Chapter 7

CONCLUSION

7.1 After formality?

At the beginning of *After Virtue*, Alasdair MacIntyre asks us to imagine a future dark ages in which the natural sciences have been lost and only partially recovered. Only fragments of science remain:

> . . . a knowledge of experiments detached from any knowledge of the theoretical context which gave them significance; parts of theories unrelated either to the other bits and pieces of theory which they possess or to experiment; instruments whose use has been forgotten; half-chapters from books, single pages from articles, not always fully legible because torn and charred. (1981:1)

Nonetheless, the people who concern themselves with this lore take themselves to be doing physics, chemistry, and biology, not realizing that "... what they are doing is not natural science in any proper sense at all" (1).

> In such a culture men would use expressions such as 'neutrino', 'mass', 'specific gravity', 'atomic weight' in systematic and often interrelated ways which would resemble in lesser or greater degrees the ways in which such expressions had been used in earlier times before scientific knowledge had been so largely lost. But many of the beliefs presupposed by the use of these expressions would have been lost and there would appear to be an element of arbitrariness and even of choice in their application which would appear very surprising to us. What would appear to be rival and competing premises for which no further argument could be given would abound. (1)

In such a world, MacIntyre claims, the language of science would be "in a grave state of disorder" (2). Yet this disorder would be invisible from the perspective of the people who *use* the language. We could see how disordered the conceptual scheme is only by looking at its *history*.

MacIntyre's thought experiment is preparation for a bold proposal:

> . . . in the actual world which we inhabit the language of morality is in the same state of grave disorder as the language of natural science in the imaginary world which I described. What we possess, if this view is true, are the fragments of a conceptual scheme, parts which now lack the contexts from which their significance derived. (2)

I want to suggest that the language of logical hylomorphism is in a similar state of disorder, one that we can only see clearly by looking at its history. Philosophers still *use* this language in distinguishing the logical from the non-logical. They say that logic is "formal," that it "abstracts from (or lacks) content," that it "excludes material considerations." They use these claims to argue for and against candidate demarcations of logic and to give a significance to projects like logicism and structuralism in the philosophy of mathematics. And they construct technical criteria to make these claims more precise. Yet they lack the system of beliefs that gives the hylomorphic language its point. Hence their language slurs over important distinctions, engenders equivocation, and produces fruitless debates that founder on opposing but equally brute "intuitions" about formality or logicality.

If I am right, then in order to make progress in the philosophy of logic (especially on the demarcation issue), we must look to the *history* of logical hylomorphism, with the aim of understanding and rectifying language that has become "disordered."

7.2 How we got where we are

In chapter 4, I argued that Kant is the source of modern logical hylomorphism.[1] In support of this claim, I offered the following evidence:

- None of Kant's predecessors think of logic as distinctively formal.
- However, many of his successors recognize Kant as the source of the idea.
- The idea does not appear in Kant's own works until 1773-5, the period during which Kant first articulates what will become his critical philosophy.
- The texts show Kant defining logic by its *generality* (in my terminology, 1-formality, section 3.1), then *inferring* that it is formal (i.e., 3-formal, section 3.3).
- This inference can be underwritten by an argument that would only have become available to Kant in 1773-5.

By 1773-5, Kant had excellent theoretical reasons for thinking that a general (i.e., 1-formal) logic must also be formal in the sense of abstracting from all semantic content (i.e., 3-formal). His insistence on the formality of logic was never part of a definitional characterization of the subject; it was a substantive thesis ("Kant's Thesis") intimately bound up with his deeper philosophical commitments.

Kant's philosophy of logic had as enduring an influence on later philosophy as his transcendental idealism. By 1837, it has become so common on the continent to characterize logic as "formal" that Bolzano devotes several sections of his *Wissenschaftslehre* to debunking this way of talking. And after Sir William Hamilton's 1833 *Edinburgh Review* article, many British writers on logic use the hylomorphic terminology.[2] Its Kantian roots largely

[1] I say "modern" because there is a medieval tradition (with sources in antiquity) of distinguishing formal from material consequence. I discuss this tradition in appendix A. I do not believe that this earlier tradition has much to do with the one that starts with Kant; it had nearly died out by the eighteenth century, and Kant never refers to it in any of his discussions of the formality of logic. For more on this, see section 4.2.4, above.

