Professor Schlick to illustrate the same point.

4. A remark taken at random from *Appearance and Reality*, by F. H. Bradley.

verified, then it follows that he has made an utterance which has no literal significance even for himself.

A further distinction which we must make is the distinction between the 'strong' and the 'weak' sense of the term 'verifiable'. A proposition is said to be verifiable, in the strong sense of the term, if, and only if, its truth could be conclusively established in experience. But it is verifiable, in the weak sense, if it is possible for experience to render it probable. In which sense are we using the term when we say that a putative proposition is genuine only if it is verifiable?

It seems to me that if we adopt conclusive verifiability as our criterion of significance, as some positivists have proposed,[5] our argument will prove too much. Consider, for example, the case of general propositions of law – such propositions, namely, as 'arsenic is poisonous'; 'all men are mortal'; 'a body tends to expand when it is heated'. It is of the very nature of these propositions that their truth cannot be established with certainty by any finite series of observations. But if it is recognized that such general propositions of law are designed to cover an infinite number of cases, then it must be admitted that they cannot, even in principle, be verified conclusively. And then, if we adopt conclusive verifiability as our criterion of significance, we are logically obliged to treat these general propositions of law in the same fashion as we treat the statements of the metaphysician.

In face of this difficulty, some positivists[6] have adopted the heroic course of saying that these general propositions are indeed pieces of nonsense, albeit an essentially impor-

5. e.g. M. Schlick, 'Positivismus und Realismus', *Erkenntnis*, Vol. I, 1930. F. Waismann, 'Logische Analyse des Warscheinlichkeitsbegriffs', *Erkenntnis*, Vol. I, 1930.

6. e.g. M. Schlick, 'Die Kausalität in der gegenwärtigen Physik', *Naturwissenschaft*, Vol. 19, 1931.

tant type of nonsense. But here the introduction of the term 'important' is simply an attempt to hedge. It serves only to mark the authors' recognition that their view is somewhat too paradoxical, without in any way removing the paradox. Besides, the difficulty is not confined to the case of general propositions of law, though it is there revealed most plainly. It is hardly less obvious in the case of propositions about the remote past. For it must surely be admitted that, however strong the evidence in favour of historical statements may be, their truth can never become more than highly probable. And to maintain that they also constituted an important, or unimportant, type of nonsense would be unplausible, to say the very least. Indeed, it will be our contention that no proposition, other than a tautology, can possibly be anything more than a probable hypothesis. And if this is correct, the principle that a sentence can be factually significant only if it expresses what is conclusively verifiable is self-stultifying as a criterion of significance. For it leads to the conclusion that it is impossible to make a significant statement of fact at all.

Nor can we accept the suggestion that a sentence should be allowed to be factually significant if, and only if, it expresses something which is definitely confutable by experience.[7] Those who adopt this course assume that, although no finite series of observations is ever sufficient to establish the truth of a hypothesis beyond all possibility of doubt, there are crucial cases in which a single observation, or series of observations, can definitely confute it. But, as we shall show later on, this assumption is false. A hypothesis cannot be conclusively confuted any more than it can be conclusively verified. For when we take the occurrence of certain observations as proof that a given

7. This has been proposed by Karl Popper in his *Logik der Forschung*.

hypothesis is false, we presuppose the existence of certain conditions. And though, in any given case, it may be extremely improbable that this assumption is false, it is not logically impossible. We shall see that there need be no self-contradiction in holding that some of the relevant circumstances are other than we have taken them to be, and consequently that the hypothesis has not really broken down. And if it is not the case that any hypothesis can be definitely confuted, we cannot hold that the genuineness of a proposition depends on the possibility of its definite confutation.

Accordingly, we fall back on the weaker sense of verification. We say that the question that must be asked about any putative statement of fact is not, Would any observations make its truth or falsehood logically certain? but simply, Would any observations be relevant to the determination of its truth or falsehood? And it is only if a negative answer is given to this second question that we conclude that the statement under consideration is nonsensical.

To make our position clearer, we may formulate it in another way. Let us call a proposition which records an actual or possible observation an experiential proposition. Then we may say that it is the mark of a genuine factual proposition, not that it should be equivalent to an experiential proposition, or any finite number of experiential propositions, but simply that some experiential propositions can be deduced from it in conjunction with certain other premises without being deducible from those other premises alone.[8]

This criterion seems liberal enough. In contrast to the principle of conclusive verifiability, it clearly does not

8. This is an over-simplified statement, which is not literally correct. I give what I believe to be the correct formulation in the Introduction, p. 16

deny significance to general propositions or to propositions about the past. Let us see what kinds of assertion it rules out.

A good example of the kind of utterance that is condemned by our criterion as being not even false but nonsensical would be the assertion that the world of sense-experience was altogether unreal. It must, of course, be admitted that our senses do sometimes deceive us. We may, as the result of having certain sensations, expect certain other sensations to be obtainable which are, in fact, not obtainable. But, in all such cases, it is further sense-experience that informs us of the mistakes that arise out of sense-experience. We say that the senses sometimes deceive us, just because the expectations to which our sense-experiences give rise do not always accord with what we subsequently experience. That is, we rely on our senses to substantiate or confute the judgements which are based on our sensations. And therefore the fact that our perceptual judgements are sometimes found to be erroneous has not the slightest tendency to show that the world of sense-experience is unreal. And, indeed, it is plain that no conceivable observation, or series of observations, could have any tendency to show that the world revealed to us by sense-experience was unreal. Consequently, anyone who condemns the sensible world as a world of mere appearance, as opposed to reality, is saying something which, according to our criterion of significance, is literally nonsensical.

An example of a controversy which the application of our criterion obliges us to condemn as fictitious is provided by those who dispute concerning the number of substances that there are in the world. For it is admitted both by monists, who maintain that reality is one substance, and by pluralists, who maintain that reality is many, that it is impossible to imagine any empirical situation which

would be relevant to the solution of their dispute. But if we are told that no possible observation could give any probability either to the assertion that reality was one substance or to the assertion that it was many, then we must conclude that neither assertion is significant. We shall see later on[9] that there are genuine logical and empirical questions involved in the dispute between monists and pluralists. But the metaphysical question concerning 'substance' is ruled out by our criterion as spurious.

A similar treatment must be accorded to the controversy between realists and idealists, in its metaphysical aspect. A simple illustration, which I have made use of in a similar argument elsewhere,[10] will help to demonstrate this. Let us suppose that a picture is discovered and the suggestion made that it was painted by Goya. There is a definite procedure for dealing with such a question. The experts examine the picture to see in what way it resembles the accredited works of Goya, and to see if it bears any marks which are characteristic of a forgery; they look up contemporary records for evidence of the existence of such a picture, and so on. In the end, they may still disagree, but each one knows what empirical evidence would go to confirm or discredit his opinion. Suppose, now, that these men have studied philosophy, and some of them proceed to maintain that this picture is a set of ideas in the perceiver's mind, or in God's mind, others that it is objectively real. What possible experience could any of them have which would be relevant to the solution of this dispute one way or the other? In the ordinary sense of the term 'real', in which it is opposed to 'illusory', the reality of the picture is not in doubt. The disputants have satisfied themselves that the picture is real, in this sense, by

9. In Chapter VIII.

10. Vide 'Demonstration of the Impossibility of Metaphysics', *Mind*, 1934, p. 339.

obtaining a correlated series of sensations of sight and sensations of touch. Is there any similar process by which they could discover whether the picture was real, in the sense in which the term 'real' is opposed to 'ideal'? Clearly there is none. But, if that is so, the problem is fictitious according to our criterion. This does not mean that the realist–idealist controversy may be dismissed without further ado. For it can legitimately be regarded as a dispute concerning the analysis of existential propositions and so as involving a logical problem which, as we shall see, can be definitively solved.[11] What we have just shown is that the question at issue between idealists and realists becomes fictitious when, as is often the case, it is given a metaphysical interpretation.

There is no need for us to give further examples of the operation of our criterion of significance. For our object is merely to show that philosophy, as a genuine branch of knowledge, must be distinguished from metaphysics. We are not now concerned with the historical question how much of what has traditionally passed for philosophy is actually metaphysical. We shall, however, point out later on that the majority of the 'great philosophers' of the past were not essentially metaphysicians, and thus reassure those who would otherwise be prevented from adopting our criterion by considerations of piety.

As to the validity of the verification principle, in the form in which we have stated it, a demonstration will be given in the course of this book. For it will be shown that all propositions which have factual content are empirical hypotheses; and that the function of an empirical hypothesis is to provide a rule for the anticipation of experience.[12] And this means that every empirical hypothesis must be relevant to some actual, or possible, experience, so that a statement which is not relevant to any experience

11. Vide Chapter VIII. 12. Vide Chapter V.

is not an empirical hypothesis, and accordingly has no factual content. But this is precisely what the principle of verifiability asserts.

It should be mentioned here that the fact that the utterances of the metaphysician are nonsensical does not follow simply from the fact that they are devoid of factual content. It follows from that fact, together with the fact that they are not *a priori* propositions. And in assuming that they are not *a priori* propositions, we are once again anticipating the conclusions of a later chapter in this book.[13] For it will be shown there that *a priori* propositions, which have always been attractive to philosophers on account of their certainty, owe this certainty to the fact that they are tautologies. We may accordingly define a metaphysical sentence as a sentence which purports to express a genuine proposition, but does, in fact, express neither a tautology nor an empirical hypothesis. And as tautologies and empirical hypotheses form the entire class of significant propositions, we are justified in concluding that all metaphysical assertions are nonsensical. Our next task is to show how they come to be made.

The use of the term 'substance', to which we have already referred, provides us with a good example of the way in which metaphysics mostly comes to be written. It happens to be the case that we cannot, in our language, refer to the sensible properties of a thing without introducing a word or phrase which appears to stand for the thing itself as opposed to anything which may be said about it. And, as a result of this, those who are infected by the primitive superstition that to every name a single real entity must correspond assume that it is necessary to distinguish logically between the thing itself and any, or all, of its sensible properties. And so they employ the term 'substance' to refer to the thing itself. But from the fact

13. Chapter IV.

that we happen to employ a single word to refer to a thing, and make that word the grammatical subject of the sentences in which we refer to the sensible appearances of the thing, it does not by any means follow that the thing itself is a 'simple entity', or that it cannot be defined in terms of the totality of its appearances. It is true that in talking of 'its' appearances we appear to distinguish the thing from the appearances, but that is simply an accident of linguistic usage. Logical analysis shows that what makes these 'appearances' the 'appearances of' the same thing is not their relationship to an entity other than themselves, but their relationship to one another. The metaphysician fails to see this because he is misled by a superficial grammatical feature of his language.

A simpler and clearer instance of the way in which a consideration of grammar leads to metaphysics is the case of the metaphysical concept of Being. The origin of our temptation to raise questions about Being, which no conceivable experience would enable us to answer, lies in the fact that, in our language, sentences which express existential propositions and sentences which express attributive propositions may be of the same grammatical form. For instance, the sentences 'Martyrs exist' and 'Martyrs suffer' both consist of a noun followed by an intransitive verb, and the fact that they have grammatically the same appearance leads one to assume that they are of the same logical type. It is seen that in the proposition 'Martyrs suffer', the members of a certain species are credited with a certain attribute, and it is sometimes assumed that the same thing is true of such a proposition as 'Martyrs exist'. If this were actually the case, it would, indeed, be as legitimate to speculate about the Being of martyrs as it is to speculate about their suffering. But, as Kant pointed out,[14]

14. Vide *The Critique of Pure Reason*, 'Transcendental Dialectic', Book II, Chapter iii, section 4.

existence is not an attribute. For, when we ascribe an attribute to a thing, we covertly assert that it exists: so that if existence were itself an attribute, it would follow that all positive existential propositions were tautologies, and all negative existential propositions self-contradictory; and this is not the case.[15] So that those who raise questions about Being which are based on the assumption that existence is an attribute are guilty of following grammar beyond the boundaries of sense.

