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LETTERS TO NATURE

A proof of the impossibility of inductive probability

Karl Popper

Penn, High Wycombe, Buckinghamshire, UK

David Miller

Department of Philosophy, University of Warwick, Coventry CV4 7AL, UK

Proofs of the impossibility of induction have been falling 'dead-born from the Press' ever since the first of them (in David Hume's *Treatise of Human Nature*) appeared in 1739. One of us (K.P.) has been producing them for more than 50 years. The present proof strikes us both as pretty.

Let h be a hypothesis, and e (possible) evidence in favour of it. In a simple case e is deducible from h in the presence of background knowledge b (b includes the initial conditions needed to derive the predictions e from h, and it may for the time being be regarded as unproblematic). Thus we have

$$p(e, hb) = 1 \qquad (1)$$

which can be read: the probability of e, given h, in the presence of b, equals 1. We also have

$$p(he, b) = p(h, b) \qquad (2)$$

since e is logically contained in h (in the presence of b). On the other hand we have the product law,

$$p(he, b) = p(h, eb)p(e, b) \qquad (3)$$

which means that, provided

$$0 < p(e, b) < 1 \qquad (4)$$

we obtain from equations (2)–(4)

$$p(h, eb) > p(h, b) \qquad (5)$$

provided $p(h, b) > 0$.

Now this explains, and seems to justify, the belief in induction. For if (in the presence of b) e follows from h, then the probability of h, given e, is greater than the prior probability of h, before e was given. So we may say that e supports h.

It is the support of h by e, the increase in the probability of h generated by the evidence e, that looks like a probabilistic inductive effect. We now show that this is an illusion.

Any proposition, for example, the hypothesis h, can be factorized with respect to any other proposition, for example, the empirical evidence statement e, into two factors:

$$h = (h \leftarrow e)(h \vee e) \qquad (6)$$

where $h \leftarrow e$ (read 'h if e') is the same as $e \rightarrow h$ (read 'if e then h').

This factorization of h has important properties, (i) and (ii).

(i) Any weakening of the content of either of the two factors makes the right-hand side of equation (6) weaker than the left-hand side. (A proposition x is logically weaker than a proposition y if, and only if, x follows from y but not the other way round.)

(ii) Any proposition x that can replace one of the two factors in the equation is logically at least as strong as the factor that it replaces: the factor follows deductively from x.

Thus each of the two factors is the weakest proposition strong enough in the presence of the other factor to entail the proposition h.

Given the evidence e the probability of the second factor, $h \vee e$, equals 1 (since it follows from e); moreover, given e, the probability of the first factor, $h \leftarrow e$, equals the probability of h, given e: for

$$p(h \leftarrow e, e) = p((h \leftarrow e)e, e) = p(he, e) = p(h, e) \qquad (7)$$

Thus

$$p((h \leftarrow e)(h \vee e), e) = p(h, e) = p(h \leftarrow e, e)$$
$$= p(h \leftarrow e, e)p(h \vee e, e) \qquad (8)$$

that is: given e, the factors of h are probabilistically independent: the probability of their product is equal to the product of their probabilities.

Thus we have split h into two factors, the second of which, $h \vee e$, deductively follows from the evidence e: it is the logically strongest part of h (or of the content of h) that follows from e. The first factor, $h \leftarrow e$, contains all of h that goes beyond e.

In the presence of e, the first factor, $h \leftarrow e$, is deductively equivalent to h itself. (All this, from equation (7) on, holds also if we assume the presence of the background knowledge b.)

To sum up: we have split h with respect to e into two factors. One of them follows deductively from e, and it can be, in the presence of e (and of b), ignored; the other is, in the presence of e (or of e and b), equivalent to h.

The whole question of induction can now be put as follows: Let us assume that e supports h; that is, the probability of

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h, given e, is greater than its probability without e:

$$p(h,e)>p(h)$$

or, making the background knowledge explicit,

$$p(h,eb)>p(h,b)$$

Does e, in this case, provide any support for the factor $h\leftarrow e$, which in the presence of e is alone needed to obtain h? The answer is: No. It never does. Indeed e countersupports $h\leftarrow e$ unless either $p(h,e)=1$ or $p(e)=1$ (which are possibilities of no interest).

This can be shown by:

Theorem 1: If

$$p(h,e)\neq 1\neq p(e)$$

then

$$p(h\leftarrow e,e)<p(h\leftarrow e)$$

This means: e countersupports $h\leftarrow e$.

Our theorem can be strengthened: the degree of countersupport equals $Exc(h,e)$, that is the excess of the probability of the conditional over the conditional probability.

