

Our Axiom (1) Can't Be Derived from Skyrms's Six "Rules"

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The argument has three steps:

Step 1. Skyrms's six rules can be derived from Axioms (2) and (3). You will show this on your first problem set.

Step 2. Axiom (1) can't be derived from Axioms (2) and (3). To show this, it is sufficient to give an interpretation on which Axioms (2) and (3) are true but Axiom (1) is false. Consider an interpretation with a Boolean algebra composed of the propositions $\mathcal{B} = \{p, \sim p, p \vee \sim p, p \& \sim p\}$. [Convince yourself that \mathcal{B} is a Boolean algebra!]. And, with a function $\text{Pr}(\cdot)$ defined over \mathcal{B} as follows:

$$\begin{aligned}\text{Pr}(p) &= 2 \\ \text{Pr}(\sim p) &= -1 \\ \text{Pr}(p \vee \sim p) &= 1 \\ \text{Pr}(p \& \sim p) &= 0\end{aligned}$$

Axiom (2): This interpretation contains one tautology, $p \vee \sim p$, and the interpretation assigns it a Pr-value of 1. So Axiom (2) is true on this interpretation.

Axiom (3): There are four pairs of mutually exclusive propositions on this interpretation: $\{p, \sim p\}$, $\{p, p \& \sim p\}$, $\{\sim p, p \& \sim p\}$, and $\{p \vee \sim p, p \& \sim p\}$. You can convince yourself that in each case the Pr-value of the disjunction of the pair is equal to the sum of the Pr-values of the disjuncts. So Axiom 3 is true on this interpretation.

Axiom (1): It is not the case that on this interpretation the Pr-value of every proposition is non-negative. So Axiom (1) is false on this interpretation.

Step 3. Derivation is transitive. Given that Skyrms's six rules can be derived from our Axioms (2) and (3), then if Axiom (1) could be derived from the six rules, it could be derived from Axioms (2) and (3). But we just showed that Axiom (1) can't be derived from Axioms (2) and (3). Therefore Axiom (1) can't be derived from Skyrms's six rules. *QED.*

An interesting exercise: Convince yourself that Kolmogorov's axioms are all independent. This will require showing for each of the three axioms that it can't be derived from the other two.