A similar mistake has been made in connexion with such propositions as 'Unicorns are fictitious'. Here again the fact that there is a superficial grammatical resemblance between the English sentences 'Dogs are faithful' and 'Unicorns are fictitious', and between the corresponding sentences in other languages, creates the assumption that they are of the same logical type. Dogs must exist in order to have the property of being faithful, and so it is held that unless unicorns in some way existed they could not have the property of being fictitious. But, as it is plainly self-contradictory to say that fictitious objects exist, the device is adopted of saying that they are real in some non-empirical sense – that they have a mode of real being which is different from the mode of being of existent things. But since there is no way of testing whether an object is real in this sense, as there is for testing whether it is real in the ordinary sense, the assertion that fictitious objects have a special non-empirical mode of real being is devoid of all literal significance. It comes to be made as a result of the assumption that being fictitious is an attribute. And this is a fallacy of the same order as the fallacy of supposing that existence is an attribute, and it can be exposed in the same way.

In general, the postulation of real non-existent entities

15. This argument is well stated by John Wisdom, *Interpretation and Analysis*, pp. 62, 63.

results from the superstition, just now referred to, that, to every word or phrase that can be the grammatical subject of a sentence, there must somewhere be a real entity corresponding. For as there is no place in the empirical world for many of these 'entities', a special non-empirical world is invoked to house them. To this error must be attributed, not only the utterances of a Heidegger, who bases his metaphysics on the assumption that 'Nothing' is a name which is used to denote something peculiarly mysterious,[16] but also the prevalence of such problems as those concerning the reality of propositions and universals whose senselessness, though less obvious, is no less complete.

These few examples afford a sufficient indication of the way in which most metaphysical assertions come to be formulated. They show how easy it is to write sentences which are literally nonsensical without seeing that they are nonsensical. And thus we see that the view that a number of the traditional 'problems of philosophy' are metaphysical, and consequently fictitious, does not involve any incredible assumptions about the psychology of philosophers.

Among those who recognize that if philosophy is to be accounted a genuine branch of knowledge it must be defined in such a way as to distinguish it from metaphysics, it is fashionable to speak of the metaphysician as a kind of misplaced poet. As his statements have no literal meaning, they are not subject to any criteria of truth or falsehood: but they may still serve to express, or arouse, emotion, and thus be subject to ethical or aesthetic standards. And it is suggested that they may have considerable value, as means of moral inspiration, or even as works of art. In

16. Vide *Was ist Metaphysik*, by Heidegger: criticized by Rudolf Carnap in his 'Überwindung der Metaphysik durch logische Analyse der Sprache', *Erkenntnis*, Vol. II, 1932.

this way, an attempt is made to compensate the metaphysician for his extrusion from philosophy.[17]

I am afraid that this compensation is hardly in accordance with his deserts. The view that the metaphysician is to be reckoned among the poets appears to rest on the assumption that both talk nonsense. But this assumption is false. In the vast majority of cases the sentences which are produced by poets do have literal meaning. The difference between the man who uses language scientifically and the man who uses it emotively is not that the one produces sentences which are incapable of arousing emotion, and the other sentences which have no sense, but that the one is primarily concerned with the expression of true propositions, the other with the creation of a work of art. Thus, if a work of science contains true and important propositions, its value as a work of science will hardly be diminished by the fact that they are inelegantly expressed. And similarly, a work of art is not necessarily the worse for the fact that all the propositions comprising it are literally false. But to say that many literary works are largely composed of falsehoods, is not to say that they are composed of pseudo-propositions. It is, in fact, very rare for a literary artist to produce sentences which have no literal meaning. And where this does occur, the sentences are carefully chosen for their rhythm and balance. If the author writes nonsense, it is because he considers it most suitable for bringing about the effects for which his writing is designed.

The metaphysician, on the other hand, does not intend to write nonsense. He lapses into it through being deceived by grammar, or through committing errors of reasoning, such as that which leads to the view that the sensible

17. For a discussion of this point, see also C. A. Mace, 'Representation and Expression', *Analysis*, Vol. I, No. 33; and 'Metaphysics and Emotive Language', *Analysis*, Vol. II, Nos. 1 and 2.

world is unreal. But it is not the mark of a poet simply to make mistakes of this sort. There are some, indeed, who would see in the fact that the metaphysician's utterances are senseless a reason against the view that they have aesthetic value. And, without going so far as this, we may safely say that it does not constitute a reason for it.

It is true, however, that although the greater part of metaphysics is merely the embodiment of humdrum errors, there remain a number of metaphysical passages which are the work of genuine mystical feeling; and they may more plausibly be held to have moral or aesthetic value. But, as far as we are concerned, the distinction between the kind of metaphysics that is produced by a philosopher who has been duped by grammar, and the kind that is produced by a mystic who is trying to express the inexpressible, is of no great importance: what is important to us is to realize that even the utterances of the metaphysician who is attempting to expound a vision are literally senseless; so that henceforth we may pursue our philosophical researches with as little regard for them as for the more inglorious kind of metaphysics which comes from a failure to understand the workings of our language.

CHAPTER 2

THE FUNCTION OF PHILOSOPHY

AMONG the superstitions from which we are freed by the abandonment of metaphysics is the view that it is the business of the philosopher to construct a deductive system. In rejecting this view we are not, of course, suggesting that the philosopher can dispense with deductive reasoning. We are simply contesting his right to posit certain first principles, and then offer them with their consequences as a complete picture of reality. To discredit this procedure, one has only to show that there can be no first principles of the kind it requires.

As it is the function of these first principles to provide a certain basis for our knowledge, it is clear that they are not to be found among the so-called laws of nature. For we shall see that the 'laws of nature', if they are not mere definitions, are simply hypotheses which may be confuted by experience. And, indeed, it has never been the practice of the system-builders in philosophy to choose inductive generalizations for their premises. Rightly regarding such generalizations as being merely probable, they subordinate them to principles which they believe to be logically certain.

This is illustrated most clearly in the system of Descartes. It is commonly said that Descartes attempted to derive all human knowledge from premises whose truth was intuitively certain: but this interpretation puts an undue stress on the element of psychology in his system. I think he realized well enough that a mere appeal to intuition was insufficient for his purpose, since men are not all equally credulous, and that what he was really trying to do was to base all our knowledge on propositions which

it would be self-contradictory to deny. He thought he had found such a proposition in '*cogito*', which must not here be understood in its ordinary sense of 'I think', but rather as meaning 'there is a thought now'. In fact he was wrong, because '*non cogito*' would be self-contradictory only if it negated itself: and this no significant proposition can do. But even if it were true that such a proposition as 'there is a thought now' was logically certain, it still would not serve Descartes's purpose. For if '*cogito*' is taken in this sense, his initial principle, '*cogito ergo sum*', is false. 'I exist' does not follow from 'there is a thought now'. The fact that a thought occurs at a given moment does not entail that any other thought has occurred at any other moment, still less that there has occurred a series of thoughts sufficient to constitute a single self. As Hume conclusively showed, no one event intrinsically points to any other. We infer the existence of events which we are not actually observing, with the help of general principles. But these principles must be obtained inductively. By mere deduction from what is immediately given we cannot advance a single step beyond. And, consequently, any attempt to base a deductive system on propositions which describe what is immediately given is bound to be a failure.

The only other course open to one who wished to deduce all our knowledge from 'first principles', without indulging in metaphysics, would be to take for his premises a set of *a priori* truths. But, as we have already mentioned, and shall later show, an *a priori* truth is a tautology. And from a set of tautologies, taken by themselves, only further tautologies can be validly deduced. But it would be absurd to put forward a system of tautologies as constituting the whole truth about the universe. And thus we may conclude that it is not possible to deduce all our knowledge from 'first principles'; so that those who hold that it is the function of philosophy to carry

out such a deduction are denying its claim to be a genuine branch of knowledge.

The belief that it is the business of the philosopher to search for first principles is bound up with the familiar conception of philosophy as the study of reality as a whole. And this conception is one which it is difficult to criticize, because it is so vague. If it is taken to imply, as it sometimes is, that the philosopher somehow projects himself outside the world, and takes a bird's-eye view of it, then it is plainly a metaphysical conception. And it is also metaphysical to assert, as some do, that 'reality as a whole' is somehow generically different from the reality which is investigated piecemeal by the special sciences. But if the assertion that philosophy studies reality as a whole is understood to imply merely that the philosopher is equally concerned with the content of every science, then we may accept it, not indeed as an adequate definition of philosophy, but as a truth about it. For we shall find, when we come to discuss the relationship of philosophy to science, that it is not, in principle, related to any one science more closely than to any other.

In saying that philosophy is concerned with each of the sciences, in a manner which we shall indicate,[1] we mean also to rule out the supposition that philosophy can be ranged alongside the existing sciences, as a special department of speculative knowledge. Those who make this supposition cherish the belief that there are some things in the world which are possible objects of speculative knowledge and yet lie beyond the scope of empirical science. But this belief is a delusion. There is no field of experience which cannot, in principle, be brought under some form of scientific law, and no type of speculative knowledge about the world which it is, in principle, beyond the power of science to give. We have already gone some way to sub-

1. Vide Chapter III and Chapter VIII.

stantiate this proposition by demolishing metaphysics; and we shall justify it to the full in the course of this book.

With this we complete the over throw of speculative philosophy. We are now in a position to see that the function of philosophy is wholly critical. In what exactly does its critical activity consist?

One way of answering this question is to say that it is the philosopher's business to test the validity of our scientific hypotheses and everyday assumptions. But this view, though very widely held, is mistaken. If a man chooses to doubt the truth of all the propositions he ordinarily believes, it is not in the power of philosophy to reassure him. The most that philosophy can do, apart from seeing whether his beliefs are self-consistent, is to show what are the criteria which are used to determine the truth of falsehood of any given proposition: and then, when the sceptic realizes that certain observations would verify his propositions, he may also realize that he could make those observations, and so consider his original beliefs to be justified. But in such a case one cannot say that it is philosophy which justifies his beliefs. Philosophy merely shows him that experience can justify them. We may look to the philosopher to show us what we accept as constituting sufficient evidence for the truth of any given empirical proposition. But whether the evidence is forthcoming or not is in every case a purely empirical question.

If anyone thinks that we are here taking too much for granted, let him refer to the chapter on 'Truth and Probability', in which we discuss how the validity of synthetic propositions is determined. He will see there that the only sort of justification that is necessary or possible for self-consistent empirical propositions is empirical verification. And this applies just as much to the laws of science as to the maxims of common sense. Indeed there is no difference in kind between them. The superiority of

the scientific hypothesis consists merely in its being more abstract, more precise, and more fruitful. And although scientific objects such as atoms and electrons seem to be fictitious in a way that chairs and tables are not, here, too, the distinction is only a distinction of degree. For both these kinds of objects are known only by their sensible manifestations and are definable in terms of them.