[2] Before Hamilton's article, no British writer of whom I am aware characterizes logic by its "formality."

forgotten, "formality" comes to be seen more and more as an essential part of the *definition* of logic, and less and less as a substantive (and potentially contentious) property of it. As it becomes obligatory to find something to mean by the sentence "logic is formal," "formal" acquires some new meanings. Some philosophers continue to use it in the Kantian sense of 3-formality. But others—some of whom would reject Kant's claim that logic is 3-formal—use "formal" to mean schematic-formal (section 2.2), 2-formal (section 3.2), or 1-formal—or a confused blend of these. These semantic changes are made possible, in part, by the fact that in Kant, logic is formal in all of these senses, so that it is easy to read one's favored sense of formality back into Kant's use of "formal."

In chapter 5, I showed how Frege gradually but self-consciously rejects the Kantian philosophy of logic, and with it, the hylomorphic terminology. Frege sees clearly that he need not and ought not accept Kant's Thesis: that he can take logic to be "general," in Kant's sense (i.e., 1-formal), without taking it to be "formal," in Kant's sense (i.e., 3-formal). He can do this because he rejects the auxiliary premises from which Kant derives Kant's Thesis. Logic, for Frege, is a substantive science; what distinguishes it from geometry and physics is not that it "abstracts from all content" but that it is normative for thought as such (i.e., 1-formal), not just for thought about a particular domain, such as the physical.

Frege's clarity could have brought order back to the language of logical hylomorphism, but it didn't catch on. The logical positivist tradition, influenced by Wittgenstein and neo-Kantianism, takes logic to be 3-formal and appeals to linguistic convention to explain formality. Since the conventions at issue can be syntactically specified, 3-formality begins to be confused with syntactic formality (section 2.1). Meanwhile, "formal" continues to be used to mean 2-formality, 1-formality, and schematic formality. All of these notions are connected *historically*, but they are by no means equivalent. The language of logical hylomorphism is, to use MacIntyre's words, "in a grave state of disorder."

I suggest that this history can shed new light on the intractability of contemporary debates about the bounds of logic. These debates can often come down to an unsatisfying battle of "my intuitions against yours" (see section 1.4, above). Perhaps this is because the antagonists are operating with different conceptions of formality (or topic-neutrality: see section 3.5). The historical analysis I am recommending offers a way to get past the appeal to brute intuitions by *identifying* the different concepts at work and *explaining* how they all came to be thought of as explications of logical formality. Historical self-consciousness can help us avoid appealing blindly to intuitions that come from different strands of the tradition and are held together by historical causes, not (any longer) by reasons.

7.3 Applications

Let me offer a few examples of how the conceptual distinctions in chapters 2–3 and the historical analysis in chapters 4–5 can be of use in current debates about the demarcation of logic. Since this is a conclusion, I will be brief and sketchy. Some of these examples are covered in more detail elsewhere in the dissertation; the others should be taken as research proposals.

7.3.1 Topic-neutrality

Since at least Ryle 1954, it has been popular to demarcate logic and the logical notions by their "topic-neutrality" or maximal generality (see section 3.5, above). On its face, this looks like a relatively uncontentious characterization. But it is notoriously difficult to apply: disputes about what counts as logic become disputes about what is topic-neutral. From one point of view, arithmetic and set theory are paradigms of topic-neutrality: they can be applied to virtually any domain, since things of any sort can be counted and collected. From another point of view, however, they have their own special topics: sets and numbers, respectively. From yet a third point of view, there is no such thing as absolute topic-neutrality: topic-neutrality is always a matter of degree.

