# Philosophy 148 — Announcements & Such

- Administrative Stuff
  - I'll be using a straight grading scale for this course. Here it is:
    - A+ > 97, A (94,97], A- (90,94], B+ (87,90], B (84,87], B- (80,84], C+ (77,80], C (74,77], C- (70,74], D [50,70], F < 50.
  - People did very well on the quiz ($\mu = 93$). HW #1 assigned (due 2/28).
  - Today's Agenda
    - Back to sketching our "guiding analogy":

$$\frac{\text{truth-on-}\mathcal{I}}{\text{truth}} :: \frac{\text{probability-in-}\mathcal{M}}{\text{probability}}$$

    - Today, this will involve the following:
      - Finishing-up our overview of the basic framework of the analogy
      - Looking (a little) at different "theories" of *truth*
      - Looking (harder) at different "theories" of *probability*
      - We'll start with Objective Theories … then Subjective ones …

## T-on-$\mathcal{I}$ : Truth :: Pr-on-$\mathcal{M}$ : Probability (V)

- Initially, we have only systematic constraints. Specifically, we have no systematic logical relations between atomic sentences, and the only systematic probabilistic constraints are $a_i \in [0,1]$ and $\sum_i a_i = 1$. *E.g.*:

| $X$ | $Y$ | Interpretations/S.D.'s | Models ($\mathcal{M}$) |
|---|---|---|---|
| T | T | $\mathcal{I}_1$ / $s_1$ | $a_1 \in [0,1]$ |
| T | F | $\mathcal{I}_2$ / $s_2$ | $a_2 \in [0,1]$ |
| F | T | $\mathcal{I}_3$ / $s_3$ | $a_3 \in [0,1]$ |
| F | F | $\mathcal{I}_4$ / $s_4$ | $1 - (a_1 + a_2 + a_3)$ |

- Then (in context) we associate extra-systematic contents with atoms, *e.g.*:
  - $X \overset{\text{def}}{=}$ John is unmarried.
  - $Y \overset{\text{def}}{=}$ John is a bachelor.
- In this case, we can *conceptually rule-out* interpretation $\mathcal{I}_3$ on extra-sysetmatic grounds. In other words, $s_3$ is (necessarily) false.

- That leads to the following extra-systematic revision of our initial STT:

| $X$ | $Y$ | Interpretations/S.D.'s | Models ($\mathcal{M}$) |
|---|---|---|---|
| T | T | $\mathcal{I}_1$ / $s_1$ | $a_1 \in [0,1]$ |
| T | F | $\mathcal{I}_2$ / $s_2$ | $a_2 \in [0,1]$ |
| F | T | $\mathcal{I}_3$ / $s_3$ | 0 |
| F | F | $\mathcal{I}_4$ / $s_4$ | $1-(a_1+a_2)$ |

- In other cases, we will *not* be able to *rule-out* any interpretations. But, we will be able to *rule-out* certain *probability assignments/models*. *E.g.*:
  - $X \overset{\text{def}}{=}$ The coin will land heads when it is tossed.
  - $Y \overset{\text{def}}{=}$ The coin is heavily biased in favor of heads.
- In this case, let's assume the right constraint is $\Pr(X \mid Y) \approx 1$. Then, this will impose the following extra-systematic constraint on our initial STT:

$$\frac{a_1}{a_1+a_3} \approx 1$$

- This doesn't *rule-out* any *interpretations*, but it does *rule-out* some *probability models*. Finally, there is a third "grade of ruling-out"...