It is time, therefore, to abandon the supersition that natural science cannot be regarded as logically respectable until philosophers have solved the problem of induction. The problem of induction is, roughly speaking, the problem of finding a way to prove that certain empirical generalizations which are derived from past experience will hold good also in the future. There are only two ways of approaching this problem on the assumption that it is a genuine problem, and it is easy to see that neither of them can lead to its solution. One may attempt to deduce the proposition which one is required to prove either from a purely formal principle or from an empirical principle. In the former case one commits the error of supposing that from a tautology it is possible to deduce a proposition about a matter of fact; in the latter case one simply assumes what one is setting out to prove. For example, it is often said that we can justify induction by invoking the uniformity of nature, or by postulating a 'principle of limited independent variety'.[2] But, in fact, the principle of the uniformity of nature merely states, in a misleading fashion, the assumption that past experience is a reliable guide to the future; while the principle of limited independent variety presupposes it. And it is plain that any other empirical principle which was put forward as a justification of induction would beg the question in the same way. For the only grounds which one could have for believing such a principle would be inductive grounds.

2. cf. J. M. Keynes, *A Treatise on Probability*, Part III.

Thus it appears that there is no possible way of solving the problem of induction, as it is ordinarily conceived. And this means that it is a fictitious problem, since all genuine problems are at least theoretically capable of being solved: and the credit of natural science is not impaired by the fact that some philosophers continue to be puzzled by it. Actually, we shall see that the only test to which a form of scientific procedure which satisfies the necessary condition of self-consistency is subject, is the test of its success in practice. We are entitled to have faith in our procedure just so long as it does the work which it is designed to do – that is, enables us to predict future experience, and so to control our environment. Of course, the fact that a certain form of procedure has always been successful in practice affords no logical guarantee that it will continue to be so. But then it is a mistake to demand a guarantee where it is logically impossible to obtain one. This does not mean that it is irrational to expect future experience to conform to the past. For when we come to define 'rationality' we shall find that for us 'being rational' entails being guided in a particular fashion by past experience.

The task of defining rationality is precisely the sort of task that it is the business of philosophy to undertake. But in achieving this it does not justify scientific procedure. What justifies scientific procedure, to the extent to which it is capable of being justified, is the success of the predictions to which it gives rise: and this can be determined only in actual experience. By itself, the analysis of a synthetic principle tells us nothing whatsoever about its truth.

Unhappily, this fact is generally disregarded by philosophers who concern themselves with the so-called theory of knowledge. Thus it is common for writers on the subject of perception to assume that, unless one can give a

satisfactory analysis of perceptual situations, one is not entitled to believe in the existence of material things. But this is a complete mistake. What gives one the right to believe in the existence of a certain material thing is simply the fact that one has certain sensations: for, whether one realizes it or not, to say that the thing exists is equivalent to saying that such sensations are obtainable. It is the philosopher's business to give a correct definition of material things in terms of sensations. But his success of failure in this task has no bearing whatsoever on the validity of our perceptual judgements. That depends wholly on actual sense-experience.

It follows that the philosopher has no right to despise the beliefs of common sense. If he does so, he merely displays his ignorance of the true purpose of his inquiries. What he is entitled to despise is the unreflecting analysis of those beliefs, which takes the grammatical structure of the sentence as a trustworthy guide to its meaning. Thus, many of the mistakes made in connexion with the problem of perception can be accounted for by the fact, already referred to in connexion with the metaphysical notion of 'substance', that it happens to be impossible in an ordinary European language to mention a thing without appearing to distinguish it generically from its qualities and states. But from the fact that the common-sense analysis of a proposition is mistaken it by no means follows that the proposition is not true. The philosopher may be able to show us that the propositions we believe are far more complex than we suppose; but it does not follow from this that we have no right to believe them.

It should now be sufficiently clear that if the philosopher is to uphold his claim to make a special contribution to the stock of our knowledge, he must not attempt to formulate speculative truths, or to look for first principles, or to make *a priori* judgements about the validity of our

empirical beliefs. He must, in fact, confine himself to works of clarification and analysis of a sort which we shall presently describe.

In saying that the activity of philosophizing is essentially analytic, we are not, of course, maintaining that all those who are commonly called philosophers have actually been engaged in carrying out analyses. On the contrary, we have been at pains to show that a great deal of what is commonly called philosophy is metaphysical in character. What we have been in search of, in inquiring into the function of philosophy, is a definition of philosophy which should accord to some extent with the practice of those who are commonly called philosophers, and at the same time be consistent with the common assumption that philosophy is a special branch of knowledge. It is because metaphysics fails to satisfy this second condition that we distinguish it from philosophy, in spite of the fact that it is commonly referred to as philosophy. And our justification for making this distinction is that it is necessitated by our original postulate that philosophy is a special branch of knowledge, and our demonstration that metaphysics is not.

Although this procedure is logically unassailable, it will perhaps be attacked on the ground that it is inexpedient. It will be said that the 'history of philosophy' is, almost entirely, a history of metaphysics; and, consequently, that although there is no actual fallacy involved in our using the word 'philosophy' in the sense in which philosophy is incompatible with metaphysics, it is dangerously misleading. For all our care in defining the term will not prevent people from confusing the activities which we call philosophical with the metaphysical activities of those whom they have been taught to regard as philosophers. And therefore it would surely be advisable for us to abandon the term 'philosophy' altogether, as a name for a

distinctive branch of knowledge, and invent some new description for the activity which we were minded to call the activity of philosophizing.

Our answer to this is that it is not the case that the 'history of philosophy' is almost entirely a history of metaphysics. That it contains some metaphysics is undeniable. But I think it can be shown that the majority of those who are commonly supposed to have been great philosophers were primarily not metaphysicians but analysts. For example, I do not see how anyone who follows the account which we shall give of the nature of philosophical analysis and then turns to Locke's *Essay Concerning Human Understanding* can fail to conclude that it is essentially an analytic work. Locke is generally regarded as being one who, like G. E. Moore at the present time, puts forward a philosophy of common sense.[3] But he does not, any more than Moore, attempt to give an *a priori* justification of our common-sense beliefs. Rather does he appear to have seen that it was not his business as a philosopher to affirm or deny the validity of any empirical propositions, but only to analyse them. For he is content, in his own words, 'to be employed as an under-labourer in clearing the ground a little, and removing some of the rubbish that lies in the way of knowledge'; and so devotes himself to the purely analytic tasks of defining knowledge, and classifying propositions, and displaying the nature of material things. And the small portion of his work which is not philosophical, in our sense, is not given over to metaphysics, but to psychology.

Nor is it fair to regard Berkeley as a metaphysician. For he did not, in fact, deny the reality of material things, as we are still too commonly told. What he denied was the adequacy of Locke's analysis of the notion of a material

3. Vide G. E. Moore, 'A Defence of Common Sense', *Contemporary British Philosophy*, Vol. II.

thing. He maintained that to say of various 'ideas of sensation' that they belonged to a single material thing was not, as Locke thought, to say that they were related to a single unobservable underlying 'somewhat', but rather that they stood in certain relations to one another. And in this he was right. Admittedly he made the mistake of supposing that what was immediately given in sensation was necessarily mental; and the use, by him and by Locke, of the word 'idea' to denote an element in that which is sensibly given is objectionable, because it suggests this false view. Accordingly we replace the word 'idea' in this usage by the neutral word 'sense-content', which we shall use to refer to the immediate data not merely of 'outer' but also of 'introspective' sensation, and say that what Berkeley discovered was that material things must be definable in terms of sense-contents. We shall see, when we come finally to settle the conflict between idealism and realism, that his actual conception of the relationship between material things and sense-contents was not altogether accurate. It led him to some notoriously paradoxical conclusions, which a slight emendation will enable us to avoid. But the fact that he failed to give a completely correct account of the way in which material things are constituted out of sense-contents does not invalidate his contention that they are so constituted. On the contrary, we know that it must be possible to define material things in terms of sense-contents, because it is only by the occurrence of certain sense-contents that the existence of any material thing can ever be in the least degree verified. And thus we see that we have not to inquire whether a phenomenalist 'theory of perception' or some other sort of theory is correct, but only what form of phenomenalist theory is correct. For the fact that all causal and representative theories of perception treat material things as if they were unobservable entities entitles us, as Berkeley saw,

to rule them out *a priori*. The unfortunate thing is that, in spite of this, he found it necessary to postulate God as an unobservable cause of our 'ideas'; and he must be criticized also for failing to see that the argument which he uses to dispose of Locke's analysis of a material thing is fatal to his own conception of the nature of the self, a point which was effectively seized upon by Hume.

Of Hume we may say not merely that he was not in practice a metaphysician, but that he explicitly rejected metaphysics. We find the strongest evidence of this in the passage with which he concludes his *Enquiry Concerning Human Understanding*. 'If', he says, 'we take in our hand any volume; of divinity, or school metaphysics, for instance; let us ask, Does it contain any abstract reasoning concerning quantity or number? No. Does it contain any experimental reasoning concerning matter of fact and existence? No. Commit it then to the flames. For it can contain nothing but sophistry and illusion.' What is this but a rhetorical version of our own thesis that a sentence which does not express either a formally true proposition or an empirical hypothesis is devoid of literal significance? It is true that Hume does not, so far as I know, actually put forward any view concerning the nature of philosophical propositions themselves, but those of his works which are commonly accounted philosophical are, apart from certain passages which deal with questions of psychology, works of analysis. If this is not universally conceded, it is because his treatment of causation, which is the main feature of his philosophical work, is often misinterpreted. He has been accused of denying causation, whereas in fact he was concerned only with defining it. So far is he from asserting that no causal propositions are true that he is himself at pains to give rules for judging of the existence of causes and effects.[4] He realized well

4. Vide *A Treatise of Human Nature*, Book I, Part III, section 15.

enough that the question whether a given causal proposition was true or false was not one that could be settled *a priori*, and accordingly confined himself to discussing the analytic question, What is it that we are asserting when we assert that one event is causally connected with another? And in answering this question he showed, I think, conclusively, first that the relation of cause and effect was not logical in character, since any proposition asserting a casual connexion could be denied without self-contradiction, secondly that causal laws were not analytically derived from experience, since they were not deducible from any finite number of experiential propositions, and, thirdly, that it was a mistake to analyse propositions asserting causal connexions in terms of a relation of necessitation which held between particular events, since it was impossible to conceive of any observations which would have the slightest tendency to establish the existence of such a relation. He thus laid the way open for the view, which we adopt, that every assertion of a particular causal connexion involves the assertion of a causal law, and that every general proposition of the form 'C causes E' is equivalent to a proposition of the form 'whenever C, then E', where the symbol 'whenever' must be taken to refer, not to a finite number of actual instances of C, but to the infinite number of possible instances. He himself defines a cause as 'an object, followed by another, and where all the objects similar to the first are followed by objects similar to the second', or, alternatively, as 'an object followed by another, and whose appearance always conveys the thought to that other';[5] but neither of these definitions is acceptable as it stands. For, even if it is true that we should not, according to our standards of rationality, have good reason to believe that an event C was the cause of an event E unless we had observed a constant conjunction of events

5. *An Enquiry Concerning Human Understanding*, section 7.

like C with events like E, still there is no self-contradiction involved in asserting the proposition 'C is the cause of E' and at the same time denying that any events like C or like E ever have been observed; and this would be self-contradictory if the first of the definitions quoted was correct. Nor is it inconceivable, as the second definition implies, that there should be causal laws which have never yet been thought of. But although we are obliged, for these reasons, to reject Hume's actual definitions of a cause, our view of the nature of causation remains substantially the same as his. And we agree with him that there can be no other justification for inductive reasoning than its success in practice, while insisting more strongly than he did that no better justification is required. For it is his failure to make this second point clear that has given his views the air of paradox which has caused them to be so much undervalued and misunderstood.

