Chapter 3

THREE NOTIONS OF LOGICAL FORMALITY

Logic, it is often said, is distinctively *formal*: it concerns itself with those relations of implication and consistency that turn only on the "forms" or "formal features" of thoughts or statements, abstracting from the "matter" or "content." This kind of talk is so common as to be nearly invisible. Asked what it means, philosophers will often appeal to one or more of the senses of formality canvassed in the previous chapter. As we have seen, however, these senses are decoys. Although they are clear and philosophically innocuous, they are not fit for the task of carving out the province of logic. Logic can be treated syntactically—but so can other disciplines. Logical laws are schematic—but so are non-logical laws. Logical validity turns on grammatical form—but only if we set up the grammar to reveal and systematize logical validity. What, then, *is* meant by "formal" when philosophers use it to delineate logic (e.g., in the passages quoted in section 1.3.3, above)?

In this chapter, I will distinguish three different notions of logical formality. All three have played a role in the literature since Kant, and all three continue to shape our intuitions about logicality. I make no claim to completeness: there may be other notions of logical formality. I am confident, however, that these are the main notions in play:

> To say that logic is **1-formal** is to say that its norms are *constitutive* of concept use *as such* (as opposed to a particular kind of concept use). 1-formal laws are the norms to which any conceptual activity—asserting, inferring, supposing, judging, and so on—must be held responsible.
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> To say that logic is **2-formal** is to say that its characteristic notions and laws are indifferent to the particular identities of different objects. 2-formal notions and laws treat each object the same (whether it is a cow, a peach, a shadow, or a number). Mathematically, 2-formality can be spelled out as invariance under all permutations of the domain of objects.
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> To say that logic is **3-formal** is to say that it abstracts entirely from the semantic content or "matter" of concepts—that it considers thought in abstraction from its relation to the world and is therefore entirely free of substantive presuppositions.

We can get at these three notions by construing "formal" as "independent of content or subject matter." What does it mean to say that logic is independent of content or subject matter? That depends on what we mean by "content" or "subject matter."

1. We might mean "particular domain of application." In that case, to say that logic abstracts from content or subject matter is to say that it is applicable without qualification, in any domain—that it is normative for thought or concept use *as such*, or 1-formal.

2. We might mean "particular object or individual." In that case, to say that logic abstracts from content or subject matter is to say that it pays no heed to distinguishing features of individuals, but treats them all the same—i.e., that it is 2-formal.

3. We might mean "semantic content." In that case, to say that logic abstracts from content or subject matter is to say that it abstracts entirely from the semantic contents of concepts, claims, and inferences—i.e., that it is 3-formal.

In the next three sections (3.1– 3.3), I flesh out these three notions of formality in more detail, with examples. Then, in section 3.4, I show that the three notions of formality are conceptually independent of each other, and in section 3.5, I show how they provide three ways of spelling out the "generality" or "topic-neutrality" of logic. Finally, in section 3.6, I consider the significance of each of the notions for logicism.

3.1 1-formality

Consider two kinds of norms for playing chess. On the one hand, there are prescriptions for playing chess well, for carrying out the Queen's Indian defense, or for playing an endgame with two rooks. These are norms that are applicable only in specific situations, or only given certain goals or interests: they are "hypothetical," in the sense of Kant's "hypothetical imperatives." On the other hand, there are the rules of the game. These norms apply to chess playing *as such*, because they are *constitutive* of chess playing. One might violate them (intentionally or inadvertently) and still count as playing chess. One might even be ignorant of some of them and still count as playing chess. But unless these norms are *binding* on one's moves, one is not playing chess, but some other game. They are "categorical" norms for chess.

We can make the same distinction among norms for *thought* (which, following Kant, I will take as a blanket term for all concept use, including judging, inferring, supposing, asserting, and even perceiving, insofar as it involves the application of concepts). On the one hand, there will be norms for certain kinds of concept use (asserting, laying down legal decisions), or for the use of certain kinds of concepts (physical concepts, chemical concepts, moral concepts). These will be "hypothetical" norms for thought, norms with limited or conditional application. On the other hand, there may be norms for concept use *as such*: norms that are *constitutive* of concept use. If there are any such norms, they will be "categorical" norms for thought, and they will be universally and unconditionally applicable.

Kant appeals to this distinction in distinguishing the laws of formal logic from the laws of the special sciences. The laws of the special sciences are *contingent* laws of the understanding: laws "...without which a certain determinate use of the understanding would not occur" (JL:12). They are contingent not in the sense that they could have been otherwise (among them Kant includes the laws of geometry), but in the sense that they are only *conditionally* applicable to thought: "...it is contingent whether I think of this or that object, to which these particular rules relate" (JL:12).[1] Thus, for example, the laws of physics are norms of thought *about matter and energy.* They are "contingent" in the sense that one can think without being constrained by them—provided one does not think about matter and energy. To interpret an activity as thought about matter and energy is to hold it subject to evaluation by these laws.

The laws of logic, by contrast, are defined as the "...necessary laws of the understanding and of reason in general, or what is one and the same, of the mere form of thought as such ..." (JL:13). By "necessary laws of the understanding," Kant means "...those [laws] without which no use of the understanding would be possible at all..." (JL:12), that is, the norms *constitutive* of thought. Similarly, in the first *Critique* he says that general logic "...contains the absolutely necessary rules of thought without which there can be no employment whatsoever of the understanding" (KrV:A52/B76). In my terminology, this is a demarcation of logic by its *1-formality.*

As I will show in chapter 4, Kant himself does not use "formal" to mean *1-formal* (instead, he uses "general"), but many later writers do. This use of "formal" is common in nineteenth century works on logic, especially those influenced by Kant. For example, Hyslop writes:

> ...Logic is a science of the *formal* laws of thought. They are the laws which are not only essential to it, but which are the same whatever the subject-matter involved in our reasoning. The laws of thought remain the same in the reasonings of Astronomy, Physics, Politics, or Ethics, but the 'matter' changes and does not affect the validity of the process. The 'form' of our reasoning in all these cases is essential to its being such a process. Hence Logic, as a science, is 'formal,' and deals only with the 'formal' principles of thought in distinction from the material objects of reason. (Hyslop 1892:12)

Frege, who also distinguishes logic from the special sciences by its 1-formality, usually uses

[1] Kant's use of "necessary" and "contingent" in JL:12 must be distinguished from his use of the same words in JL:14. In JL:12, necessary and contingent laws of the understanding are two different brands of *norms.* The difference is that contingent laws are not applicable to thought as such, but only to thought about some particular objects. In JL:14, contingent laws are *non-normative* psychological laws of "how we do think," while necessary laws are norms governing "how we ought to think." The distinction of JL:12 corresponds to the first *Critique*'s distinction between general and special logics (A52/B76), while the distinction of JL:14 corresponds to the distinction between pure and applied logic (A53/B77).