If we stop at our "intuitions" about topic-neutrality, we are not likely to get far. The solution is to make conceptual distinctions. In section 3.5, I argued that there are three distinct notions of topic-neutrality in play, corresponding to the three notions of formality

(1-, 2-, and 3-formality). Arithmetic and set theory are (arguably) topic-neutral in one sense (1-formal), while they fail to be topic-neutral in another (2-formal). In yet a third sense (3-formal), it is plausible to maintain that no science is absolutely topic-neutral; there is just more and less.

But beyond helping us to *make* these conceptual distinctions, history can help us diagnose the confusion between them. Once we see that our intuitions about formality or topic-neutrality are tracking three different concepts of formality which have come to be confused for *historical* reasons, we can begin to sort and critically evaluate these intuitions as we could not before. We can see, for instance, that historically logic was demarcated by its 1-formality, to which 2-formality and 3-formality were connected (if at all) only by additional philosophical premises. In particular, we can see that 3-formality was taken to characterize logic for reasons deeply bound up with Kant's transcendental idealism (see section 4.4, above). This historical perspective may give us a reason to discount certain of our intuitions about formality or topic-neutrality and to emphasize others.

7.3.2 Permutation invariance

Recently, many philosophers and logicians have undertaken to demarcate logical notions by their invariance under all permutations of the domain of objects (see section 3.2 and chapter 6, above). But the notion of formality to which this proposal answers—2-formality—is of little importance historically in the demarcation of logic. Kant did take logic to be 2-formal, but he did not distinguish it in this respect from arithmetic and algebra. And Frege did not take logic to be 2-formal at all. So the key question for the permutation invariance approach to logicality concerns motivation. Why is this an appropriate criterion for *logicality*? The equivocity of "formal" disguises the lack of motivation here. Sher can connect her proposal to the tradition by observing that many philosophers (e.g., Russell and Tarski) have demarcated logic by its formality (cf. 1991:133, 1996:683-4). But it is not clear that there is anything more than a word in common.

The permutation invariance criterion has also been motivated as a way of spelling out the generality or topic-neutrality of logic (e.g., in McCarthy 1981). But as we have just seen, there are at least three distinct notions of topic-neutrality in play. Why should *this* one be used? Proponents of the permutation invariance criterion may be able to answer this question, but they need to address it.

Finally, it is not clear how 2-formality applies to sentential and intensional operators. In chapter 6, I suggested that the the permutation invariance criterion for logical notions can be extended in a natural way to sentential and intensional operators. However, this extension requires us to distinguish "intrinsic" from "non-intrinsic" structure on basic semantic types, and here 2-formality gives us no guidance. For this purpose, I suggested, we need to invoke 1-formality. The fact that 2-formality (unsupplemented) only yields a criterion for logicality for extensional quantifiers and predicates suggests that it is not sufficiently fundamental to our understanding of logicality.

7.3.3 The debate over second-order logic

One of the most intractable debates about the bounds of logic concerns the status of second-order logic. There are two main reasons for taking second-order logic to be non-logical:

(1) Second-order logic lacks a complete proof procedure (Kneale and Kneale 1962:724, Wagner 1987, Resnik 1988, Jané 1993).

(2) Second-order logic appears to be committed to an ontology of sets (Quine 1986:66).[3]

Proponents of second-order logic can urge, on the other hand, that

(3) The semantics and proof theory of second-order quantifiers are natural extensions of the semantics and proof theory of first-order quantifiers (Boolos 1975:514, Shapiro 1991).

[3] At any rate, our best explanation of the semantics of the second-order quantifiers requires quantification over subsets of the domain; moreover, whether certain second-order sentences are logically true depends on the truth of substantive set-theoretic claims, like the Continuum Hypothesis (Shapiro 1991:104-5). Boolos 1984 has suggested that English plural quantification can be used to explain the semantics of the second-order quantifiers; if he is right, then this second objection may lapse. See Resnik 1988 for a critique of Boolos's proposal.

(4) Unlike first-order logic, second-order logic has sufficient expressive power to characterize mathematical notions like infinity and well-ordering (Boolos 1975:521, Shapiro 1991).