When we consider, also, that Hobbes and Bentham were chiefly occupied in giving definitions, and that the best part of John Stuart Mill's work consists in a development of the analyses carried out by Hume, we may fairly claim that in holding that the activity of philosophizing is essentially analytic we are adopting a standpoint which has always been implicit in English empiricism. Not that the practice of philosophical analysis has been confined to members of this school. But it is with them that we have the closest historical affinity.

If I refrain from discussing these questions in detail, and make no attempt to furnish a complete list of all the 'great philosophers' whose work is predominantly analytic – a list which would certainly include Plato and Aristotle and Kant – it is because the point to which this discussion is relevant is one of minor importance in our inquiry. We have been maintaining that much of 'traditional philosophy' is genuinely philosophical, by our

standards, in order to defend ourselves against the charge that our retention of the word 'philosophy' is misleading. But even if it were the case that none of those who are commonly called philosophers had ever been engaged in what we call the activity of philosophizing, it would not follow that our definition of philosophy was erroneous, given our initial postulates. We may admit that our retention of the word 'philosophy' is causally dependent on our belief in the historical propositions set forth above. But the validity of these historical propositions has no logical bearing on the validity of our definition of philosophy, nor on the validity of the distinction between philosophy, in our sense, and metaphysics.

It is advisable to stress the point that philosophy, as we understand it, is wholly independent of metaphysics, inasmuch as the analytic method is commonly supposed by its critics to have a metaphysical basis. Being misled by the associations of the word 'analysis', they assume that philosophical analysis is an activity of dissection; that it consists in 'breaking up' objects into their constituent parts, until the whole universe is ultimately exhibited as an aggregate of 'bare particulars', united by external relations. If this were really so, the most effective way of attacking the method would be to show that its basic presupposition was nonsensical. For to say that the universe was an aggregate of bare particulars would be as senseless as to say that it was Fire or Water or Experience. It is plain that no possible observation would enable one to verify such an assertion. But, so far as I know, this line of criticism is in fact never adopted. The critics content themselves with pointing out that few, if any, of the complex objects in the world are simply the sum of their parts. They have a structure, an organic unity, which distinguishes them, as genuine wholes, from mere aggregates. But the analyst, so it is said, is obliged by his atomistic

metaphysics to regard an object consisting of parts *a*, *b*, *c*, and *d*, in a distinctive configuration as being simply $a+b+c+d$, and thus gives an entirely false account of its nature.

If we follow the Gestalt psychologists, who of all men talk most constantly about genuine wholes, in defining such a whole as one in which the properties of every part depend to some extent on its position in the whole, then we may accept it as an empirical fact that there exist genuine, or organic, wholes. And if the analytic method involved a denial of this fact, it would indeed be a faulty method. But, actually, the validity of the analytic method is not dependent on any empirical, much less any metaphysical, presupposition about the nature of things. For the philosopher, as an analyst, is not directly concerned with the physical properties of things. He is concerned only with the way in which we speak about them.

In other words, the propositions of philosophy are not factual, but linguistic in character – that is, they do not describe the behaviour of physical, or even mental, objects; they express definitions, or the formal consequences of definitions. Accordingly, we may say that philosophy is a department of logic. For we shall see that the characteristic mark of a purely logical inquiry is that it is concerned with the formal consequences of our definitions and not with questions of empirical fact.

It follows that philosophy does not in any way compete with science. The difference in type between philosophical and scientific propositions is such that they cannot conceivably contradict one another. And this makes it clear that the possibility of philosophical analysis is independent of any empirical assumptions. That it is independent of any metaphysical assumptions should be even more obvious still. For it is absurd to suppose that the provision of definitions, and the study of their formal consequences,

involves the nonsensical assertion that the world is composed of bare particulars, or any other metaphysical dogma.

What has contributed as much as anything to the prevalent misunderstanding of the nature of philosophical analysis is the fact that propositions and questions which are really linguistic are often expressed in such a way that they appear to be factual.[6] A striking instance of this is provided by the proposition that a material thing cannot be in two places at once. This looks like an empirical proposition, and is constantly invoked by those who desire to prove that it is possible for an empirical proposition to be logically certain. But a more critical inspection shows that it is not empirical at all, but linguistic. It simply records the fact that, as a result of certain verbal conventions, the proposition that two sense-contents occur in the same visual or tactual sense-field is incompatible with the proposition that they belong to the same material thing.[7] And this is indeed a necessary fact. But it has not the least tendency to show that we have certain knowledge about the empirical properties of objects. For it is necessary only because we happen to use the relevant words in a particular way. There is no logical reason why we should not so alter our definitions that the sentence 'A thing cannot be in two places at once' comes to express a self-contradiction instead of a necessary truth.

Another good example of linguistically necessary proposition which appears to be a record of empirical fact is the proposition, 'Relations are not particulars, but

6. Carnap has stressed this point. Where we speak of 'linguistic' propositions expressed in 'factual' or 'pseudo-factual' language he speaks of 'Pseudo-Objektsätze' or 'quasi-syntaktische Sätze' as being expressed in the 'Inhaltliche', as opposed to the 'Formale Redeweise'. Vide *Logische Syntax der Sprache*, Part V.

7. cf. my article 'On Particulars and Universals', *Proceedings of the Aristotelian Society*, 1933–4, pp. 54, 55.

universals.' One might suppose that this was a proposition of the same order as, 'Armenians are not Mohammedans, but Christians': but one would be mistaken. For, whereas the latter proposition is an empirical hypothesis relating to the religious practices of a certain group of people, the former is not a proposition about 'things' at all, but simply about words. It records the fact that relation-symbols belong by definition to the class of symbols for characters, and not to the class of symbols for things.

The assertion that relations are universals provokes the question, 'What is a universal?'; and this question is not, as it has traditionally been regarded, a question about the character of certain real objects, but a request for a definition of a certain term. Philosophy, as it is written, is full of questions like this, which seem to be factual but are not. Thus, to ask what is the nature of a material object is to ask for a definition of 'material object', and this, as we shall shortly see, is to ask how propositions about material objects are to be translated into propositions about sense-contents. Similarly, to ask what is a number is to ask some such question as whether it is possible to translate propositions about the natural numbers into propositions about classes.[8] And the same thing applies to all the philosophical questions of the form, 'What is an *x*?' or, 'What is the nature of *x*?' They are all requests for definitions, and, as we shall see, for definitions of a peculiar sort.

Although it is misleading to write about linguistic questions in 'factual' language, it is often convenient for the sake of brevity. And we shall not always avoid doing it ourselves. But it is important that no one should be deceived by this practice into supposing that the philosopher is engaged on an empirical or a metaphysical inquiry. We

8. cf. Rudolf Carnap, *Logische Syntax der Sprache*, Part V, 79B, and 84.

may speak loosely of him as analysing facts, or notions, or even things. But we must make it clear that these are simply ways of saying that he is concerned with the definition of the corresponding words.

CHAPTER 3

THE NATURE OF PHILOSOPHICAL ANALYSIS

FROM our assertion that philosophy provides definitions, it must not be inferred that it is the function of the philosopher to compile a dictionary, in the ordinary sense. For the definitions which philosophy is required to provide are of a different kind from those which we expect to find in dictionaries. In a dictionary we look mainly for what may be called *explicit* definitions; in philosophy, for definitions *in use*. A brief explanation should suffice to make the nature of this distinction clear.

We define a symbol *explicitly* when we put forward another symbol, or symbolic expression which is synonymous with it. And the word 'synonymous' is here used in such a way that two symbols belonging to the same language can be said to be synonymous if, and only if, the simple substitution of one symbol for the other, in any sentence in which either can significantly occur, always yields a new sentence which is equivalent to the old. And we say that two sentences of the same language are equivalent if, and only if, every sentence which is entailed by any given group of sentences in conjunction with one of them is entailed by the same group in conjunction with the other. And, in this usage of the word 'entail', a sentence s is said to entail a sentence t when the proposition expressed by t is deducible from the proposition expressed by s; while a proposition p is said to be deducible from, or to follow from, a proposition q when the denial of p contradicts the assertion of q.

The provision of these criteria enables us to see that the

vast majority of the definitions which are given in ordinary discourse are *explicit* definitions. In particular, it is worth remarking that the process of defining *per genus et differentiam*, to which Aristotelian logicians devote so much attention, always yields definitions which are explicit in the foregoing sense. Thus, when we define an oculist as an eye-doctor, what we are asserting is that, in the English language, the two symbols 'oculist' and 'eye-doctor' are synonymous. And, generally speaking, all the questions that are discussed by logicians in connexion with this mode of definition are concerned with the possible ways of finding synonyms in a given language for any given term. We shall not enter into these questions ourselves, because they are irrelevant to our present purpose, which is to expound the method of philosophy. For the philosopher, as we have already said, is primarily concerned with the provision, not of *explicit* definitions, but of definitions *in use*.[1]

We define a symbol *in use*, not by saying that it is synonymous with some other symbol, but by showing how the sentences in which it significantly occurs can be translated into equivalent sentences, which contain neither the *definiendum* itself, nor any of its synonyms. A good illustration of this process is provided by Bertrand Russell's so-called theory of definite descriptions, which is not a theory at all in the ordinary sense, but an indication of the way in which all phrases of the form 'the so-and-so' are to be defined.[2] It proclaims that every sentence which contains a symbolic expression of this form can be translated into a sentence which does not contain any such expression, but does contain a sub-sentence asserting

1. That this statement needs to be qualified is shown in the Introduction. pp. 30ff.

2. Vide *Principia Mathematica*, Introduction, Chapter iii, and *Introduction to Mathematical Philosophy*, Chapter xvi.

that one, and only one, object possesses a certain property, or else that no one object possesses a certain property. Thus, the sentence 'The round square cannot exist' is equivalent to 'No one thing can be both square and round'; and the sentence 'The author of *Waverley* was Scotch' is equivalent to 'One person, and one person only, wrote *Waverley*, and that person was Scotch.'[3] The first of these examples provides us with a typical illustration of the way in which any definite descriptive phrase which occurs as the subject of a negative existential sentence can be eliminated; and the second, with a typical illustration of the way in which any definite descriptive phrase which occurs anywhere in any other type of sentence can be eliminated. Together, therefore, they show us how to express what is expressed by any sentence which contains a definite descriptive phrase without employing any such phrase. And thus they furnish us with a definition of these phrases in use.

The effect of this definition of descriptive phrases, as of all good definitions, is to increase our understanding of certain sentences. And this is a benefit which the author of such a definition confers not only on others, but also on himself. It might be objected that he must already understand the sentences in order to be able to define the symbols which occur in them. But this initial understanding need not amount to anything more than an ability to tell, in practice, what sort of situations verify the propositions they express. Such an understanding of sentences containing definite descriptive phrases may be possessed even by those who believe that there are subsistent entities, such as the round square, or the present King of France. But the fact that they do maintain this shows that their understanding of these sentences is imperfect. For their lapse into metaphysics is the outcome of the

33. This is not quite accurate, vide Introduction, pp. 28–30.

naïve assumption that definite descriptive phrases are demonstrative symbols. And in the light of the clearer understanding which is afforded by Russell's definition, we see that this assumption is false. Nor could this end have been achieved by an explicit definition of any descriptive phrase. What was required was a translation of sentences containing such phrases which would reveal what may be called their logical complexity. In general, we may say that it is the purpose of a philosophical definition to dispel those confusions which arise from our imperfect understanding of certain types of sentence in our language, where the need cannot be met by the provision of a synonym for any symbol, either because there is no synonym, or else because the available synonyms are unclear in the same fashion as the symbol to which the confusion is due.