"general" to indicate this feature.[2] But in at least one passage, he uses "formal" in the sense of *1-formal*:

> . . . the basic propositions on which arithmetic is based cannot apply merely to a limited area whose peculiarities they express in the way in which the axioms of geometry express the peculiarities of what is spatial; rather, these basic propositions must extend to everything that can be thought. And surely we are justified in ascribing such extremely general propositions to logic. I shall now deduce several conclusions from this logical *or formal* nature of arithmetic. (Frege FTA:95, emphasis added)[3]

It is important to be clear about the sense in which logical laws are, on this tradition, normative for thought as such. The point is *not* that nothing can count as thought unless it *conforms to* the laws of logic. That would make logical error impossible, and it would make nonsense of Kant's claim that logic is a *normative* discipline, since the way we *ought* to think would turn out to be the only way we *could* think. When Kant says that we cannot think except "according to" the laws of logic (JL:12), he means that our thought must be *responsible to* the laws of logic for its assessment. Just as the throwing of a baseball does not count as a *pitch* unless it is liable to assessment in light of the rules of baseball, so no cognitive activity counts as *thought* unless it is liable to assessment in light of the laws of logic. And just as there can be an illegal pitch, so there can be an illogical thought. What makes it a thought is not that it *conforms to* the laws of logic, but that the laws of logic are normative for it. To say that the laws of logic are norms for thought as such, then, is not to say that it is impossible to think illogically, but only that it is impossible to think illogically and be thinking *correctly*.[4]

Moreover, as every teenage driver soon discovers, one can be normatively constrained by laws one does not even *acknowledge*. The laws to which thought about matter and energy

[2]The laws of logic, he says, ". . . are the most general laws, which prescribe universally the way in which one ought to think if one is to think at all" (GGZ:xv).

[3]Joan Weiner picks up this language in her 1986 article: "[Frege's logic]. . . is meant to be nothing more nor less than a means of setting out the formal rules of all thought, the rules which make the use of the understanding possible." Curiously, in the corresponding discussion in her 1990 book, she replaces "formal rules of all thought" with "true rules of all thought" (79). As we will see in chapter 5, Frege too gives up this hylomorphic language in his later writing, as he gets clearer about his differences with Kant.

[4]This, I take it, is the main upshot of Frege's "logical aliens" thought experiment (GGZ:xvi-xvii).

is responsible are the *true* laws of physics, whether we know them are not. The same point applies to the laws of logic. Thus, to say that they are norms for thought *as such* is not yet to say that we know them *a priori*. It is true that Kant and Frege held that logical laws had both these characteristics, but this is a substantive thesis. The two characteristics may go together, but let us not conflate them.

On the other hand, it *does* follow from logic's being normative for thought as such that it is *necessary* in a strong sense. We can *think* about possible worlds or situations in which the physical laws of the actual world do not hold—"where animals speak and stars stand still, where men are turned to stone and trees turn into men, where the drowning haul themselves up out of swamps by their own topknots..." (Frege FA:§14). Such thought violates certain contingent norms for thought—norms for correct thought about animals, stars, forces, and so on—and thus does not count as correct thought about animals, stars, forces, and so on. But considered merely as *thought*, it cannot be faulted, because it does not violate the norms for thought as such. What makes it possible for us to think *correctly* about such counterfactual worlds, then, is that we can prescind from *some* of the norms of thought—by acknowledging our thought as counterfactual, as concerning mere possibility—while continuing to acknowledge others. In the limiting case of *logical* possibility, we prescind from *all* contingent norms of thought, acknowledging only the norms for thought *as such*. But if we try to prescind from these norms, too—say, by thinking about a possible world in which contradictions are true—then *no* norms remain to which our concept use can be held responsible. In this case it is no longer recognizable as concept use at all, since concept use is *essentially* evaluable as correct or incorrect. We can correctly think about what the world would be like if the laws of physics were different, but not about what it would be like if the laws of logic were different. This is the sense in which the norms for thought *as such* are necessary: it is impossible to think at all, even counterfactually, without being constrained by them.

In addition to necessity, 1-formality carries with it another feature that has been important in the philosophy of mathematics: independence from intuition or sensibility. Kant saw the dependence of mathematics on intuition as a restriction on its domain of applicability: mathematical norms were binding only on reasoning about objects capable of being given to the senses (and famously led to antinomies outside this realm). In this framework, to show that some mathematical laws are 1-formal—that is, normative for thought *as such*—would be to show that they apply to objects independently of whether these objects can be given in intuition, and thus that they do not depend on experience or pure intuition for their *justification*. Frege argued backwards along this path, from the applicability of arithmetic outside the realm of the intuitable to its 1-formality or logicality (FTA:94-5, FA:§14).

It is worth emphasizing that the claim that there *are* norms for concept use *as such* is nontrivial. It would take argument to rule out the possibility that *all* norms for concept use are contingent in the sense of Kant's JL:12. Even if all thought must be subject to norms, it is not obvious that there must be any one norm to which all thought is subject. It is not obvious that, as Frege says, "[t]hought is in essentials the same everywhere: it is not true that there are different kinds of laws of thought to suit the different kinds of objects thought about" (FA: iii). It might be that thought is *compartmentalized*, so that there are norms for thought about different kinds of objects, but no global norms for thought as such. To the extent that it rules out this possibility, the thesis that logic is 1-formal is hardly trivial.

3.2 2-formality

To say that logic is formal is to say that it does not concern itself with specific content. One way this might be cashed out—the way we have just explored—is to say that logical norms are applicable to thought about any subject matter whatever—to thought *qua* thought. But there is also another approach, one that eschews talk of norms and domains of applicability. This approach starts from the idea that what makes content *specific* is its concern with *particular individuals*. It is clear that the concept *horse*, the relation *is taller than*, and the quantifier *every animal* have specific content, because they all distinguish between Lucky Feet, on the one hand, and the Statue of Liberty, on the other:

- "Lucky Feet is a horse" is true; "The Statue of Liberty is a horse" is false.
- "The Statue of Liberty is taller than Lucky Feet" is true; "Lucky Feet is taller than the Statue of Liberty" is false.
- The truth of "Every animal is healthy" depends on whether Lucky Feet is healthy, but not on whether the Statue of Liberty is healthy.