## T-on-$\mathcal{I}$ : Truth :: Pr-on-$\mathcal{M}$ : Probability (VI)

- Here is another example of a pair of sentences:
  - $X \overset{\text{def}}{=}$ The ball is black.
  - $Y \overset{\text{def}}{=}$ The ball is either black or white.
- Some philosophers claim that there is *some* sense in which we should have $\Pr(X \mid Y) = \frac{1}{2}$ here — as an extra-systematic constraint $\left[\frac{a_1}{a_1+a_3} = \frac{1}{2}\right]$.
- But, intuitively, it's a different sort of constraint than the one in our last example. In our last example "biased" was *itself* a *probabilistic* concept.
- Here, there is no *probabilistic extra-systematic content* involved.
- As such, if some extra-systematic probabilistic constraint is called for here, it's not for purely conceptual reasons. I will call this an *epistemic* extra-systematic constraint (an instance of the "Principle of Indifference").
- This can be motivated by unpacking $\Pr(X \mid Y)$ as (something like) "the degree of confidence one should have in $X$ — *if $Y$ were all one knew*."
- We'll come back to this *epistemic* understanding of probabilities shortly.

## T-on-$\mathcal{I}$ : Truth : : Pr-on-$\mathcal{M}$ : Probability (VII)

- We'll focus extensively on probabilistic issues shortly. First, we will take a short detour through a discussion of (extra-systematic) *truth* (simpliciter).
- There are various "Theories" or "Philosophical Explications" of *truth*. I have posted a nice overview by Haack (and the SEP entry by Glanzberg).
- I will separate the philosophical theories of truth into two categories:
  - Objective Theories of Truth.
    - Correspondence theories.
  - Subjective Theories of Truth.
    - Epistemic theories.
      - Coherence theories.
    - Pragmatic theories.
- There are also theories that are neutral on the subjective/objective question. For instance, deflationary theories (which I will skip over).

## T-on-$\mathcal{I}$ : Truth :: Pr-on-$\mathcal{M}$ : Probability (VIII)

- According to correspondence theories of truth, $p$ is true if $p$ *corresponds to* some *truthmaker* $\mathfrak{t}_p$ (that is, if there exists a truthmaker $\mathfrak{t}_p$ for $p$).
- There are different views on the bearers of truth-values (sentences, propositions, beliefs) and truthmakers (facts, states of affairs).
- Moreover, there are different views about whether truthmakers must exist in some mind-independent or "transcendent" realm. *Realists* will require that the realm of truthmakers is mind-independent. Anti-realists will not.

**Sentence** ($s$) (in context): "John loves Mary."

$\downarrow$ expresses (in context)

**Proposition** ($p$): The proposition that John loves Mary.

$\downarrow$ corresponds to

**Truthmaker** ($\mathfrak{t}_p$): The fact that John loves Mary.

- If $p$ is *false*, there is no corresponding $\mathfrak{t}_p$ at the bottom of the diagram.

## T-on-$\mathcal{I}$ : Truth : : Pr-on-$\mathcal{M}$ : Probability (IX)

- Subjective theories of truth do not involve any sort of correspondence between sentences/propositions/beliefs and some realm of truthmakers.
- The *epistemic* theory of truth, for instance, holds that (Alston):

  > The truth of a truth bearer consists not in its relation to some "transcendent" state of affairs, but in the epistemic virtues the former displays within our thought, experience, and discourse. Truth value is a matter of whether, or the extent to which, a belief is justified, warranted, rational, well grounded, or the like.

- The coherence theory of truth is a instance of the epistemic theory (where coherence with one's other beliefs is the salient "epistemic virtue").
- The pragmatic theory of truth holds that "truth is satisfactory to believe". Basically, a belief is true if believing it "works" for its believer.

☞ We will adopt an objective/realist stance toward truth in this course. This is largely for simplicity. Also, "subjective" theories seem *unstable/false*.