A complete philosophical elucidation of any language would consist, first, in enumerating the types of sentence that were significant in that language, and then in displaying the relations of equivalence that held between sentences of various types. And here it may be explained that two sentences are said to be of the same type when they can be correlated in such a way that to each symbol in one sentence there corresponds a symbol of the same type in the other; and that two symbols are said to be of the same type when it is always possible to substitute one for the other without changing a significant sentence into a piece of nonsense. Such a system of definitions in use would reveal what may be called the structure of the language in question. And thus we may regard any particular philosophical 'theory', such as Russell's 'theory of definite descriptions', as a revelation of part of the structure of a given language. In Russell's case, the language is the everyday English language; and any other language, such as French or German, which has the same structure

as English.[4] And, in this context, it is not necessary to draw a distinction between the spoken and the written language. As far as the validity of a philosophical definition is concerned, it does not matter whether we regard the symbol defined as being constituted by visible marks or by sounds.

A factor which complicates the structure of a language such as English is the prevalence of ambiguous symbols. A symbol is said to be ambiguous when it is constituted by signs which are identical in their sensible form, not only with one another, but also with signs which are elements of some other symbol. For what makes two signs elements of the same symbol is not merely an identity of form, but also an identity of usage. Thus, if we were guided merely by the form of the sign, we should assume that the 'is' which occurs in the sentence 'He is the author of that book' was the same symbol as the 'is' which occurs in the sentence 'A cat is a mammal'. But, when we come to translate the sentences, we find that the first is equivalent to 'He, and no one else, wrote that book', and the second to 'The class of mammals contains the class of cats'. And this shows that, in this instance, each 'is' is an ambiguous symbol which must not be confused with the other, nor with the ambiguous symbols of existence, and class-membership, and identity, and entailment, which are also constituted by signs of the form 'is'.

To say that a symbol is constituted by signs which are identical with one another in their sensible form, and in their significance, and that a sign is a sense-content, or a series of sense-contents, which is used to convey literal meaning, is not to say that a symbol is a collection, or system, of sense-contents. For when we speak of certain

4. This must not be taken to imply that all English-speaking people actually employ a single precise system of symbols. Vide pp. 92–4

objects, *b*, *c*, *d* ... as being elements of an object *e*, and of *e* as being constituted by *b*, *c*, *d* ... we are not saying that they form part of *e*, in the sense in which my arm is a part of my body, or a particular set of books on my shelf is part of my collection of books. What we are saying is that all the sentences in which the symbol *e* occurs can be translated into sentences which do not contain *e* itself, or any symbol which is synonymous with *e*, but do contain symbols *b*, *c*, *d*.... In such a case we say that *e* is a logical construction out of *b*, *c*, *d*.... And, in general, we may explain the nature of logical constructions by saying that the introduction of symbols which denote logical constructions is a device which enables us to state complicated propositions about the elements of these constructions in a relatively simple form.

What one must not say is that logical constructions are fictitious objects. For while it is true that the English State, for example, is a logical construction out of individual people, and that the table at which I am writing is a logical construction out of sense-contents, it is not true that either the English State or this table is fictitious, in the sense in which Hamlet or a mirage is fictitious. Indeed, the assertion that tables are logical constructions out of sense-contents is not a factual assertion at all, in the sense in which the assertion that tables were fictitious objects would be a factual assertion, albeit a false one. It is, as our explanation of the notion of a logical construction should have made clear, a linguistic assertion, to the effect that the symbol 'table' is definable in terms of certain symbols which stand for sense-contents, not explicitly, but in use. And this, as we have seen, is tantamount to saying that sentences which contain the symbol 'table', or the corresponding symbol in any language which has the same structure as English, can all be translated into sentences of the same language which do not contain that symbol,

nor any of its synonyms, but do contain certain symbols which stand for sense-contents; a fact which may be loosely expressed by saying that to say anything about a table is always to say something about sense-contents. This does not, of course, imply that to say something about a table is ever to say the same thing about the relevant sense-contents. For example, the sentence, 'I am now sitting in front of a table' can, in principle, be translated into a sentence which does not mention tables, but only sense-contents. But this does not mean that we can simply substitute a sense-content symbol for the symbol 'table' in the original sentence. If we do this, our new sentence, so far from being equivalent to the old, will be a mere piece of nonsense. To obtain a sentence which is equivalent to the sentence about the table, but refers to sense-contents instead, the whole of the original sentence has to be altered. And this, indeed, is implied by the fact that to say that tables are logical constructions out of sense-contents is to say, not that the symbol 'table' can be explicitly defined in terms of symbols which stand for sense-contents, but only that it can be so defined in use. For, as we have seen, the function of a definition in use is not to provide us with a synonym for any symbol, but to enable us to translate sentences of a certain type.

The problem of giving an actual rule for translating sentences about a material thing into sentences about sense-contents, which may be called the problem of the 'reduction' of material things to sense-contents, is the main philosophical part of the traditional problem of perception. It is true that writers on perception who set out to describe 'the nature of a material thing' believe themselves to be discussing a factual question. But, as we have already pointed out, this is a mistake. The question, 'What is the nature of a material thing?' is, like any other question of that form, a linguistic question, being a demand

for a definition. And the propositions which are set forth in answer to it are linguistic propositions, even though they may be expressed in such a way that they seem to be factual. They are propositions about the relationship of symbols, and not about the properties of the things which the symbols denote.

It is necessary to emphasize this point in connexion with the 'problem of perception', since the fact that we are unable, in our everyday language, to describe the properties of sense-contents with any great precision, for lack of the requisite symbols, makes it convenient to give the solution of this problem in factual terminology. We express the fact that to speak about material things is, for each of us, a way of speaking about sense-contents by saying that each of us 'constructs' material things out of sense-contents: and we reveal the relationship between the two sorts of symbols by showing what are the principles of this 'construction'. In other words, one answers the question, 'What is the nature of a material thing?' by indicating, in general terms, what are the relations that must hold between any two of one's sense-contents for them to be elements of the same material thing. The difficulty, which here seems to arise, of reconciling the subjectivity of sense-contents with the objectivity of material things will be dealt with in a later chapter of this book.[5]

The solution which we shall now give of this 'problem of perception' will serve as a further illustration of the method of philosophical analysis. To simplify the question, we introduce the following definitions. We say that two sense-contents directly resemble one another when there is either no difference, or only an infinitesimal difference, of quality between them; and that they resemble one another indirectly when they are linked by a series of direct resemblances, but are not themselves directly

5. Chapter 7.

resemblant, a relationship whose possibility depends on the fact that the relative product[6] of infinitesimal differences in quality is an appreciable difference in quality. And we say that two visual, or tactual, sense-contents are directly continuous when they belong to successive members of a series of actual, or possible sense-fields, and there is no difference, or only an infinitesimal difference, between them, with respect to the position of each in its own sense-field; and that they are indirectly continuous when they are related by an actual, or possible, series of such direct continuities. And here it should be explained that to say of a sense-experience, or a sense-field which is a part of a sense-experience, or a sense-content which is a part of a sense-field, that it is possible, as opposed to actual, is to say, not that it ever has occurred or will occur in fact, but that it would occur if certain specifiable conditions were fulfilled. So when it is said that a material thing is constituted by both actual and possible sense-contents, all that is being asserted is that the sentences referring to sense-contents, which are the translations of the sentences referring to any material thing, are both categorical and hypothetical. And thus the notion of a possible sense-content, or sense-experience, is as unobjectionable as the familiar notion of a hypothetical statement.

Relying on these preliminary definitions, one may assert with regard to any two of one's visual sense-contents, or with regard to any two of one's tactual sense-contents, that they are elements of the same material thing if, and only if, they are related to one another by a relation of direct, or indirect, resemblance in certain respects, and by a relation of direct, or indirect, continuity. And as each

6. 'The *relative product* of two relations R and S is the relation which holds between x and z when there is an intermediate term y such that x has the relation R to y and y has the relation S to z.' *Principia Mathematica, Introduction*, Chapter I.

of these relations is symmetrical – that is to say, a relation which cannot hold between any terms A and B without also holding between B and A – and also transitive – that is, a relation which cannot hold between a term A and another term B, and between B and another term C, without holding between A and C – it follows that the groups of visual and tactual sense-contents which are constituted by means of these relations cannot have any members in common. And this means that no visual, or tactual, sense-content can be an element of more than one material thing.

The next step in the analysis of the notion of a material thing is to show how these separate groups of visual and tactual sense-contents are correlated. And this may be effected by saying that any two of one's visual and tactual groups belong to the same material thing when every element of the visual group which is of minimal visual depth forms part of the same sense-experience as an element of the tactual group which is of minimal tactual depth. We cannot here define visual or tactual depth otherwise than ostensively. The depth of a visual or tactual sense-content is as much a sensible property of it as its length or breadth.[7] But we may describe it by saying that one visual or tactual sense-content has a greater depth than another when it is farther from the observer's body, provided that we make it clear that this is not intended to be a definition. For it would clearly vitiate any 'reduction' of material things to sense-contents if the defining sentences contained references to human bodies, which are themselves material things. We, however, are obliged to mention material things when we wish to describe certain sense-contents, because the poverty of our language is such that we have no other verbal means of explaining what their properties are.

7. See H. H. Price, *Perception*, p. 218.

As for the sense-contents of taste, or sound, or smell, which are assigned to particular material things, they may be classified by reference to their association with tactual sense-contents. Thus, we assign sense-contents of taste to the same material things as the simultaneously occurring sense-contents of touch which are experienced by the palate, or the tongue. And in assigning an auditory or olfactory sense-content to a material thing, we remark that it is a member of a possible series of temporarily continuous sounds, or smells, of uniform quality but gradually increasing intensity; the series, namely, which one would ordinarily be said to experience in the course of moving towards the place from which the sound, or the smell, came; and we assign it to the same material thing as the tactual sense-content which is experienced at the same time as the sound, or the smell, of maximum intensity in the series.

What is next required of us, who are attempting to analyse the notion of a material thing, is the provision of a rule for translating sentences which refer to the 'real' qualities of material things. Our answer is that to say of a certain quality that it is the real quality of a given material thing is to say that it characterizes those elements of the thing which are the most conveniently measured of all the elements which possess qualities of the kind in question. Thus, when I look at a coin and assert that it is really round in shape, I am not asserting that the shape of the sense-content, which is the element of the coin that I am actually observing, is round, still less that the shape of all the visual, or tactual, elements of the coin is round; what I am asserting is that roundness of shape characterizes those elements of the coin which are experienced from the point of view from which measurements of shape are most conveniently carried out. And similarly I assert that the

real colour of the paper on which I am writing is white, even though it may not always appear to be white, because whiteness of colour characterizes those visual elements of the paper which are experienced in the conditions in which the greatest discrimination of colours is possible. And, finally, we define relations of quality, or position, between material things in terms of the relations of quality, or position, which obtain between such 'priviliged' elements.

This definition, or, rather, this outline of a definition, of symbols which stand for material things is intended to have the same sort of effect as the definition of descriptive phrases which we gave as our original example of the process of philosophical analysis. It serves to increase our understanding of the sentences in which we refer to material things. In this case also, there is, of course, a sense in which we already understand such sentences. Those who use the English language have no difficulty, in practice, in identifying the situations which determine the truth or falsehood of such simple statements as 'This is a table', or 'Pennies are round'. But they may very well be unaware of the hidden logical complexity of such statements which our analysis of the notion of a material thing has just brought to light. And, as a result, they may be led to adopt some metaphysical belief, such as the belief in the existence of material substances or invisible substrata, which is a source of confusion in all their speculative thought. And the utility of the philosophical definition which dispels such confusions is not to be measured by the apparent triviality of the sentences which it translates.