On the other hand, the concept *is a thing*, the relation *is identical with*, and the quantifier *everything* do not distinguish between Lucky Feet and the Statue of Liberty. In fact, they do not distinguish between *any* two particular objects. As far as they are concerned, one object is as good as another and might just as well be switched with it. Notions with this kind of indifference to the particular identities of objects might reasonably be said to abstract from specific content—to be "formal." Tarski seems to have something like this in mind when he says:

> ...since we are concerned here with the concept of logical, i.e., *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence X or the sentences of the class K refer. The consequence relation *cannot be affected by replacing the designations of the objects referred to in these sentences by the designations of any other objects.* (Tarski 1936:414-5, emphasis added)

Call this notion of formality "2-formality." I will argue in chapters 4 and 5 that 2-formality did not historically play a significant role in the demarcation of logic. It has become important as a way of spelling out the distinctive "formality" of logic only in the twentieth century, probably because it is so much clearer than 1- or 3-formality. What is particularly appealing is that the notion of *indifference* by which 2-formality is defined can be cashed out in a mathematically precise way as invariance under a group of transformations. The technique goes back to Felix Klein, who proposed it in 1872 as a way of defining different geometries, and it is helpful to see first how it applies to geometry.

Klein observed that we can distinguish the various geometries by the groups of transformations under which their characteristic notions are invariant. Consider first the familiar case of Euclidean geometry. Its characteristic notions—*parallel*, *similar*, *congruent*, etc.—are indifferent to the absolute spatial location and orientation of the figures to which they apply. If a Euclidean sentence is true of a particular figure on a plane, it will remain true no matter how we move the figure across the plane, rotate it, proportionally stretch or shrink it, or reflect it across a line. That is, Euclidean notions are invariant under the group of similarity transformations. Similarly, the notions of affine geometry are invariant under the group of affine transformations (which preserve straightness of lines, but not angle size). The fact that affine notions are invariant under more transformations than Euclidean notions captures the fact that they are indifferent to more features of figures than the Euclidean notions. For example, affine geometry does not distinguish between different kinds of triangles (equilateral, isosceles, scalene). We can take this process even further. Topology, which is invariant under the group of bicontinuous transformations (transformations that preserve connectedness, but not the straightness of lines), is indifferent to the differences between squares and circles, treating both as simple closed curves.

A number of philosophers have suggested that Klein's method of demarcating geometries by their invariance under different groups of transformations can be extended to logic. One can capture the indifference of logical notions to the particular identities of objects by demanding that they be invariant under the group of *all* transformations of the objects in a domain. Proposals of this kind can be found in Mautner 1946, Mostowski 1957:13, Tarski 1966, Scott 1970:161, Dummett 1973:22 n, McCarthy 1981, van Benthem 1989, Sher 1991 and 1996, McGee 1996, and Shapiro 1998:99.[5]

For present purposes, this approach can best be illustrated through examples.[6] Unary first-order quantifiers can be modeled semantically as functions from sets (predicate exten-

[5]Tarski seems to have arrived at his proposal independently of a similar proposal by Mautner 1946. The work on permutation invariance in McCarthy 1981 and Sher 1991 was, in turn, independent of Tarski, whose lecture was not published until 1986. (Sher reports being pleasantly surprised to find her proposal seconded by such a luminary.)

[6]Technical definitions and a full discussion can be found in chapter 6, below.

sions) to truth values. Now consider the quantifier "all chickens," which takes every set that contains all the chickens in the domain to True and every other set to False. Suppose the domain of objects includes two chickens and two cows. Let K be a set containing two chickens, so that "all chickens" takes the value True on K. If we permute the objects in the domain by switching the chickens for the cows, so that K is now a set containing two cows, then "all chickens" takes the value False on K. Since the truth of sentences containing "all chickens" is sensitive to permutations of the domain, "all chickens" is not permutation-invariant. On the other hand, the numerical quantifier "at least three things," which takes every set with three or more members to True and every other set to False, *is* permutation-invariant. No matter how the elements of a set K are permuted, it will always contain the same number of members, so no permutation can affect the value of "at least three things" on K.

Permutation invariance can be regarded as a precise technical gloss on the idea of indifference to the particular identities of objects. Whereas "all chickens" is sensitive to the difference between chickens and cows, permutation-invariant notions—like quantifiers and identity—are insensitive to the particular identities of the objects to which they apply: "there are at least three Fs" will be true whether the Fs are numbers, people, places, or diamonds, provided there are at least three of them. The same is not true of the proprietary notions of arithmetic and set theory: addition, for instance, is sensitive to the *differences* between particular numbers.

Which notions are permutation-invariant? The notions we regard as basic to (extensional) logic—universal and existential quantifiers, identity, and (trivially) the truth functions—are all permutation-invariant. But there are also a few others, most prominently the cardinality quantifiers: "there exist at least α things such that ...," and "there exist exactly α things such that ...," for every cardinal α. Where α is finite, these quantifiers are already definable in standard first-order logic. But the addition of cardinality quantifiers with infinite α yields a significant expansion of the expressive power of first-order logic (see Tharp 1975). Indeed, as Feferman (A) shows (drawing on McGee 1996), the addition of all the first-order permutation-invariant quantifiers to a language gives it the power of full second-order logic.

We motivated 2-formality as a way of thinking about abstraction from *specific* content. Thus the claim that logic is 2-formal leaves open the possibility that logic is concerned with *general* content and very general facts about the world. 3-formality, to which we now turn, precludes *any* concern with content.

3.3 3-formality

To say that logic is 3-formal is to say that it abstracts entirely from the semantic content of thoughts (or interpreted sentences). The word "entirely" is essential here: a logic that abstracts from the contents of *some* concepts ("specific" ones, like "horse" or "red"), but not from the contents of others ("general" or logical ones, like existence, identity, and conjunction), does not count as 3-formal.

What is left to consider when we abstract entirely from the semantic content of a thought? Some philosophers would say "nothing." But some have held that there is more to thought than the concepts it employs: there is also the way they are put together. On Kant's view, for example, "...all judgments are functions of unity among our representations" (KrV:A69/B93-4), and the various modes of unity determine the possible *forms* of judgments (A70/B95):

> The *matter* of the judgment consists in the given representations that are combined in the unity of consciousness in the judgment, the *form* in the determination of the way that the various representations belong, as such, to one consciousness. (JL:101)

General logic, on Kant's view, abstracts from the matter and considers only the form of judgments: it considers only the way in which concepts are united in judgments.[7]

> General logic... abstracts from all content of knowledge, that is, from all relation of knowledge to the object, and considers only the logical form in the relation

[7]This is a bit of a simplification. Kant applies the matter/form dichotomy at three levels—concepts, judgments, inferences—and general logic concerns itself with the formal element at each level (see JL:§2, §18, §59).