## T-on-$\mathcal{I}$ : Truth :: Pr-on-$\mathcal{M}$ : Probability (X)

- According to subjective theories of truth, we have something like:
  - $p$ is true iff some subjective condition $C_p$ involving $p$ obtains.
- But, what could the subjective theorist *mean* when they say that "$C_p$ obtains"? Presumably, this too must receive a subjectivist gloss.
- That is, they can't mean $C_p$ *really* obtains (with an *objective* "*really*").
- So, for instance, take the coherence theory of truth. According to it, a belief $p$ is true iff $p$ coheres with some salient body of beliefs $B_p$.
- Let $p' \stackrel{\text{def}}{=} p$ coheres with $B_p$. What can a coherentist mean when they say that $p'$ is true? Presumably, they must mean that $p'$ coheres with $B_{p'}$.
- Now, it appears that we have a regress. Let $p'' \stackrel{\text{def}}{=} p'$ coheres with $B_{p'}$. Then, for $p''$ to be true, we must have that $p''$ coheres with $B_{p''}$, *etc*...
- Similarly for the pragmatic theory. Let $p^* \stackrel{\text{def}}{=}$ it is useful to believe $p$. When is $p^*$ true? Presumably, when it is useful to believe $p^*$, *etc*...

## T-on-$\mathcal{I}$ : Truth :: Pr-on-$\mathcal{M}$ : Probability (XI)

- There are less worrisome subjective theories. For instance, one might have an epistemic theory which says that a belief is true iff it is *justified*.
- On such an account, the belief that $p$ is justified will be true when the belief that $p$ is justified it *itself* justified. This leads to a regress, but…
- This is a regress that many epistemic theories of justification must *already* face head-on. So, there is no *special/new* instability here.
- Another problem with subjective theories is that they just seem *false*.
- One thing about truth that seems clear is that it is *redundant*. When I assert "$p$ is true", this is just like asserting $p$ itself. For instance, if I say "*it is true that* it is raining", this is equivalent to just saying "*it is raining*".
- An immediate problem with subjective theories is that this redundancy property seems to be violated. Intuitively, when I say that $p$ is justified (or useful or coherent, *etc.*), this is *not* equivalent to just saying $p$. [Intuitively, evidence can be *misleading*, and *wishful thinking* may be useful, *etc.*]

## T-on-$\mathcal{I}$ : Truth : : Pr-on-$\mathcal{M}$ : Probability (XII)

- Just as we can talk about $p$ being *true-on-$\mathcal{I}_i$*, which is synonymous with $s_i \vDash p$, we can also talk about $p$ having *probability-$r$-on-$\mathcal{M}$*.
- And, like *truth-on-$\mathcal{I}_i$*, *probability-on-$\mathcal{M}$* is a *logical/formal* concept.
- That is, once we have *specified* a probability model $\mathcal{M}$, this *logically determines* the *probability-on-$\mathcal{M}$* values of all sentences in $\mathcal{L}$.
- Moreover, just as the *truth-on-$\mathcal{I}_i$* of sentence $p$ does not imply anything about $p$'s *truth* (*simpliciter*), neither does the *probability-on-$\mathcal{M}$* of $p$ imply anything about $p$'s *probability* (*simpliciter*) — *if there be such a thing*.
- Just as we have different philosophical "theories" of truth, we will also have different (and analogous) philosophical "theories" of probability.
- And, as in the case of truth, there will be objective theories and subjective theories of probability. However, there will be more compelling reasons for "going subjective" in the probability case than in the truth case.
- Let's begin by looking at some objective theories of probability.