With the exception of the issue of ontological commitment, none of these points are controversial. To make a move in the debate, then, one must argue that the other camp's considerations are irrelevant for the demarcation of logic. But this is tricky: *all* of these considerations seem relevant.

The beginning of wisdom here, I suggest, is to see that our conflicting intuitions may be responsive to competing conceptions of logicality. Consider the question whether second-order logic is formal or topic-neutral. Consideration (2) suggests that it is not; consideration (3) suggests that it is. But perhaps both are right: second-order logic is formal or topic-neutral in some senses (1-formal, 2-formal, schematic-formal) but not in others (3-formal, syntactic-formal). The considerations pushed by both parties in the debate might *all* be relevant to the logicality of second-order logic—on different conceptions of logicality.

If this is right, then in order to make progress, we must distinguish the notions of formality in play and decide which of them ought to be connected with logicality. If all of them had equal claim, then perhaps the correct conclusion would be that there is no such thing as "logic"—or rather, that what was called "logic" turns out to be several distinct disciplines, each properly characterized by one of the (inequivalent) notions of formality. The history suggests, however, that not all of the notions of formality have equal claim to demarcate logic. In chapter 4, I argued that Kant characterizes logic as 3-formal as a *consequence* of his wider philosophical views, and that 3-formality is taken to be *definitional* of logic only later, as the Kantian philosophy of logic becomes entrenched. If this is right, then we should think twice before demanding that our logic "lack substantial content." Similarly, in chapter 2, I argued that syntactic and schematic formality are not capable of demarcating logic. If this is right, then we should be wary of appealing to them in debating the logicality of second-order logic. Finally, in chapter 5, I showed how Frege could reasonably take his Begriffsschrift to be a logic, despite its non-permutation-invariant concepts and existential commitments. This ought to make us think twice both about arguing *for* the logicality of second-order logic on the basis of the permutation invariance of the second-order quantifiers, and about arguing *against* the logicality of second-order logic on the basis of its putative existential commitments.

7.3.4 The logicality of Hume's Principle

Wright 1983 shows that the Peano postulates for arithmetic can be derived in second-order logic from "Hume's Principle"

> **(HP)** the number of Fs = the number of Gs iff there is a one-one mapping from the Fs onto the Gs,[4]

without any appeal to "extensions" (see section 1.2.1, above).[5] Wright's demonstration raises the possibility of a partial vindication of logicism. If (HP) were a principle of *logic*, then Wright's proof would show that all truths of arithmetic are logical truths. Of course, no one takes (HP) to be *logically* true. The most Wright claims is that (HP) is *analytic.* But it is instructive to ask *on what grounds* (HP) is denied to be a logical truth.

It is not sufficient to point out that (HP) contains a primitive functor, "the number of," which is not among the traditional "logical constants." For sometimes it is necessary to broaden the scope of a discipline in order to do the job assigned to it. The theory of relations was not always part of traditional logic, and Aristotle's logic did not even contain rules for the basic sentential connectives. Frege's iterable quantifiers were certainly an innovation. We should leave room for claims that the bounds of logic are wider than was previously thought, provided such claims can be justified on the basis of an antecedently acceptable characterization of logic.

Tradition aside, numbers would seem to have as strong a claim as Frege's extensions to be "logical objects." Frege's arguments that numbers are logical objects are largely

[4]Wright calls this principle ($N^=$) (see section 1.2.1, above). The name "Hume's Principle" is due to Boolos, after the citation in FA:§63.