> of any knowledge to other knowledge; that is, it treats of the form of thought in general. (Kant KrV:A55/B79)

For example, general logic treats "all horses are mammals" simply as the unification of two concepts in a universal, affirmative, categorical, and assertoric judgment. It abstracts entirely from the content of the concepts. The way in which the concepts are united in thought is not, for Kant, a further constituent of the thought (a "binding" concept), but a feature of the thought's *form*.

As a result of this abstraction from content—that is, from the relation of thought to its objects[8]—logic cannot yield any knowledge about the world or any real truths:

> . . . since the mere form of knowledge, however completely it may be in agreement with logical laws, is far from being sufficient to determine the material (objective) truth of knowledge, no one can venture with the help of logic alone to judge regarding objects, or to make any assertion. (A60/B85)

But this limitation of 3-formal logic is also an advantage. Precisely because logic abstracts from all relation to objects in the world, there is no substantive question regarding its *adequacy* to these objects, as there is in the case of mathematics:

> That logic should have been thus succesful is an advantage which it owes entirely to its limitations, whereby it is justified in abstracting—indeed, it is under obligation to do so—from all objects of knowledge and their differences, leaving the understanding nothing to deal with save itself and its form. But for reason to enter on the sure path of science is, of course, much more difficult, since it has to deal not with itself alone but also with objects. (B ix)

Kant wastes no time explaining the possibility of our *a priori* knowledge of the laws of logic: he takes this to be unproblematic (JL:15). Indeed, his explanation of our *a priori* knowledge of mathematics *presupposes* that the understanding has transparent knowledge of its own forms, considered independently of their relation to objects. Kant's Copernican turn—the doctrine that ". . . we can know *a priori* of things only what we ourselves put into them" (KrV:B xviii)—could not help to explain our *a priori* knowledge of mathematics if

[8] Since for Kant the content of a concept depends on its relation to an object (A69/B94, A139/B178, A155/B194-5, A239/B298, JL:§2), these formulations are equivalent (A59/B83, A63/B87).

it were problematic how we could have *a priori* knowledge of "what we ourselves put into" things.

The claim that logic is 3-formal presupposes that we can distinguish between the constituents and the form of thought, in such a way that the latter can be understood apart from the former. Kant's idealist successors reject 3-formality because they reject this distinction.[9] But many twentieth century philosophers accept the distinction, demarcate logic by its concern with the *form* of thought or conceptual inquiry, and draw the same consequences as Kant:

1. that logic alone tells us nothing about the world;

2. that the world does not *constrain* logic, so that logical knowledge and knowledge about the world are fundamentally different; and

3. that "logical truths," if there are such things, do not state facts.

I will give three examples.

3.3.1 Schlick and Einstein

Schlick and other early logical empiricists with backgrounds in neo-Kantianism invoke a sharp form/content distinction to explain how pure geometry can be an *a priori* science without invoking Kant's "pure forms of intuition." In the absence of pure forms of intuition, Schlick reasons, the only way to avoid making pure geometry empirical is to disconnect it entirely from the real world. He does this by regarding geometric concepts as *implicitly defined* in terms of their logical relations to each other, as specified by a system of axioms in which only the logical constants are interpreted. Implicitly defined concepts, Schlick says, "have no association or connection with reality at all..." (1925:37, quoted in Coffa

[9]Hegel writes: "Logic is usually said to be concerned with forms *only* and to derive the material for them from elsewhere. But this 'only,' which assumes that the logical thoughts are nothing in comparison with the rest of the contents, is not the word to use about forms which are the absolutely real ground of everything" (1827:49, 51). Cf. Bradley 1883:519-24.

1991:176).[10] Einstein articulates this view lucidly:

> As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. It seems to me that complete clearness as to this state of things first became common property through that new departure in mathematics which is known by the name of mathematical logic or 'Axiomatics.' The progress achieved by axiomatics consists in its having neatly separated the logical-formal from its objective or intuitive content; according to axiomatics the logical-formal alone forms the subject-matter of mathematics, which is not concerned with the intuitive or other content associated with the logical-formal. ... [On this view it is clear that] mathematics as such cannot predicate anything about perceptual objects or real objects. In axiomatic geometry the words 'point,' 'straight line,' etc., stand only for empty conceptual schemata. (Einstein 1921:28, 31)

But this way of accounting for our *a priori* knowledge of pure geometry presupposes that the "logical-formal" notions in terms of which geometrical concepts are implicitly defined are themselves devoid of all "objective or intuitive content." In other words, it presupposes that logic is 3-formal. For if logic had objective content, then the implicitly defined pure geometrical concepts would have objective content too, and pure geometry would "refer to reality." It is not simply because these concepts are *implicitly defined* that Schlick and Einstein can take them to be "logical-formal," but because they are implicitly defined in terms of their *logical* relations to each other. Concepts implicitly defined in a *non*-logical background language would have the same kind of objective content as the resources used to define them.[11]

3.3.2 Carnap

Carnap's influential distinction between formal and factual sciences is motivated by similar concerns about the problem of mathematical knowledge:

> In this distinction we [the Vienna Circle] had seen the way out of the difficulty which had prevented the older empiricism from giving a satisfactory account of the nature of knowledge in logic and mathematics. ... Our solution, based on

[10] See Friedman 1990 for an interesting discussion of the difficulties into which Schlick is led by his strict form/content distinction.

[11] Cf. section 1.2.2, above.

> Wittgenstein's conception, consisted in asserting the thesis of empiricism only for factual truth. By contrast, the truths in logic and mathematics are not in need of confirmation by observations, because they do not state anything about the world of facts, they hold for any possible combination of facts. (Schilpp 1963:64)[12]

Note that if this solution is to be non-trivial, "factual truth" must not simply *mean* "truth about the empirical world." Carnap is therefore committed to the view that the true sentences of logic and pure mathematics do not state facts of any kind. They "...do not express any matters of fact, actual or nonactual" (Carnap 1934:126). Echoing Kant's characterization of logic, Carnap says: "*The formal sciences do not have any objects at all*; they are systems of auxiliary statements without objects and without content" (128). But precisely *because* they do not have content, they are not constrained by facts about the world. Instead, they define the "linguistic framework" in terms of which such facts can be stated. This linguistic framework can be regarded as the *way* in which contentful concepts are constructed and related to one another: the *form* of thought, or as Carnap would prefer to put it, of scientific inquiry.