## Brief Digression on Basic Set Theory

- The statement "$a \in \mathbf{S}$" means that the object $a$ is a member of the set $\mathbf{S}$:
  - $1 \in \{1, 2, 3\}$, but $4 \notin \{1, 2, 3\}$.
- The statement "$\mathbf{X} \subseteq \mathbf{Y}$" means that the set $\mathbf{X}$ is a subset of the set $\mathbf{Y}$ (in other words, all members of the set $\mathbf{X}$ are members of the set $\mathbf{Y}$):
  - $\{1, 2\} \subseteq \{1, 2, 3\}$, but $\{1, 4\} \nsubseteq \{1, 2, 3\}$.
  - We use "$\mathbf{X} \subset \mathbf{Y}$" to say that $\mathbf{X}$ is a *proper* subset of $\mathbf{Y}$.
    - $\{1, 2, 3\} \subseteq \{1, 2, 3\}$, but $\{1, 2, 3\} \not\subset \{1, 2, 3\}$.
- We can characterize sets using the $\{\cdot \mid \cdot\}$ notation, for instance:
  - $\{a \mid a > 0 \,\&\, a \in \mathbb{Z}\}$ denotes the set of positive integers.
- $\mathbf{X} \cap \mathbf{Y}$ denotes the *intersection* of the sets $\mathbf{X}$ and $\mathbf{Y}$.
  - $\{1, 2, 3\} \cap \{2, 4, 6\} = \{2\}$, and $\{2, 4, 8\} \cap \{8, 2, 1\} = \{2, 8\}$.
- $\mathbf{X} \cup \mathbf{Y}$ denotes the *union* of the sets $\mathbf{X}$ and $\mathbf{Y}$.
  - $\{1, 2, 3\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 6\}$, and $\{2, 4, 8\} \cup \{8, 2, 1\} = \{4, 2, 8, 1\}$.

# Objective Theories of Probability (I)

- The simplest objective theory is the *actual (finite) frequency* theory.
- First, we must verify that actual frequencies in finite populations satisfy the probability axioms (otherwise, they aren't *probabilities* at all).
- Let $\mathbf{P}$ be an actual (non-empty, finite) population, let $\chi$ be a property, and let $\boldsymbol{\chi}$ denote the set of (all) objects that actually have property $\chi$.
- Let $\#(S) \stackrel{\text{def}}{=}$ the number of objects in a set $S$. Using $\#(\cdot)$, we can define the actual frequency of $\chi$ in such a population $\mathbf{P}$ in the following way:
  - $f_{\mathbf{P}}(\chi) \stackrel{\text{def}}{=} \dfrac{\#(\boldsymbol{\chi} \cap \mathbf{P})}{\#(\mathbf{P})}$
- Next, let $X$ be the proposition that an (arbitrary) object $a \in \mathbf{P}$ has property $\chi$. Using $f_{\mathbf{P}}(\chi)$, we can define $\Pr_{\mathbf{P}}(X)$, as follows:
  - $\Pr_{\mathbf{P}}(X) \stackrel{\text{def}}{=} f_{\mathbf{P}}(\chi)$.
- We need to show that $\Pr_{\mathbf{P}}(X)$ is in fact a *probability* function. There are various ways to do this. I will show that $\Pr_{\mathbf{P}}(X)$ satisfies our three axioms.

# Objective Theories of Probability (II)

- **Axiom 1.** We need to show that $\Pr_{\mathbf{P}}(X) \geq 0$, for any property $\chi$. This is easy, since the ratio $\frac{\#(\boldsymbol{\chi} \cap \mathbf{P})}{\#(\mathbf{P})}$ must be non-negative, for any property $\chi$. This is because $\mathbf{P}$ is non-empty [$\#(\mathbf{P}) > 0$], and $\#(\boldsymbol{\chi} \cap \mathbf{P})$ must be non-negative.
- **Axiom 2.** We need to show that, if $X \dashv\vDash \top$, then $\Pr_{\mathbf{P}}(X) = 1$. In this context, we're taking about properties $\chi$ that — by logic alone — must be satisfied by all objects in the universe (*e.g.*, $\chi x = Fx \vee \sim Fx$). In this case, we have $\boldsymbol{\chi} \cap \mathbf{P} = \mathbf{P}$, since *every* object is in $\boldsymbol{\chi}$. Therefore, $\Pr_{\mathbf{P}}(X) = \frac{\#(\mathbf{P})}{\#(\mathbf{P})} = 1$.
- **Axiom 3.** To be shown: If $X \,\&\, Y \dashv\vDash \bot$, then $\Pr_{\mathbf{P}}(X \vee Y) = \Pr_{\mathbf{P}}(X) + \Pr_{\mathbf{P}}(Y)$. In this context, $X \,\&\, Y \dashv\vDash \bot$ means we are talking about properties $\chi$ and $\psi$ such that — by logic alone — no object can satisfy both properties at once (*e.g.*, $\chi a \,\&\, \psi a \vDash \bot$). In such a case, we will have the following:

$$\Pr_{\mathbf{P}}(X \vee Y) = \frac{\#[(\boldsymbol{\chi} \cup \boldsymbol{\psi}) \cap \mathbf{P}]}{\#(\mathbf{P})} = \frac{\#[(\boldsymbol{\chi} \cap \mathbf{P}) \cup (\boldsymbol{\psi} \cap \mathbf{P})]}{\#(\mathbf{P})} = \frac{\#(\boldsymbol{\chi} \cap \mathbf{P}) + \#(\boldsymbol{\psi} \cap \mathbf{P})}{\#(\mathbf{P})}$$
$$= \Pr_{\mathbf{P}}(X) + \Pr_{\mathbf{P}}(Y)$$

## Objective Theories of Probability (III)

- OK, so actual frequencies in populations determine *probabilities*. But, they are rather peculiar probabilities, in several respects.
- First, they are *population-relative*. If an object $a$ is a member of multiple populations $\mathbf{P}_1, \ldots, \mathbf{P}_n$, then this may yield different values for $\Pr_{\mathbf{P}_1}(X)$, $\ldots$, $\Pr_{\mathbf{P}_n}(X)$. This is related to the *reference class problem* from last time.
- Another peculiarity of finite actual frequencies is that they sometimes seem to be misleading about intuitive objective probabilities.
- For instance, imagine tossing a coin $n$ times. This gives a population $\mathbf{P}$ of size $n$, and we can compute the $\mathbf{P}$-frequency-probability of heads $\Pr_{\mathbf{P}}(H)$.
- As $n$ gets larger, the value of this frequency tends to "settle down" to some small range of values (see *Mathematica* notebook). Intuitively, none of these finite actual frequencies is exactly equal to the bias of the coin.
- So, finite frequencies seem, at best, to provide "estimates" of probabilities in some deeper objective sense. What might such a "deeper sense" be?

## Objective Theories of Probability (IV)

- The *law of large numbers* ensures that (given certain underlying assumptions about the coin) the "settling down" we observe in many actual frequency cases (coin-tossing) will converge *in the limit* ($n \to \infty$).
- If we do have convergence to some value (say $\frac{1}{2}$ for a fair coin), then this value seems a better candidate for the "intuitive" objective probability. This leads to the *hypothetical limiting frequency theory* of probability.
- According to the hypothetical limiting frequency theory, probabilities are frequencies we *would* observe in a population — *if* that population were extended indefinitely (*e.g.*, if we were to toss the coin $\infty$ times).
- There are various problems with this theory. First, convergence is not always guaranteed. In fact, there are *many* hypothetical infinite extensions of any $\mathbf{P}$ for which the frequencies do *not* converge as $n \to \infty$.
- Second, even among those extensions that *do* converge, there can be *many different* possible convergent values. Which is "the" probability?

## Objective Theories of Probability (V)

- *Propensity* or *chance* theories of probability posit the existence of a deeper kind of physical probability, which manifests itself empirically in finite frequencies, and which constrains limiting frequencies.
- Having a theory that makes sense of quantum mechanical probabilities was one of the original inspirations of propensity theorists (Popper).
- In quantum mechanics, probability seems to be a fundamental physical property of certain systems. The theory entails exact *probabilities* of certain token events in certain experimental set-ups/contexts.
- These probabilities seem to transcend both finite and infinite frequencies. They seem to be basic *dispositional properties* of certain physical systems.
- In classical (deterministic) physics, all token events are *determined* by the physical laws + initial conditions of the universe. In quantum mechanics, only *probabilities* of token events are determined by the laws + i.c.'s.
- This leaves room for (non-extreme) *objective chances* of token events.