[5]Boolos 1987 has shown that Frege's FA already contains the general lines of the derivation: "[o]nce Hume's principle is proved, *Frege makes no further use of extensions*" (191).

independent of his identification of numbers with extensions: they turn instead on the inferential role of number words (for the "object" part) and the general applicability of arithmetic (for the "logical" part) (see sections 5.2.2 and 5.2.7, above). Thus it would not have been unreasonable for Frege to have reacted to the problems with his theory of extensions by taking "the number of" to be a *primitive* logical functor.[6] Given his antecedent commitment to the logicality of arithmetic,[7] he could have taken his failure to reduce the concept of number to more basic notions as proof that his logic was incomplete and needed to be supplemented by a new primitive.[8] Heck 1997 comes close to considering this proposal: "the question arises why, upon receiving Russell's famous letter, Frege did not simply drop Axiom V, install Hume's Principle as an axiom, and claim himself to have established logicism anyway" (274). But Heck, like every other commentator, takes it for granted that (HP) would not be a "principle of logic," though "perhaps it has a similarly privileged epistemological position."[9] What is the basis for this universal assumption?

I suggest that the basis for this universal assumption is a shared commitment to

(NE) Logic alone does not imply the existence of any objects.[10]

(NE) is certainly incompatible with the logicality of (HP). And (NE) may well be true. But how is it argued for? Boolos writes:

> We firmly believe that the existence of even two objects, let alone infinitely many,

[6] The fact that Frege *doesn't* do this does not show that it would have been unreasonable for him to do so.

[7] See especially his claim in FTA that "...we have *no choice* but to acknowledge the purely logical nature of arithmetical modes of inference" (96, emphasis added)—even in advance of actually carrying out the technical reduction (FA:§90 sounds a more cautious note).

[8] Such a primitive would not be *entirely* alien to the logical tradition. *Quantity* has always been a concern of logic, and the traditional quantifiers ("all," "at least one," "none") might be regarded as special cases of numerical quantifiers ("there are exactly two," "there are more than thirty"). In fact, Boole's 1868 paper on "numerically definite propositions" has a primitive operator "Nx", interpreted as "the number of individuals contained in the class x." In a sketch of a logic of probabilities, Boole argues that "...the idea of Number is not solely confined to Arithmetic, but ... it is an element which may properly be combined with the elements of every system of language which can be employed for the purposes of general reasoning, whatsoever may be the nature of the subject" (1952:166).

[9] Although Bird 1997 argues that (HP) is "broadly logical," he appears to mean by "broadly logical" what is usually meant by "analytic."

[10] Of course, standard first-order logic does imply the existence of *one* object, inasmuch as "$(\exists x)(x{=}x)$" is a theorem. But this is a technical simplification (see Quine 1961:160-2).

> cannot be guaranteed by logic alone. After all, logical truth is just *truth no matter what things we may be talking about and no matter what our (nonlogical) words mean*. Since there might be fewer than two items that we happen to be talking about, we cannot take even $\exists x \exists y (x \neq y)$ to be valid. (1987:199)

Presumably Boolos is not making the (uncontroversial) claim that quantificational logic as we now understand it does not imply the existence of even two objects. For what is at issue is precisely whether our present understanding of quantificational logic is adequate. Instead, Boolos must be making a *conceptual* claim about logic: logical truth is, *by definition*, "truth no matter what things we may be talking about and no matter what our (nonlogical) words mean."

But if a logic with existential commitments can be ruled out on *conceptual* grounds, just by thinking about what "logic" *means*, then we are going to have trouble making Frege's Platonist logicism seem so much as coherent. Boolos immediately raises this question:

> How then, we might now think, *could* logicism ever have been thought to be a mildly plausible philosophy of mathematics? Is it not obviously demonstrably inadequate? (1987:199-200)

Note that the "inadequacy" to which Boolos is pointing here is independent of the *technical* problems that led Frege to abandon his logicism. Boolos is claiming that Frege's project can be ruled out *from the start* on the basis of a general characterization of logic. This is surely an intolerable result. In discussing logicism, we ought to use the word "logic" in such a way that it is at least intelligible how someone could have thought that arithmetic (Platonistically construed) could be reduced to logic. In particular, we should not demarcate logic by its 3-formality or 2-formality. If we think that (HP) cannot be logical because we are committed to (NE), then we should *argue for* (NE) on the basis of a conception of logic Frege could have accepted, not one that makes logicism look like a round square.