Carnap diverges from Kant in maintaining that the form of thought is, in some sense, a matter of *choice* or *convention*, and that consequently there can be many equally legitimate logical frameworks (Schilpp 1963:64). Conventionalism was Carnap's answer to a question that naturally arises for the view that logic is 3-formal: if logic is not constrained by the *world*, what is the source of its objectivity? Carnap's answer has drawn heavy fire (e.g., from Quine 1963 and Prior 1976:123-4). But here I am primarily interested in showing *that* Carnap holds logic to be 3-formal, not *how* he articulates this position or whether he succeeds in defending it.

[12] Carnap acknowledges the influence of Wittgenstein's *Tractatus* here, and Wittgenstein does seem to be advocating a version of the view that logic is 3-formal: "The propositions of logic are tautologies.... The propositions of logic therefore say nothing.... Theories which make a proposition of logic appear substantial are always false" (1922:§§6.1, 6.11, 6.111). For a sophisticated account of the relation between Carnap's view of logic and Wittgenstein's, see Friedman 1988.

3.3.3 Nagel

Ernest Nagel defends a similar view in his article "Logic Without Ontology" (1956:ch. 4). His goal is "to make plausible the view that the role of the logico-mathematical disciplines in inquiry can be clarified without requiring the invention of a hypostatic subject matter for them..." (57). Like Carnap, he draws a sharp distinction between logical principles—"principles whose function it is to institute a desired order into inquiry"—and "statements about the explicit subject matter of inquiry" (72). Thus, for example, the principle of non-contradiction is not a claim about things in the world, but rather a criterion for the use of the logical terminology "same respect," "same attribute," "belong," and "not belong" (58):[13]

> Accordingly, the interpretation of the principle as an ontological truth neglects its function as a *norm* or *regulative principle* for introducing distinctions and for instituting appropriate linguistic usage. To maintain that the principle is *descriptive* of the structure of antecedently determinate 'facts' or 'attributes' is to convert the *outcome* of employing the principle into a *condition* of its employment. (60)

Nagel revealingly characterizes the view he is *opposing* as the view that logical laws "... are not formal or empty" and that "... they tell us something about the *actual* world" (66). We may infer that the view he is defending is a version of the claim that logic is 3-formal. Logical laws are implicit definitions that define the framework or *form* of rational inquiry about the world (80). Because they do not represent constraints on the world, they can be known *a priori*; but for the same reason, they can tell us nothing about the world.

3.4 Independence of the three notions

These, then, are three things that might be meant by calling logic "formal": first, that it provides norms constitutive of concept use as such; second, that it is indifferent to the particular identities of individuals; third, that it abstracts entirely from the semantic content of the concepts used. It is useful to think of these three notions of formality in relation to

[13] Similar ideas lie at the root of C. I. Lewis's neo-Kantian pragmatism (1929:245-6).

three Kantian dualisms identified in Brandom 1994:614-18: 1-formality can be understood in terms of the dualism of thought and sensibility, 2-formality in terms of the dualism of general and singular, and 3-formality in terms of the dualism of structure and content (see figure 3.1).

Figure 3.1: Three notions of logical formality.

Notion	Description	Dualism
1-formality	normativity for thought *as such*	thought/sensibility
2-formality	indifference to particular identity	general/singular
3-formality	abstraction from semantic content	structure/content

In certain philosophical frameworks, these three dualisms line up neatly, and logic can be said to be formal in all three senses. For example, in Kant's transcendental idealism, sensibility provides the *content* for cognition and is the source of all *singular* representations, whereas structure and generality depend on thought. It should not be surprising, then, that Kant holds logic to be formal in all three senses. (The relation between Kant's philosophy of logic and his transcendental idealism will be discussed in more depth in chapter 4.)

But the three notions need not line up in this way. For example, Frege rejects the Kantian connection between sensibility and content. Thus, although like Kant he holds that logic is 1-formal, he rejects the Kantian idea that logic is 3-formal. (Frege's rejection of 3-formality will be discussed in chapter 5.) On Frege's view, logic is about the world in just the same sense as physics, only its concepts are more general. Frege also severs the Kantian connection between sensibility and singularity: for Frege, thought has its own objects (extensions and truth values). Thus, Frege's logic (unlike Kant's) is not 2-formal: it respects differences between particular objects and employs concepts that are not permutation-invariant. 1-formality implies neither 2-formality nor 3-formality.

Conversely, 2-formality does not imply 1-formality. There may be general truths expressed in permutation-invariant vocabulary that are not binding on thought *as such*. For example, a finitist might claim that (as a matter of contingent fact) the world contains no more than n objects.[14] Since "there are no more than $n \ldots$" is a permutation-invariant quantifier, the finitist will have to count this claim as 2-formally true. But she need not count it as 1-formally true: it is coherent for a finitist to suppose that the number of objects in the world could have been larger than it is.

Indeed, Kant holds that arithmetic and algebra are 2-formal, but not 1-formal or 3-formal.[15] These sciences do not, on Kant's view, have their own objects; rather, they are are directed toward "...objects without distinction" (A63/B88), from the particular features of which they abstract completely (A717/B745). But although the characteristic concepts of arithmetic and algebra do not respect individual differences between objects, the laws of arithmetic and algebra are not normative for thought *as such*; they are binding only on thought about objects capable of being exhibited in sense. Nor do they abstract entirely from semantic content: they do not abstract from the concept of *magnitude*, a concept that (in Kant's view) can be given content only through construction in pure intuition.

3-formality does not imply 1-formality, either. On the positivists' view of logic, for instance, the necessity of logic is simply a reflection of the rules for the use of a particular language (cf. Ayer 1946:77). Thus logical laws are not normative for thought as such, but only for thought in a particular framework. If we do not want to be bound by them, we can simply use another language. In this respect, they are analogous to Kant's "contingent laws of the understanding," which are only binding on us if we intend our thought to relate to certain objects.