It is sometimes said that the purpose of such philosophical definitions is to reveal the meaning of certain symbols, or combinations of symbols. The objection to this way of speaking is that it does not give an unequivocal description of the philosopher's practice, because it em-

ploys, in 'meaning', a highly ambiguous symbol. It is for this reason that we defined the relation of equivalence between sentences, without referring to 'meaning'. And, indeed, I doubt whether all the sentences which are equivalent, according to our definition, would ordinarily be said to have the same meaning. For I think that although a complex sign of the form 'the sentences *s* and *t* have the same meaning' is sometimes used, or taken, to express what we express by saying 'the sentences *s* and *t* are equivalent', this is not the way in which such a sign is most commonly used or interpreted. I think that if we are to use the sign 'meaning' in the way in which it is most commonly used, we must not say that two sentences have the same meaning for anyone, unless the occurrence of one always has the same effect on his thoughts and actions as the occurrence of the other. And, clearly, it is possible for two sentences to be equivalent, by our criterion, without having the same effect on anyone who employs the language. For instance, '*p* is a law of nature' is equivalent to '*p* is a general hypothesis which can always be relied on': but the associations of the symbol 'law' are such that the former sentence tends to produce a very different psychological effect from its equivalent. It gives rise to a belief in the orderliness of nature, and even in the existence of a power 'behind' that orderliness, which is not evoked by the equivalent sentence, and has, indeed, no rational warrant. Thus there are many people for whom these sentences do, in this common sense of 'meaning', have different meanings. And this, I suspect, accounts for the widespread reluctance to admit that the laws of nature are merely hypotheses, just as the failure of some philosophers to recognize that material things are reducible to sense-contents is very largely due to the fact that no sentence which refers to sense-contents ever has the same psychological effect on them as a sentence which

refers to a material thing. But, as we have seen, this is not a valid ground for denying that any two such sentences are equivalent.

Accordingly, one should avoid saying that philosophy is concerned with the meaning of symbols, because the ambiguity of 'meaning' leads the undiscerning critic to judge the result of a philosophical inquiry by a criterion which is not applicable to it, but only to an empirical inquiry concerning the psychological effect which the occurrence of certain symbols has on a certain group of people. Such empirical inquiries are, indeed, an important element in sociology and in the scientific study of a language; but they are quite distinct from the logical inquiries which constitute philosophy.

It is misleading, also, to say, as some do, that philosophy tells us how certain symbols are actually used. For this suggests that the propositions of philosophy are factual propositions concerning the behaviour of a certain group of people; and this is not the case. The philosopher who asserts that, in the English language, the sentence 'The author of *Waverley* was Scotch' is equivalent to 'One person, and one person only, wrote *Waverley*, and that person was Scotch' is not asserting that all, or most, English-speaking people use these sentences interchangeably. What he is asserting is that, in virtue of certain rules of entailment, namely those which are characteristic of 'correct' English, every sentence which is entailed by 'The author of *Waverley* was Scotch', in conjunction with any given group of sentences, is entailed also by that group, in conjunction with 'One person, and one person only, wrote *Waverley*, and that person was Scotch.' That English-speaking people should employ the verbal conventions that they do is, indeed, an empirical fact. But the deduction of relations of equivalence from the rules of entailment which characterize the English, or any other,

language is a purely logical activity; and it is in this logical activity, and not in any empirical study of the linguistic habits of any group of people, that philosophical analysis consists.[8]

Thus, in specifying the language to which he intends his definitions to apply, the philosopher is simply describing the conventions from which his definitions are deduced; and the validity of the definitions depends solely on their compatibility with these conventions. In most cases, indeed, the definitions are obtained from conventions which do, in fact, correspond to the conventions which are actually observed by some group of people. And it is a necessary condition of the utility of the definitions, as a means of clarification, that this should be so. But it is a mistake to suppose that the existence of such a correspondence is ever part of what the definitions actually assert.[9]

It is to be remarked that the process of analysing a language is facilitated if it is possible to use for the classification of its forms an artificial system of symbols whose structure is known. The best-known example of such a symbolism is the so-called system of logistic which was employed by Russell and Whitehead in their *Principia Mathematica*. But it is not necessary that the language in which analysis is carried out should be different from the

8. There is a ground for saying that the philosopher is always concerned with an artificial language. For the conventions which we follow in our actual usage of words are not altogether systematic and precise.

9. Thus if I wish to refute a philosophical opponent I do not argue about people's linguistic habits. I try to prove that his definitions involve a contradiction. Suppose, for example, that he is maintaining that 'A is a free agent' is equivalent to 'A's actions are uncaused'. Then I refute him by getting him to admit that 'A is a free agent' is entailed by 'A is morally responsible for his actions' whereas 'A's actions are uncaused' entails 'A is not morally responsible for his actions'.

language analysed. If it were, we should be obliged to suppose, as Russell once suggested, 'that every language has a structure concerning which, *in the language*, nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit'.[10] This was written presumably in the belief that an attempt to refer to the structure of a language in the language itself would lead to the occurrence of logical paradoxes.[11] But Carnap, by actually carrying out such an analysis, has subsequently shown that a language can without self-contradiction be used in the analysis of itself.[12]

10. Introduction to L. Wittgenstein's *Tractatus Logico-Philosophicus*, p. 23.

11. Concerning logical paradoxes, see Russell and Whitehead, *Principia Mathematica*, Introduction, Chapter ii; F. P. Ramsey, *Foundations of Mathematics* pp. 1–63; and Lewis and Langford, *Symbolic Logic*, Chapter xiii.

12. Vide *Logische Syntax der Sprache*, Parts I and II.

CHAPTER 4

THE *A PRIORI*

THE view of philosophy which we have adopted may, I think, fairly be described as a form of empiricism. For it is characteristic of an empiricist to eschew metaphysics, on the ground that every factual proposition must refer to sense-experience. And even if the conception of philosophizing as an activity of analysis is not to be discovered in the traditional theories of empiricists, we have seen that it is implicit in their practice. At the same time, it must be made clear that, in calling ourselves empiricists, we are not avowing a belief in any of the psychological doctrines which are commonly associated with empiricism. For, even if these doctrines were valid, their validity would be independent of the validity of any philosophical thesis. It could be established only by observation, and not by the purely logical considerations upon which our empiricism rests.

Having admitted that we are empiricists, we must now deal with the objection that is commonly brought against all forms of empiricism; the objection, namely, that it is impossible on empiricist principles to account for our knowledge of necessary truths. For, as Hume conclusively showed, no general proposition whose validity is subject to the test of actual experience can ever be logically certain. No matter how often it is verified in practice, there still remains the possibility that it will be confuted on some future occasion. The fact that a law has been substantiated in $n-1$ cases affords no logical guarantee that it will be substantiated in the nth case also, no matter how large we take n to be. And this means that no general proposition referring to a matter of fact can ever be shown

to be necessarily and universally true. It can at best be a probable hypothesis. And this, we shall find, applies not only to general propositions, but to all propositions which have a factual content. They can none of them ever become logically certain. This conclusion, which we shall elaborate later on, is one which must be accepted by every consistent empiricist. It is often thought to involve him in complete scepticism; but this is not the case. For the fact that the validity of a proposition cannot be logically guaranteed in no way entails that it is irrational for us to believe it. On the contrary, what is irrational is to look for a guarantee where none can be forthcoming; to demand certainty where probability is all that is obtainable. We have already remarked upon this, in referring to the work of Hume. And we shall make the point clearer when we come to treat of probability, in explaining the use which we make of empirical propositions. We shall discover that there is nothing perverse or paradoxical about the view that all the 'truths' of science and common sense are hypotheses; and consequently that the fact that it involves this view constitutes no objection to the empiricist thesis.

Where the empiricist does encounter difficulty is in connexion with the truths of formal logic and mathematics. For whereas a scientific generalization is readily admitted to be fallible, the truths of mathematics and logic appear to everyone to be necessary and certain. But if empiricism is correct no proposition which has a factual content can be necessary or certain. Accordingly the empiricist must deal with the truths of logic and mathematics in one of the two following ways: he must say either that they are not necessary truths, in which case he must account for the universal conviction that they are; or he must say that they have no factual content, and then he must explain how a proposition which is empty of all factual content can be true and useful and surprising.

If neither of these courses proves satisfactory, we shall be obliged to give way to rationalism. We shall be obliged to admit that there are some truths about the world which we can know independently of experience; that there are some properties which we can ascribe to all objects, even though we cannot conceivably observe that all objects have them. And we shall have to accept it as a mysterious inexplicable fact that our thought has this power to reveal to us authoritatively the nature of objects which we have never observed. Or else we must accept the Kantian explanation which, apart from the epistemological difficulties which we have already touched on, only pushes the mystery a stage further back.

It is clear that any such concession to rationalism would upset the main argument of this book. For the admission that there were some facts about the world which could be known independently of experience would be incompatible with our fundamental contention that a sentence says nothing unless it is empirically verifiable. And thus the whole force of our attack on metaphysics would be destroyed. It is vital, therefore, for us to be able to show that one or other of the empiricist accounts of the propositions of logic and mathematics is correct. If we are successful in this, we shall have destroyed the foundations of rationalism. For the fundamental tenet of rationalism is that thought is an independent source of knowledge, and is moreover a more trustworthy source of knowledge than experience; indeed some rationalists have gone so far as to say that thought is the only source of knowledge. And the ground for this view is simply that the only necessary truths about the world which are known to us are known through thought and not through experience. So that if we can show either that the truths in question are not necessary or that they are not 'truths about the world', we shall be taking away the support on which rationalism

rests. We shall be making good the empiricist contention that there are no 'truths of reason' which refer to matters of fact.

The course of maintaining that the truths of logic and mathematics are not necessary or certain was adopted by Mill. He maintained that these propositions were inductive generalizations based on an extremely large number of instances. The fact that the number of supporting instances was so very large accounted, in his view, for our believing these generalizations to be necessarily and universally true. The evidence in their favour was so strong that it seemed incredible to us that a contrary instance should ever arise. Nevertheless it was in principle possible for such generalizations to be confuted. They were highly probable, but, being inductive generalizations, they were not certain. The difference between them and the hypotheses of natural science was a difference in degree and not in kind. Experience gave us very good reason to suppose that a 'truth' of mathematics or logic was true universally; but we were not possessed of a guarantee. For these 'truths' were only empirical hypotheses which had worked particularly well in the past; and, like all empirical hypotheses, they were theoretically fallible.

I do not think that this solution of the empiricist's difficulty with regard to the propositions of logic and mathematics is acceptable. In discussing it, it is necessary to make a distinction which is perhaps already enshrined in Kant's famous dictum that, although there can be no doubt that all our knowledge begins with experience, it does not follow that it all arises out of experience.[1] When we say that the truths of logic are known independently of experience, we are not of course saying that they are innate, in the sense that we are born knowing them. It is obvious that mathematics and logic have to be learned in the same

1. *Critique of Pure Reason*, 2nd ed., Introduction, section i.

way as chemistry and history have to be learned. Nor are we denying that the first person to discover a given logical or mathematical truth was led to it by an inductive procedure. It is very probable, for example, that the principle of the syllogism was formulated not before but after the validity of syllogistic reasoning had been observed in a number of particular cases. What we are discussing, however, when we say that logical and mathematical truths are known independently of experience, is not a historical question concerning the way in which these truths were originally discovered, nor a psychological question concerning the way in which each of us comes to learn them, but an epistemological question. The contention of Mill's which we reject is that the propositions of logic and mathematics have the same status as empirical hypotheses; that their validity is determined in the same way. We maintain that they are independent of experience in the sense that they do not owe their validity to empirical verification. We may come to discover them through an inductive process; but once we have apprehended them we see that they are necessarily true, that they hold good for every conceivable instance. And this serves to distinguish them from empirical generalizations. For we know that a proposition whose validity depends upon experience cannot be seen to be necessarily and universally true.