By demarcating logic by its *1-formality*, we can leave both logicism and (NE) open as conceptual possibilities. There may still be a good argument for (NE): i.e., an argument that the norms for thought as such cannot imply or presuppose the existence of any objects. If so, the argument deserves to be made explicit. The point is not obvious. The rules of chess presuppose the existence of a board and pieces; might not the norms governing thought *as such* also presuppose the existence of certain objects?[11]

7.4 The centrality of 1-formality

One thing chapters 4 and 5 show is that 1-formality is central to how logic is understood in at least one tradition—the one reaching from the neo-Leibnizians through Kant to Frege. Different philosophers in this tradition have different views on the scope of logic and different views about whether logic is 2-formal or 3-formal, but they all agree in demarcating logic by its 1-formality, and that is what allows us to see them as disagreeing about a single subject matter. Nor is this just one tradition of many. It is a particularly important one for those concerned with the demarcation of logic, since it is within this tradition that many of the projects for which the demarcation of logic is important arise (e.g., logicism, structuralism in the philosophy of mathematics: see section 1.2, above).

This suggests that 1-formality ought to have a central place in any answer to the question "what is logic." However, as I observed at the end of chapter 5, 1-formality has more or less dropped out of twentieth century discussions of the demarcation of logic. There are several reasons for this.

First, the dominant event in twentieth century philosophy of logic—Quine's refutation of conventionalism—has influenced how people see the options for demarcating logic. There is the Old View, on which logic is distinctively formal (that is, 3-formal); and there is the New View, on which there is no such thing as "Logic" as the tradition conceives it—no philosophically privileged discipline that is different in kind (not just in degree) from the empirical sciences and (perhaps) mathematics. Everything in between these two views gets flattened out. Principled demarcations of logic in general (in the sense of section 1.3.1, above) get conflated with the (failed) conventionalist approaches. And where the choice is between a principled conventionalist demarcation and a pragmatic demarcation,

[11] For a view of this kind, see Tennant 1997.

the pragmatic demarcation looks like the only sound alternative.

Second, the appeal to "thought" in 1-formality seems mired in the nineteenth century. Hasn't philosophy had its "linguistic turn"? Depending on one's predilections, talk of "thought" is apt to seem either too psychologistic (despite the fact that Kant and Frege were staunch foes of psychologism) or too unscientific to be of much use in demarcating logic.

Third, there are well-known difficulties in making sense of logic as providing "norms for thought." Harman 1984 argues that

> ...even the rule "Avoid inconsistency!" has exceptions, if it requires one not to believe things one knows to be jointly inconsistent. On discovering one has inconsistent beliefs, one might not see any easy way to modify one's beliefs so as to avoid the inconsistency, and one may not have the time or ability to figure out the best response. In that case, one should (at least sometimes) simply acquiesce in the contradiction while trying to keep it fairly isolated. (108)

For example, the set containing all of of my beliefs about Cambodia plus the belief that at least one of these beliefs is false is inconsistent, but if there is no way of deciding *which* of my beliefs about Cambodia is false, it may be reasonable to keep them all. It is not clear, then, how the claim that logic provides norms for thought should be understood. Why should we not say that there's just one norm here—the norm to think what is true—and that logic is simply a very general body of truths?

The fourth difficulty concerns the "as such" in "norms for thought *as such*." Why should we think that quantifier theory is normative for thought as such, as opposed to thought involving quantifiers, or thought about collections of discrete objects? Suppose there were a language without logical vocabulary (as Brandom 1994 supposes to be possible). In what sense would quantificational or truth-functional logic be normative for thought or reasoning in that language? The universal normative applicability of logic must be based on more than just the ubiquity of the logical constants.

Though I will not try to meet these difficulties here, I do not think they are insoluble. And provided we can solve them, we have good reason to bring 1-formality back to the center of our thinking about the demarcation of logic. Not only is 1-formality central to the tradition of thinking about logic out of which most of the philosophical projects for which the demarcation of logic matters emerged, it also offers an approach to logic that swings free of the disputed notions of analyticity and *a prioricity*.