Finally, 2-formality and 3-formality can come apart. It is important not to overlook the difference between abstracting from all relation to objects—that is, from all content of concepts—and abstracting from all *differences* between objects. As we have seen in articulating the notion of 2-formality, it is not necessary to abstract from content entirely

[14]Some philosophers may question the very intelligibility of such a claim, on the grounds that it makes sense to talk of the number of objects only relative to a sortal concept. But the finitist's claim can be understood as: "there are no more than n objects of all sorts." Such a claim would be true if there were m *basic* sortal concepts ($S_1 \ldots S_m$) such that (a) the sum of the number of S_i's, $1 \leq i \leq m$, is less than or equal to n, and (b) for every sortal concept S, every S (i.e., everything that is an S) is identical with some S_i for some $1 \leq i \leq m$.

[15]See section 4.1.2, below.

in order to abstract from *specific* content. Many contemporary advocates of the 2-formality of logic (e.g., Sher, Shapiro) would repudiate the view that logic is 3-formal.[16] On the other hand, the Carnapian brand of 3-formality allows 3-formal logic (broadly construed) to contain singular terms (such as "2" and "4") and non-permutation invariant function terms (such as "+") (1934:124). For example, Carnap 1934 describes a language in which "2+2=4" is an "auxiliary statement" with "no factual content" (126). Such a language would be 3-formal but not 2-formal. It appears, then, that neither 2-formality nor 3-formality entails the other.

If the three notions of formality are equivalent in certain philosophical contexts, then, that is because of special features of those contexts. The notions are conceptually independent. As long as they are not explicitly distinguished, however, there is always danger of confusion and equivocation. For example, advocates of the permutation invariance criterion for logical constants often seem to conflate distinct senses of formality in motivating their proposals and connecting them with traditional conceptions of logic. According to van Benthem 1989, "[t]he traditional idea that logical constants are not concerned with real content may be interpreted as saying that they should be *preserved* under those operations on models that change content, while leaving general structure intact" (317). But interpreting historical talk of the formality of logic as inchoate talk of permutation invariance would be a distortion. Historically, 2-formality did not play an important role in debates over the bounds of logic: the "traditional idea" was that logic is 1-formal or 3-formal. And van Benthem has done nothing to connect his proposal with *these* senses of formality or "indifference to real content." The ambiguity of "formal" allows him to dodge the question of what permutation invariance or 2-formality has to do with *logicality*, as traditionally conceived. Similar unclarity leads Sher to suggest that her account of the logical constants as isomorphism-invariant notions answers Russell's difficulty about "what is meant by saying

[16]For the claim that logic is 2-formal, see Sher 1996:672-8, Shapiro 1998b:99. Sher and Shapiro do not explicitly repudiate 3-formality, because they do not draw the distinctions I have drawn. But they both reject "foundationalist" approaches to logic: approaches that take logic to have the kind of epistemic priority over contentful mathematics that it would have were it 3-formal (Sher 1996:680, Shapiro 1991:ch. 2).

that a proposition is 'true in virtue of its form"' (1996:683-4). The first step to securing the kind of historical continuity Sher and van Benthem are seeking is to distinguish the different notions of "formality" that are in play.

3.5 Formality, generality, and topic-neutrality

Discussions of the demarcation of logic do not always make heavy weather of "formality." They commonly appeal to the *generality* or *topic-neutrality* of logic instead.[17] In this section, I will argue that these notions are trifurcated in precisely the same way as formality. By the "generality" (or "topic-neutrality") of logic, I will show, one might mean either 1-formality, 2-formality, or 3-formality. The upshot is that this dissertation has something to contribute to a whole range of work on the demarcation of logic, not just work that invokes the *word* "formal."

What does it mean to say that logic is maximally general, or topic-neutral? We might start with this suggestion:

> **(Gen-1)** To say that logic is general or topic-neutral is to say that it is not about anything in particular.

But the notion of *aboutness* is notoriously hard to pin down. We can see some of the difficulties if we consider set theory and arithmetic. On the one hand, they seem to have special subject matters (sets and numbers, respectively). On the other hand, they can be used in discourse on virtually any topic.[18] Suppose one understood only the following words in a paragraph: "set," "is a member of," "five," and "fewer than." One could confidently say that it was about sets and numbers, yet in a broader sense one would have no idea what it was about. *Any objects* can be considered as a set or numbered. So are set theory and

[17]The term "topic-neutral" is due to Ryle 1954:116. For the claim that logic is characterized by its topic-neutrality, see also Peacocke 1976:229, Benacerraf 1973:411, McCarthy 1981:504, Sainsbury 1991:314, Strawson 1952:41, Haack 1978:5-6, Wright 1983:133. The last three of these connect topic-neutrality with "formality," as does Ryle. Schroeder-Heister 1984:104 implies that a constant that is not topic-neutral has "specific material content."

[18]Indeed, *set* is Gödel's prime example of a formal or "universally applicable" concept (Wang 1996:267).

arithmetic topic-neutral or not? Are they about nothing in particular, or are they about sets and numbers (cf. Boolos 1975:517)?

Goodman 1961 defines a technical notion of "absolute aboutness" that would give definite answers to these and other questions about aboutness. On Goodman's account, a statement S is *absolutely about* an object k just in case there is a statement T such that ...

(i) T contains an expression e designating k, and

(ii) T follows logically from S,

(iii) no generalization of T with respect to e (or any part of e) follows logically from S. (253)[19]

It follows from this definition (together with Goodman's tacit assumption that the logical constants do not designate anything) that logical truths are not absolutely about anything (256) and that logically equivalent statements are absolutely about the same things (258). Goodman's notion is useless for our purposes, however, because clauses (ii) and (iii) of the definition presuppose that it is settled what counts as logic (253-4). Thus, if set theory is part of logic, set-theoretic truths will not be about anything; while if set theory is not part of logic, set-theoretic truths will be about sets and classes of sets. It would be circular, then, to appeal to Goodman's criterion in an account of logicality.

We might try something simpler:

> **(Gen-2)** To say that logic is general or topic-neutral is to say that its truths and inference rules do not mention any particular objects, i.e., contain no singular terms essentially.[20]

But this is pretty obviously inadequate. The sentence "$(\forall x)(x \text{ is a cat} \supset x \text{ is a mammal})$" does not contain any singular terms, but it is a truth of biology, not of logic (cf. Carnap 1942:232). So the generality of logic cannot consist in the fact that logical truths contain no

[19] I have reformulated Goodman's definition slightly, without altering its content.

[20] The point of the qualification "essentially" is to allow logical truths like "Ben is tall or Ben is not tall." The singular term "Ben" does not occur in this sentence essentially, because we could substitute any other singular term without changing the truth value of the sentence.

singular terms essentially. The use of particular *general terms* can spoil maximal generality as surely as the use of particular singular terms.