In rejecting Mill's theory, we are obliged to be somewhat dogmatic. We can do no more than state the issue clearly and then trust that his contention will be seen to be discrepant with the relevant logical facts. The following considerations may serve to show that of the two ways of dealing with logic and mathematics which are open to the empiricist, the one which Mill adopted is not the one which is correct.

The best way to substantiate our assertion that the truths of formal logic and pure mathematics are neces-

sarily true is to examine cases in which they might seem to be confuted. It might easily happen, for example, that when I came to count what I had taken to be five pairs of objects, I found that they amounted only to nine. And if I wished to mislead people I might say that on this occasion twice five was not ten. But in that case I should not be using the complex sign '$2 \times 5 = 10$' in the way in which is it ordinarily used. I should be taking it not as the expression of a purely mathematical proposition, but as the expression of an empirical generalization, to the effect that whenever I counted what appeared to me to be five pairs of objects I discovered that they were ten in number. This generalization may very well be false. But if it proved false in a given case, one would not say that the mathematical proposition '$2 \times 5 = 10$' had been confuted. One would say that I was wrong in supposing that there were five pairs of objects to start with, or that one of the objects had been taken away while I was counting, or that two of them had coalesced, or that I had counted wrongly. One would adopt as an explanation whatever empirical hypothesis fitted in best with the accredited facts. The one explanation which would in no circumstances be adopted is that ten is not always the product of two and five.

To take another example: if what appears to be a Euclidean triangle is found by measurement not to have angles totalling 180 degrees, we do not say that we have met with an instance which invalidates the mathematical proposition that the sum of the three angles of a Euclidean triangle is 180 degrees. We say that we have measured wrongly, or, more probably, that the triangle we have been measuring is not Euclidean. And this is our procedure in every case in which a mathematical truth might appear to be confuted. We always preserve its validity by adopting some other explanation of the occurrence.

The same thing applies to the principles of formal logic. We may take an example relating to the so-called law of excluded middle, which states that a proposition must be either true or false, or, in other words, that it is impossible that a proposition and its contradictory should neither of them be true. One might suppose that a proposition of the form 'x has stopped doing y' would in certain cases constitute an exception to this law. For instance, if my friend has never yet written to me, it seems fair to say that it is neither true nor false that he has stopped writing to me. But in fact one would refuse to accept such an instance as an invalidation of the law of excluded middle. One would point out that the proposition 'My friend has stopped writing to me' is not a simple proposition, but the conjunction of the two propositions 'My friend wrote to me in the past' and 'My friend does not write to me now': and, furthermore, that the proposition 'My friend has not stopped writing to me' is not, as it appears to be, contradictory to 'My friend has stopped writing to me', but only contrary to it. For it means 'My friend wrote to me in the past, and he still writes to me'. When, therefore, we say that such a proposition as 'My friend has stopped writing to me' is sometimes neither true nor false, we are speaking inaccurately. For we seem to be saying that neither it nor its contradictory is true. Whereas what we mean, or anyhow should mean, is that neither it nor its apparent contradictory is true. And its apparent contradictory is really only its contrary. Thus we preserve the law of excluded middle by showing that the negating of a sentence does not always yield the contradictory of the proposition originally expressed.

There is no need to give further examples. Whatever instance we care to take, we shall always find that the situations in which a logical or mathematical principle might appear to be confuted are accounted for in such a

way as to leave the principle unassailed. And this indicates that Mill was wrong in supposing that a situation could arise which would overthrow a mathematical truth. The principles of logic and mathematics are true universally simply because we never allow them to be anything else. And the reason for this is that we cannot abandon them without contradicting ourselves, without sinning against the rules which govern the use of language, and so making our utterances self-stultifying. In other words, the truths of logic and mathematics are analytic propositions or tautologies. In saying this we are making what will be held to be an extremely controversial statement, and we must now proceed to make its implications clear.

The most familiar definition of an analytic proposition, or judgement, as he called it, is that given by Kant. He said[2] that an analytic judgement was one in which the predicate B belonged to the subject A as something which was covertly contained in the concept of A. He contrasted analytic with synthetic judgements, in which the predicate B lay outside the subject A, although it did stand in connexion with it. Analytic judgements, he explains, 'add nothing through the predicate to the concept of the subject, but merely break it up into those constituent concepts that have all along been thought in it, although confusedly'. Synthetic judgements, on the other hand, 'add to the concept of the subject a predicate which has not been in any wise thought in it, and which no analysis could possibly extract from it'. Kant gives 'all bodies are extended' as an example of an analytic judgement, on the ground that the required predicate can be extracted from the concept of 'body', 'in accordance with the principle of contradiction'; as an example of a synthetic judgement, he gives 'all bodies are heavy'. He refers also to '$7+5=12$' as a synthetic judgement, on the ground that the concept

2. *Critique of Pure Reason*, 2nd ed., Introduction, sections iv and v.

of twelve is by no means already thought in merely thinking the union of seven and five. And he appears to regard this as tantamount to saying that the judgement does not rest on the principle of contradiction alone. He holds, also, that through analytic judgements our knowledge is not extended as it is through synthetic judgements. For in analytic judgements 'the concept which I already have is merely set forth and made intelligible to me'.

I think that this is a fair summary of Kant's account of the distinction between analytic and synthetic propositions, but I do not think that it succeeds in making the distinction clear. For even if we pass over the difficulties which arise out of the use of the vague term 'concept', and the unwarranted assumption that every judgement, as well as every German or English sentence, can be said to have a subject and a predicate, there remains still this crucial defect: Kant does not give one straightforward criterion for distinguishing between analytic and synthetic propositions; he gives two distinct criteria, which are by no means equivalent. Thus his ground for holding that the proposition '7+5=12' is synthetic is, as we have seen, that the subjective intension of '7+5' does not comprise the subjective intension of '12'; whereas his ground for holding that 'all bodies are extended' is an analytic proposition is that it rests on the principle of contradiction alone. That is, he employs a psychological criterion in the first of these examples, and a logical criterion in the second, and takes their equivalence for granted. But, in fact, a proposition which is synthetic according to the former criterion may very well be analytic according to the latter. For, as we have already pointed out, it is possible for symbols to be synonymous without having the same intensional meaning for anyone: and accordingly from the fact that one can think of the sum of seven and five without necessarily thinking of twelve, it by no means

follows that the proposition '7+5=12' can be denied without self-contradiction. From the rest of his argument, it is clear that it is this logical proposition, and not any psychological proposition, that Kant is really anxious to establish. His use of the psychological criterion leads him to think that he has established it, when he has not.

I think that we can preserve the logical import of Kant's distinction between analytic and synthetic propositions, while avoiding the confusions which mar his actual account of it, if we say that a proposition is analytic when its validity depends solely on the definitions of the symbols it contains, and synthetic when its validity is determined by the facts of experience. Thus, the proposition 'There are ants which have established a system of slavery' is a synthetic proposition. For we cannot tell whether it is true or false merely by considering the definitions of the symbols which constitute it. We have to resort to actual observation of the behaviour of ants. On the other hand, the proposition 'Either some ants are parasitic or none are' is an analytic proposition. For one need not resort to observation to discover that there either are or are not ants which are parasitic. If one knows what is the function of the words 'either', 'or', and 'not', then one can see that any proposition of the form 'Either *p* is true or *p* is not true' is valid, independently of experience. Accordingly, all such propositions are analytic.

It is to be noticed that the proposition 'Either some ants are parasitic or none are' provides no information whatsoever about the behaviour of ants, or, indeed, about any matter of fact. And this applies to all analytic propositions. They none of them provide any information about any matter of fact. In other words, they are entirely devoid of factual content. And it is for this reason that no experience can confute them.

When we say that analytic propositions are devoid of

factual content, and consequently that they say nothing, we are not suggesting that they are senseless in the way that metaphysical utterances are senseless. For, although they give us no information about any empirical situation, they do enlighten us by illustrating the way in which we use certain symbols. Thus if I say, 'Nothing can be coloured in different ways at the same time with respect to the same part of itself', I am not saying anything about the properties of any actual thing; but I am not talking nonsense. I am expressing an analytic proposition, which records our determination to call a colour expanse which differs in quality from a neighbouring colour expanse a different part of a given thing. In other words, I am simply calling attention to the implications of a certain linguistic usage. Similarly, in saying that if all Bretons are Frenchmen, and all Frenchmen Europeans, then all Bretons are Europeans, I am not describing any matter of fact. But I am showing that in the statement that all Bretons are Frenchmen, and all Frenchmen Europeans, the further statement that all Bretons are Europeans is implicitly contained. And I am thereby indicating the convention which governs our usage of the words 'if' and 'all'.

We see, then, that there is a sense in which analytic propositions do give us new knowledge. They call attention to linguistic usages, of which we might otherwise not be conscious, and they reveal unsuspected implications in our assertions and beliefs. But we can see also that there is a sense in which they may be said to add nothing to our knowledge. For they tell us only what we may be said to know already. Thus, if I know that the existence of May Queens is a relic of tree-worship, and I discover that May Queens still exist in England, I can employ the tautology 'If p implies q, and p is true, q is true' to show that there still exists a relic of tree-worship in England. But in saying that there are still May Queens in England, and that

the existence of May Queens is a relic of tree-worship, I have already asserted the existence in England of a relic of tree-worship. The use of the tautology does, indeed, enable me to make this concealed assertion explicit. But it does not provide me with any new knowledge, in the sense in which empirical evidence that the election of May Queens had been forbidden by law would provide me with new knowledge. If one had to set forth all the information one possessed, with regard to matters of fact, one would not write down any analytic propositions. But one would make use of analytic propositions in compiling one's encyclopedia, and would thus come to include propositions which one would otherwise have overlooked. And, besides enabling one to make one's list of information complete, the formulation of analytic propositions would enable one to make sure that the synthetic propositions of which the list was composed formed a self-consistent system. By showing which ways of combining propositions resulted in contradictions, they would prevent one from including incompatible propositions and so making the list self-stultifying. But in so far as we had actually used such words as 'all' and 'or' and 'not' without falling into self-contradiction, we might be said already to know what was revealed in the formulation of analytic propositions illustrating the rules which govern our usage of these logical particles. So that here again we are justified in saying that analytic propositions do not increase our knowledge.