In section 6.7, above, I sketched one way in which the thought that logic is 1-formal might shape a technical demarcation of logic. The idea was to use invariance methods to separate notions into two classes:

- notions sensitive to semantic structure required by the particular expressive power of a language (i.e., semantic structure required in order to give a compositional semantics for the language), and
- notions sensitive only to the semantic structure that must be invoked in a general account of the *use* of stand-alone sentences of the language, independent of its particular expressive vocabulary.

I suggested that notions in the second class are logical notions, since the norms governing them depend only on the proprieties for assertion and inference *as such*, and not on the particular expressive power of the language. Truth-functional logic comes to be applicable to reasoning in a language just by virtue of the classification of sentences into true and false for purposes of assessment of stand-alone assertions—a classification that will take place no matter what subject matter is being addressed and no matter what expressive power the language has. Set theory comes into play, by contrast, only if the language actually contains vocabulary sensitive to set-theoretic membership; it is normative only for thought employing the *concept* of set membership. More controversially, non-S5 modal operators come out as non-logical on this criterion, because they are sensitive to a structure on semantic values (the modal accessibility relation) that is required only because the language contains an operator that is sensitive to it—that is, only for the purposes of giving a compositional semantics. In section 6.7.5, I showed how one might use this criterion to argue for the logicality of *tense* operators.

I am not wedded to the chapter 6 approach. There may be other ways of using 1-formality to guide technical demarcation projects. For example, I think that demarcations that appeal to "inferential definitions," taking logical constants to be those expressions that can be introduced into a language by a conservative set of introduction and elimination rules (e.g., Popper 1947, Kneale 1956, Prawitz 1978, Hacking 1979, Schroeder-Heister 1984, Kremer 1988, Došen 1994), can be profitabily conceived as demarcations of logic by its 1-formality.[12] In these approaches, one starts with a set of *structural rules* governing sentences independently of their internal structures (and hence independently of the logical constants within them): rules like transitivity (if A implies B and B implies C, then A implies C) or weakening (if A implies B, then A and C together imply B). These can plausibly be taken to be "norms for thought *as such*," independent of topic or special vocabulary. Since the logical constants are introduced through introduction and elimination rules that conservatively extend this base of structural rules,[13] they can be conceived as auxiliaries for the study of inferential relations that hold independently of any particular expressions—"punctuation marks," to use Došen's term. I will not pursue this line of thought here; I offer it as another way in which the concepts developed in this dissertation might profitably be applied.

7.5 Methodological postscript

Many analytic philosophers will find the method of this dissertation somewhat unusual. Though the topic is the philosophy of logic, there are few of the usual hen scratches and numbered theorems. Indeed, much of the work is historical. Yet the aim is not primarily an understanding of the history of philosophy, but an understanding of the various concepts that shape contemporary projects in the philosophy of logic. I have suggested that there is something we can do to get beyond the deadlock of competing intuitions one often finds in contemporary debates about the proper bounds of logic: we can "go historical" and seek to

[12] This is not to say that proponents of these approaches *do* think of them this way: Popper, for one, is clearly aiming at 3-formality.

[13] That is, the new rules for a constant * do not allow one to prove anything in the *-free fragment of the language that could not have been proved without them.

understand the *sources* of our intuitions about logicality. This kind of inquiry can help us to articulate and evaluate these intuitions, and (I hope) to make progress where we could not before.

I will attempt no general defense of this method. Those who find the dissertation illuminating will need no advertisement for its method; those who do not will not be convinced by more verbiage. Nor do I intend to disparage work on the philosophy of logic in the classic analytic mold. Far from it: my aim is to provide a general picture within which we can see the significance of various technical projects. What I am urging is that even in the most technical corners of analytic philosophy, research ought to be informed by history. And not just recent history: if we are to fully understand the present situation in the philosophy of logic, we must go back at least as far as Kant.