The most obvious fix is to forbid mention of particular properties, functions, and relations, as well as particular objects. This is Russell's approach from 1913 on:

> A proposition which mentions any definite entity, *whether universal or particular*, is not logical: no one definite entity, of any sort or kind, is ever a constituent of any truly logical proposition. (1913:97-8, emphasis added)
>
> Certain characteristics of the subject [mathematics/logic] are clear. To begin with, we do not, in this subject, deal with particular things *or particular properties*: we deal formally with what can be said about *any* thing or *any* property. ... It is not open to us, as pure mathematicians or logicians, to mention anything at all, because, if we do so, we introduce something irrelevant and not formal. (1920:196-7, emphasis added)

Putting Russell's proposal into a less ontologically committed form, we get:

> **(Gen-3)** To say that logic is general or topic-neutral is to say that its truths and inference rules contain no singular terms, predicates, relation terms, or function terms essentially.

However, (Gen-3) will only work if the logical expressions do not count as predicates, relation terms, or function terms. "=" is especially problematic, because it is in the same grammatical category as "is taller than" (see section 2.3.2, above). Doesn't it signify a particular relation, namely, identity? And don't logical truths like "$\forall$x(x=x)" mention this relation?

One might bite the bullet and take "=" to be non-logical. But a similar question arises for the quantifiers and sentential operators. We do not usually think of these as predicates, relation terms, or function terms (though Frege thought of quantifiers as higher-order predicates, and sentential operators as ordinary function terms). So the essential presence of quantifiers and sentential operators in logical truths does not compromise their generality, according to (Gen-3). But suppose our language contains intuitively *non-logical* quantifiers or sentential operators, like "(Cx)" (for all cats ...) or "(Kn_C)" (Bill Clinton knows that ...).[21] (Gen-3) will blindly count truths containing these terms essentially as

[21]See section 2.3.3, above.

"general" or "topic-neutral," and that seems wrong. But how could we modify (Gen-3) to treat these terms differently from the standard quantifiers and truth-functional operators, without begging the question?

Russell 1913 solves this problem by denying that the logical expressions function semantically in the same way as non-logical expressions of the same grammatical categories. They do not signify constituents of propositions at all; instead, they indicate *form*:

> "Logical constants", which might seem to be entities occurring in logical propositions, are really concerned with pure *form*, and are not actually constituents of the propositions in the verbal expression of which their names occur. (1913:98; cp. 1920:199)

It is because Russell treats logical constants in this way that he can take (Gen-3) as his account of logical generality. Thus generality, for Russell, amounts to a version of 3-formality.[22]

It should be clear, however, that *Russell's* way of understanding the generality of logic is not for everyone. Many philosophers have held that logic is maximally general or topic-neutral without holding that it is 3-formal. Those who do not see a semantic difference in kind between "(∀x)" and "(Cx)" will not want to cash out the generality of logic in the way that Russell does in 1913. And even those who hold that logic is 3-formal may want to mean something else by the claim that logic is general.[23]

A natural suggestion is to spell out the generality or topic-neutrality of logic in terms of the generality or topic-neutrality of the expressions that occur essentially in logical truths or inference rules—the "logical constants."

> **(Gen-4)** To say that logic is general or topic-neutral is to say that its truths or inference rules contain only general or topic-neutral expressions essentially.[24]

[22]Russell's theory of "logical forms" is notoriously problematic. See Griffin 1993:177, Hylton 1984:389-91.

[23]In chapter 4, I will argue that Kant falls into this category.

[24]This is Russell's solution from 1903-1911: he characterizes a "formal" proposition as a "proposition which does not contain any other constant than logical constants," where the logical constants are "purely formal concepts" (1911:288).

But it turns out to be remarkably difficult to say what it is for an *expression* to be general or topic-neutral.

Let us start with Ryle's criterion for topic-neutrality:

> We may call English expressions 'topic-neutral' if a foreigner who understood them, but only them, could get no clue at all from an English paragraph containing them what that paragraph was about. (1954:116)

The problem with this criterion is that one might answer the question "what is this paragraph about?" at many different levels of generality. Consider the following paragraph:

> If *Bob Wills* had not *recorded* the *tune*, some*one* else certainly would have. Many *Texas fiddlers* were *playing* it at the time. Though it might not have become *famous*, it could not have *escaped being put on vinyl*.

This paragraph is about:

(1) "Faded Love"
(2) a particular tune recorded by Bob Wills
(3) Bob Wills and Texas fiddle music
(4) music
(5) persons and things
(6) the past
(7) historical possibilities
(8) objects

Even the original passage, out of context, does not tell us enough to give answer (1). If we delete all the proper names,

> If ... had not *recorded* the *tune*, some*one* else certainly would have. Many ... *fiddlers* were *playing* it at the time. Though it might not have become *famous*, it could not have *escaped being put on vinyl*,

we cannot give answers (1)-(3), but we can still give (4)-(8). If we now delete all the underlined words,

> If ... had not ... the ..., some ... else certainly would have. Many ... were ... it at the time. Though it might not have become ..., it could not have ...,

we cannot give answers (1)-(5), but we can still give answers (6)-(8). We still have *some* clue about what the passage is about: we know, for instance, that it is not a geometrical proof. Finally, if we delete everything but

> If ... not ... the ..., some ... else .... Many ... it .... Though it ... not ..., it ... not ...,

we can still give answer (8)—as we could not, for instance, for the sentence "To be red is to be colored." *Every* expression (except perhaps "if" and the like) gives us *some* information about the topic of a paragraph. Instead of a firm dividing line between topic-neutral and topic-specific expressions, then, Ryle's criterion gives us a spectrum of varying degrees.

Perhaps this is all we should ask for. Lycan 1989 suggests that it is a mistake to look for a line between logical and non-logical expressions, since topic-neutrality comes in degrees: truth-functional expressions are more topic-neutral than quantifiers, which are more topic-neutral than tense and modal operators, which are more topic-neutral than epistemic expressions, and so on.[25] Yet it is widely held that the generality of the paradigm logical constants is different in kind, and not just in degree, from that of "quickly," "red," or "person." Is there any way to cash this distinction out in a principled and motivated way?

One place we might look for a sharp dividing line is the permutation invariance criterion, discussed in section 3.2, above. The permutation invariance criterion might be thought to give a precise sense in which logical notions (the semantic values of logical constants) are "not about anything in particular" (cf. (Gen-1), above):

> **(Gen-5)** To say that logic is general or topic-neutral is to say that its fundamental notions are invariant under all permutations (or bijections) of the domain of objects.