The analytic character of the truths of formal logic was obscured in the traditional logic through its being insufficiently formalized. For in speaking always of judgements, instead of propositions, and introducing irrelevant psychological questions, the traditional logic gave the impression of being concerned in some specially intimate way with the workings of thought. What it was actually concerned with was the formal relationship of classes, as is

shown by the fact that all its principles of inference are subsumed in the Boolean class-calculus, which is subsumed in its turn in the propositional calculus of Russell and Whitehead.[3] Their system, expounded in *Principia Mathematica*, makes it clear that formal logic is not concerned with the properties of men's minds, much less with the properties of material objects, but simply with the possibility of combining propositions by means of logical particles into analytic propositions, and with studying the formal relationship of these analytic propositions, in virtue of which one is deducible from another. Their procedure is to exhibit the propositions of formal logic as a deductive system, based on five primitive propositions, subsequently reduced in number to one. Hereby the distinction between logical truths and principles of inference, which was maintained in the Aristotelian logic, very properly disappears. Every principle of inference is put forward as a logical truth and every logical truth can serve as a principle of inference. The three Aristotelian 'laws of thought', the law of identity, the law of excluded middle, and the law of non-contradiction, are incorporated in the system, but they are not considered more important than the other analytic propositions. They are not reckoned among the premises of the system. And the system of Russell and Whitehead itself is probably only one among many possible logics, each of which is composed of tautologies as interesting to the logician as the arbitrarily selected Aristotelian 'laws of thought'.[4]

A point which is not sufficiently brought out by Russell, if indeed it is recognized by him at all, is that every

3. Vide Karl Menger, 'Die Neue Logik', *Krise und Neuaufbau in den Exakten Wissenschaften*, pp. 94–6; and Lewis and Langford, *Symbolic Logic*, Chapter v.

4. Vide Lewis and Langford, *Symbolic Logic*, Chapter vii, for an elaboration of this point.

logical proposition is valid in its own right. Its validity does not depend on its being incorporated in a *system*, and deduced from certain propositions which are taken as self-evident. The construction of systems of logic is useful as a means of discovering and certifying analytic propositions, but it is not in principle essential even for this purpose. For it is possible to conceive of a symbolism in which every analytic proposition could be seen to be analytic in virtue of its form alone.

The fact that the validity of an analytic proposition in no way depends on its being deducible from other analytic propositions is our justification for disregarding the question whether the propositions of mathematics are reducible to propositions of formal logic, in the way that Russell supposed.[5] For even if it is the case that the definition of a cardinal number as a class of classes similar to a given class is circular, and it is not possible to reduce mathematical notions to purely logical notions, it will still remain true that the propositions of mathematics are analytic propositions. They will form a special class of analytic propositions, containing special terms, but they will be none the less analytic for that. For the criterion of an analytic proposition is that its validity should follow simply from the definition of the terms contained in it, and this condition is fulfilled by the propositions of pure mathematics.

The mathematical propositions which one might most pardonably suppose to be synthetic are the propositions of geometry. For it is natural for us to think, as Kant thought, that geometry is the study of the properties of physical space, and consequently that its propositions have factual content. And if we believe this, and also recognize that the truths of geometry are necessary and certain, then we may be inclined to accept Kant's hypothesis that

5. Vide *Introduction to Mathematical Philosophy*, Chapter ii.

space is the form of intuition of our outer sense, a form imposed by us on the matter of sensation, as the only possible explanation of our *a priori* knowledge of these synthetic propositions. But while the view that pure geometry is concerned with physical space was plausible enough in Kant's day, when the geometry of Euclid was the only geometry known, the subsequent invention of non-Euclidean geometries has shown it to be mistaken. We see now that the axioms of a geometry are simply definitions, and that the theorems of a geometry are simply the logical consequences of these definitions.[6] A geometry is not in itself about physical space; in itself it cannot be said to be 'about' anything. But we can use a geometry to reason about physical space. That is to say, once we have given the axioms a physical interpretation, we can proceed to apply the theorems to the objects which satisfy the axioms. Whether a geometry can be applied to the actual physical world or not, is an empirical question which falls outside the scope of the geometry itself. There is no sense, therefore, in asking which of the various geometries known to us are false and which are true. In so far as they are all free from contradiction, they are all true. What one can ask is which of them is the most useful on any given occasion, which of them can be applied most easily and most fruitfully to an actual empirical situation. But the proposition which states that a certain application of a geometry is possible is not itself a proposition of that geometry. All that the geometry itself tells us is that if anything can be brought under the definitions, it will also satisfy the theorems. It is therefore a purely logical system, and its propositions are purely analytic propositions.

It might be objected that the use made of diagrams in geometrical treatises shows that geometrical reasoning

6. cf. H. Poincaré, *La Science et l' Hypothèse*, Part II, Chapter iii.

is not purely abstract and logical, but depends on our intuition of the properties of figures. In fact, however, the use of diagrams is not essential to completely rigorous geometry. The diagrams are introduced as an aid to our reason. They provide us with a particular application of the geometry, and so assist us to perceive the more general truth that the axioms of the geometry involve certain consequences. But the fact that most of us need the help of an example to make us aware of those consequences does not show that the relation between them and the axioms is not a purely logical relation. It shows merely that our intellects are unequal to the task of carrying out very abstract processes of reasoning without the assistance of intuition. In other words, it has no bearing on the nature of geometrical propositions, but is simply an empirical fact about ourselves. Moreover, the appeal to intuition, though generally of psychological value, is also a source of danger to the geometer. He is tempted to make assumptions which are accidentally true of the particular figure he is taking as an illustration, but do not follow from his axioms. It has, indeed, been shown that Euclid himself was guilty of this, and consequently that the presence of the figure is essential to some of his proofs.[7] This shows that his system is not, as he presents it, completely rigorous, although of course it can be made so. It does not show that the presence of the figure is essential to a truly rigorous geometrical proof. To suppose that it did would be to take as a necessary feature of all geometries what is really only an incidental defect in one particular geometrical system.

We conclude, then, that the propositions of pure geometry are analytic. And this leads us to reject Kant's hypothesis that geometry deals with the form of intuition of our outer sense. For the ground for this hypothesis was

7. cf. M. Black, *The Nature of Mathematics*, p.154.

that it alone explained how the propositions of geometry could be both true *a priori* and synthetic: and we have seen that they are not synthetic. Similarly our view that the propositions of arithmetic are not synthetic but analytic leads us to reject the Kantian hypothesis[8] that arithmetic is concerned with our pure intuition of time, the form of our inner sense. And thus we are able to dismiss Kant's transcendental aesthetic without having to bring forward the epistemological difficulties which it is commonly said to involve. For the only argument which can be brought in favour of Kant's theory is that it alone explains certain 'facts'. And now we have found that the 'facts' which it purports to explain are not facts at all. For while it is true that we have *a priori* knowledge of necessary propositions, it is not true, as Kant supposed, that any of these necessary propositions are synthetic. They are without exception analytic propositions, or, in other words, tautologies.

We have already explained how it is that these analytic propositions are necessary and certain. We saw that the reason why they cannot be confuted in experience is that they do not make any assertion about the empirical world. They simply record our determination to use words in a certain fashion. We cannot deny them without infringing the conventions which are presupposed by our very denial, and so falling into self-contradiction. And this is the sole ground of their necessity. As Wittgenstein puts it, our justification for holding that the world could not conceivably disobey the laws of logic is simply that we could not say of an unlogical world how it would look.[9] And just as the validity of an analytic proposition is independent of the nature of the external world, so is it independent of

8. This hypothesis is not mentioned in the *Critique of Pure Reason* but was maintained by Kant at an earlier date.

9. *Tractatus Logico-Philosophicus*, 3.031.

the nature of our minds. It is perfectly conceivable that we should have employed different linguistic conventions from those which we actually do employ. But whatever these conventions might be, the tautologies in which we recorded them would always be necessary. For any denial of them would be self-stultifying.

We see, then, that there is nothing mysterious about the apodeictic certainty of logic and mathematics. Our knowledge that no observation can ever confute the proposition '$7+5=12$' depends simply on the fact that the symbolic expression '$7+5$' is synonymous with '12', just as our knowledge that every occulist is an eye-doctor depends on the fact that the symbol 'eye-doctor' is synonymous with 'oculist'. And the same explanation holds good for every other *a priori* truth.

What is mysterious at first sight is that these tautologies should on occasion be so surprising, that there should be in mathematics and logic the possibility of invention and discovery. As Poincaré says: 'If all the assertions which mathematics puts forward can be derived from one another by formal logic, mathematics cannot amount to anything more than an immense tautology. Logical inference can teach us nothing essentially new, and if everything is to proceed from the principle of identity, everything must be reducible to it. But can we really allow that these theorems which fill so many books serve no other purpose than to say in a roundabout fashion "A = A" ?'[10] Poincaré finds this incredible. His own theory is that the sense of invention and discovery in mathematics belongs to it in virtue of mathematical induction, the principle that what is true for the number 1, and true for $n+1$ when it is true for n,[11] is true for all numbers. And he claims that this is a

10. *La Science et l' Hypothèse*, Part I, Chapter i.

11. This was wrongly stated in previous editions as 'true for n when it is true for $n+1$'.

synthetic *a priori* principle. It is, in fact, *a priori*, but it is not synthetic. It is a defining principle of the natural numbers, serving to distinguish them from such numbers as the infinite cardinal numbers, to which it cannot be applied.[12] Moreover, we must remember that discoveries can be made, not only in arithmetic, but also in geometry and formal logic, where no use is made of mathematical induction. So that even if Poincaré were right about mathematical induction, he would not have provided a satisfactory explanation of the paradox that a mere body of tautologies can be so interesting and so surprising.

The true explanation is very simple. The power of logic and mathematics to surprise us depends, like their usefulness, on the limitations of our reason. A being whose intellect was infinitely powerful would take no interest in logic and mathematics.[13] For he would be able to see at a glance everything that his definitions implied, and, accordingly, could never learn anything from logical inference which he was not fully conscious of already. But our intellects are not of this order. It is only a minute proportion of the consequences of our definitions that we are able to detect at a glance. Even so simple a tautology as '$91 \times 79 = 7189$' is beyond the scope of our immediate apprehension. To assure ourselves that '7189' is synonymous with '91×79' we have to resort to calculation, which is simply a process of tautological transformation – that is, a process by which we change the form of expressions without altering their significance. The multiplication tables are rules for carrying out this process in arithmetic, just as the laws of logic are rules for the tautological trans-

12. cf. B. Russell's *Introduction to Mathematical Philosophy*, Chapter iii, p. 27.

13. cf. Hans Hahn, 'Logik, Mathematik und Naturerkennen', *Einheitswissenschaft*, Heft II, p. 18. 'Ein allwissendes Wesen braucht keine Logik und keine Mathematik.'

formation of sentences expressed in logical symbolism or in ordinary language. As the process of calculation is carried out more or less mechanically, it is easy for us to make a slip and so unwittingly contradict ourselves. And this accounts for the existence of logical and mathematical 'falsehoods', which otherwise might appear paradoxical. Clearly the risk of error in logical reasoning is proportionate to the length and the complexity of the process of calculation. And in the same way, the more complex an analytic proposition is, the more chance it has of interesting and surprising us.

It is easy to see that the danger of error in logical reasoning can be minimized by the introduction of symbolic devices, which enable us to express highly complex tautologies in a conveniently simple form. And this gives us an opportunity for the exercise of invention in the pursuit of logical inquiries. For a well-chosen definition will call our attention to analytic truths, which would otherwise have escaped us. And the framing of definitions which are useful and fruitful may well be regarded as a creative act.

Having thus shown that there is no inexplicable paradox involved in the view that the truths of logic and mathematics are all of them analytic, we may safely adopt it as the only satisfactory explanation of their *a priori* necessity. And in adopting it we vindicate the empiricist claim that there can be no *a priori* knowledge of reality. For we show that the truths of pure reason, the propositions which we know to be valid independently of all experience, are so only in virtue of their lack of factual content. To say that a proposition is true *a priori* is to say that it is a tautology. And tautologies, though they may serve to guide us in our empirical search for knowledge, do not in themselves contain any information about any matter of fact.