Theorem 2: Under the same assumptions,

$$p(h\leftarrow e)-p(h\leftarrow e,e)=p(h\leftarrow e)-p(h,e)$$
$$=p(-h,e)p(-e)=Exc(h,e)>0$$

This is again unaffected if we bring in the background knowledge b, as shown by

Corollary: If

$$p(h,eb)\neq 1\neq p(e,b)$$

then

$$p(h\leftarrow e,b)-p(h\leftarrow e,eb)=p(h\leftarrow e,b)-p(h,eb)$$
$$=p(-h,eb)p(-e,b)=Exc(h,e,b)>0$$

The proof of this corollary, of course, proves also theorem 1 and theorem 2.

Proof: We transform $p(-h,eb)p(-e,b)$ so as to obtain the differences on the left-hand side of the corollary.

$$p(-h,eb)p(-e,b)$$
$$=(1-p(h,eb))(1-p(e,b))$$
$$=1-p(h,eb)-p(e,b)+p(h,eb)p(e,b)$$
$$=1-p(h,eb)-p(e,b)+p(he,b) \quad \text{(product law)}$$
$$=1-(p(e,b)-p(he,b))-p(h,eb)$$
$$=(1-p(-he,b))-p(h,eb)$$
$$=p(h\leftarrow e,b)-p(h,eb)$$

In view of equation (7) this completes the proof.

All this means: that factor that contains all of h that does not follow deductively from e is strongly countersupported by e. It is countersupported the more the greater the content of e, which may be measured by

$$ct(e,b)=1-p(e,b) \qquad (9)$$

Indeed, the countersupport increases with the content of e, whether e supports h or not.

This result is completely devastating to the inductive interpretation of the calculus of probability. All probabilistic support is purely deductive: that part of a hypothesis that is not deductively entailed by the evidence is always strongly countersupported by the evidence—the more strongly the more the evidence asserts. This is completely general; it holds for every hypothesis h; and it holds for every evidence e, whether it supports h, is independent of h, or countersupports h.

There is such a thing as probabilistic support; there might even be such a thing as inductive support (though we hardly think so). But the calculus of probability reveals that probabilistic support cannot be inductive support.

High-resolution X-ray and radio maps of the millisecond pulsar

R. H. Becker

Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, USA

D. J. Helfand

Columbia Astrophysics Laboratory, Columbia University, New York, New York 10027, USA

The detection[1] of a new pulsar in the radio source 4C21.53 with a period of 1.5 ms has raised important questions concerning the origin and evolution of galactic neutron stars. Before its discovery, we had been conducting radio and X-ray observations of 4C21.53 designed to place in context the intriguing properties of the source (extended emission, a low-frequency excess, and interstellar scintillation[2]). We present here the results of this programme including a stringent upper limit on any X-ray emission from the pulsar and high-resolution multi-frequency radio maps of the region which provide the best available pulsar position. We argue against any connection between the pulsar and the nearby extended radio emission region and comment on the constraints which the X-ray data place on the nature of this object.

The radio images of the region were obtained with the Very Large Array (VLA) during two observing runs. In July 1982, the region was observed at 20- and 6-cm wavelength in the 'B' configuration, resulting in maps at the two wavelengths with angular resolution of ~2.6 and ~0.8 arc s and fields of view of 30 and 10 arc min, respectively. In December 1982, the same region was observed in the 'D' configuration at 2-cm wavelength with angular resolution and field size of ~3 arc s and 3 arc min, respectively. The 20-cm wavelength image is shown in Fig. 1. Three sources are easily discernible: an extended source, 4C21.53W, and two unresolved objects. The position and fluxes of all three are listed in Table 1. The southern compact object is the new fast pulsar.

In our 6-cm map of the region, the compact source to the east of 4C21.53W was detected with a flux of 2 ± 0.4 mJy. The fast pulsar was not apparent in the data and we can place a 2σ upper limit to its 6 cm flux of 0.4 mJy. This requires that the pulsar spectral index of ~2.5 between 609 and 1,415 MHz (ref. 1) steepens to ~3.5 above 1,415 MHz. Such spectral breaks are seen in several pulsars, although few have quite such a steep index. The Crab pulsar has a spectral slope of just 3.5 at 1,400 MHz.

The extended source can be described as a centrally peaked, flat-spectrum source of emission, a description which is equally appropriate for either an H II region of a Crab-like supernova remnant (SNR). This ambiguity can be broken by either the measurement of hydrogen recombination lines (indicative of H II regions) or linear polarization (indicative of Crab-like SNR). The VLA observations sample all four Stokes parameters of the radio radiation, allowing a measurement of the degree of polarization. For example, PSR1937+214 has an integrated polarization at 20 cm of $10\pm2\%$. However, we have found no evidence of polarization in 4C21.53W at any of the three wavelengths observed, with upper limits at 20-, 6- and 2-cm wavelengths of 1%, 3% and 5%, respectively. Contrary to early reports, there is no evidence in the morphology of this region for a connection to the pulsar, and 21-cm absorption profiles for both the pulsar and the extended region have now been obtained[3] which show that the two must be at different distances. We conclude that 4C21.53W is an unrelated H II region whose proximity to the pulsar is important only in that it provided the

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