Not only does the permutation invariance criterion settle the borderline cases for which the topic-neutrality criterion could give no clear verdict, it also provides a kind of *explanation*

[25]It would probably be more accurate to say that the relation *is more topic-neutral than* is a partial order: the modal operators and the tense operators, for instance, represent two different spectra of relative topic-neutrality, not two (comparable) positions on a single spectrum.

for the fact that logical expressions can be used in discourse about any topic:

> Logic, on the present conception, takes certain general laws of formal structure and, using the machinery of logical terms, turns them into general laws of reasoning, applicable in any field of discourse. The fact that biological, physical, psychological, historical, ... structures obey the general laws of formal structure explains the generality ("topic neutrality") of logic: some references to formal structure (to complements and unions of properties, identity of individuals, non-emptiness of extensions, etc.) is interwoven in all discourse, and therefore logic (the logic of negation and disjunction, identity, existential quantification, etc.) is universally applicable. (Sher 1996:674-5)

Thus 2-formality provides one way of understanding the generality of logic, just as 3-formality provides another. But some philosophers think of logic as general in neither of these senses. For example, Frege says that "... the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter," and that "... logic is the science of the most general laws of truth" (PW:128). But he does not think that logic is 3-formal (see chapter 5, below). Kit Fine 1998 suggests that permutation invariance (2-formality) "... is the formal counterpart to Frege's idea of the generality of logic" (1998:556). But this cannot be right either. As a logicist, Frege *cannot* hold that logic is indifferent to particular differences between objects: if arithmetic is to be reducible to logic, then logic must be capable of distinguishing the number 7 from the number 6.[26]

I suggest that for Frege, the generality of logic amounts to its general *applicability* to thought as such, whatever its topic.

> **(Gen-6)** To say that logic is general or topic-neutral is to say that it provides norms for thought *as such.*

That is, Frege's "generality" is 1-formality. I will argue in chapter 4 that Kant conceives of the generality of logic in this way, too.

In sum, there are three coherent ways of construing the generality or topic-neutrality of logic: as 1-formality, as 2-formality, and as 3-formality. The upshot is that everything we learn in this investigation of the "formality" of logic can be applied to discussions of logic's generality or topic-neutrality as well.

[26]For more argument that Frege's logic is not 2-formal, see section 5.3, below.

3.6 Philosophical significance of the three notions

Let us return briefly to the three philosophical *uses* for a demarcation of logic we considered in section 1.2. Here I will focus on logicism, with the aim of showing how it *matters* in what sense (if any) logic is formal.

If logic is 1-formal, then a proof that mathematics reduces to logic would show that mathematics is normative for thought as such, and hence independent of human sensibility. This is how Frege and Russell think of their logicist projects. For Russell, the point of logicism is to refute what he sees as the dangerous idealist doctrine that mathematical knowledge is conditioned by subjective facts about human sensibility, and hence only *conditionally* (not absolutely) true (see Hylton 1990a). For Frege, the point is to clarify the epistemological basis for mathematical knowledge—in particular, to establish its independence from sensibility (cf. Weiner 1990:ch. 2).[27] Neither Frege nor Russell takes logicism to show that mathematics lacks content. Russell says that Kant "rightly perceived" that mathematical truths are synthetic (1903:457), and Frege, though he takes mathematics to be analytic, redefines "analytic" so that analytic propositions can extend our knowledge and have content (FA:§88, §3).

Logicism takes on a very different significance if logic is demarcated as 3-formal, as it is by the positivists. In that case, to say that mathematics reduces to logic is to say that mathematics *lacks content* and says nothing about the real world.[28] As we have seen, Carnap and the positivists see in the 3-formality of logic and mathematics a way to account for a priori mathematical knowledge in an empiricist framework. Mathematics can be known *a priori*, on their view, because it represents no constraint on reality; it is not real knowledge at all.[29]

Finally, if logic is demarcated as 2-formal, then a Platonist logicism such as Frege's—in

[27]Demopoulos 1994 and Tappenden 1995 argue that Frege's main concern in giving logical proofs of arithmetical propositions is not to secure them from doubt, but to establish their general applicability (even outside the realm of the intuitable).

[28]For an especially vigorous statement of this view, see Ayer 1946:ch. 4.

[29]For the contrast between Russellian and positivistic logicism, see Hylton 1990a:144-5.

which numerals are taken as names for objects—becomes *impossible*, by definition. On the other hand, a non-Platonist type of logicism becomes *trivial*, since virtually any mathematical predicate or functor can be defined as a permutation-invariant logical constant (Sher 1991:132; cf. Tarski 1966:152-3, section 3.2, above). One of the accomplishments of Frege's logicism is the logical definition of *finiteness*; but on the permutation invariance approach, "there are finitely many…" is already a logical constant. Sher 1991 embraces this consequence of her view:

> The bounds of logic, on my view, are the bounds of mathematical reasoning. Any higher-order mathematical predicate or relation can function as a logical term, provided it is introduced in the right way into the syntactic-semantic apparatus of first-order logic. (xii-xiii)

But this consequence ought to make us wonder whether the notion of logicality Sher is explicating has anything to do with the notion of logicality presupposed by the original logicists. She claims that both the old logicism and the kind of logicism that would follow trivially from her account of logic "are based on the equation that being mathematical = being formal = being logical" (133). But it is not clear that anything more than the *word* "formal" connects them.

It matters a great deal, then, *in what sense* (if any) logic is formal.

3.7 A puzzle

I have proposed that talk of the "formality" of logic (and equally of its "generality" or "topic-neutrality") moves between three quite different notions. To some extent, all three notions influence our "intuitions" about logicality: the intuitions to which philosophers appeal in motivating their demarcation proposals. In arguing that second-order logic cannot be logic because of its substantive mathematical content, one supposes that logic must be 3-formal; in arguing that arithmetic cannot be logic because of its special objects, one supposes that logic must be 2-formal; in arguing that quantum logic cannot be logic because it does not provide norms for thought as such, but only for thought in a quantum world, one supposes that logic must be 1-formal. Distinguishing these three notions, I hope, will bring some light to the intractable debates about the bounds of logic.

But not just distinguishing them: it is also useful to ask how the three notions are related. And here there is a puzzle. Why are there three distinct notions of logical formality? How did they come to be confused under a single name? What are the connections between them? And how did they all come to be connected with logicality? Or are there several distinct notions of logicality in the tradition, corresponding to the different notions of formality? The only way to answer these questions is through conceptual archaeology. In chapter 4, I will argue that we need to start digging with Kant.