## Objective Theories of Probability (VI)

- We saw that finite frequencies satisfy the (classical) probability axioms.
- Infinite frequencies don't satisfy the (classical) axioms of (infinite) probability calculus, for two reasons (beyond the scope of our course).
  - The underlying (infinite) logical space is non-Boolean.
  - Infinite frequencies do not satisfy the (infinite) additivity axiom.
- Some have claimed that QM-probabilities are also non-classical, owing to the fact that the underlying "quantum *logic*" is non-Boolean. But, there are also interpretations of QM in terms of classical probabilities.
- It is often *assumed* that objective chances satisfy the probability axioms, but it is not quite clear *why* (especially, in light of the above remarks).
- Since I won't be dwelling on these sorts of objective (physical) theories of probability in this course, I won't fuss about these technical puzzles.
- Next, we'll discuss subjective probability, and we *will* dwell on that.

# Subjective Theories of Probability (I)

- I'll begin by motivating subjective probability with an example/context:

  I'm holding a coin behind my back. It is either 2-headed or 2-tailed. You do not know which kind of coin it is (and you have no reason to favor one of these possibilities over the other). I'm about to toss it. What probability (or odds) would you assign to the proposition that it will land heads?

- Many people have the intuition that $\frac{1}{2}$ (or even 50:50 odds) would be a reasonable answer to this question. However, it seems clear that $\frac{1}{2}$ *cannot* be the *objective* probability/chance of heads in this example.
- After all, we know that the coin is either 2-headed or 2-tailed. As such, the objective probability of heads is either 1 or 0 in this example.
- One might describe this as *epistemic* probability, because it seems *epistemically reasonable* to be *50% confident* that the coin will land heads.
- Also, taking a bet at even odds on heads seems *pragmatically* reasonable. This suggests a *pragmatic* theory of probability is also plausible here.

## Subjective Theories of Probability (II)

- It seems clear that there is such a thing as "degree of belief". And, it also seems clear that there are *some* sorts of constraints on such degrees.
- But, why should degrees of belief obey the *probability axioms*?
- There are arguments that epistemic and pragmatic probabilities should each be *probabilities*. We will examine examples of each type of argument.
- We will begin with *pragmatic* subjective probability.
- There are various arguments for *pragmatic probabilism*: that *pragmatically rational* degrees of belief obey the probability axioms.
- All such arguments must do two things:
  - Identify a *necessary condition* ($N$) for pragmatic rationality.
  - Show that having non-probabilistic degrees of belief *violates* $N$.
- The first argument we will examine is the Dutch Book Argument (DBA).
- But, first, a "doxastic framework" for talking about rational agents.

## A Doxastic Framework (I)

- We will assume that rational agents have attitudes toward propositions. One of these attitudes is *belief*. What is belief? This is not entirely clear.
- We will say that belief is a relation between an agent $S$ and a proposition $p$. We needn't worry too much about the precise conditions under which $S$ believes that $p$. Intuitively, belief is *dispositional* property.
- When $S$ believes $p$, this will be accompanied by various dispositions to behave in certain ways: to provide arguments in favor of $p$ should it be challenged, to act in accordance with the assumption that $p$ is true, *etc*.
- We will not assume that an agent must actively (or consciously) *attend to* $p$ in order to believe it. And, we'll make some rather strong assumptions about the *logical structure* of a rational agent's doxastic state.
- We will assume a certain kind of *logical omniscience* concerning the space of *entertainable* propositions. This is to be distinguished from logical omniscience concerning the space of *believed* propositions.

# A Doxastic Framework (II)

- We'll assume that, at each time $t$, an ideally rational agent $S$ has a *doxastic state*, which includes a *Boolean algebra of propositions* $\mathcal{B}_S^t$ — those propositions that are *entertainable* for $S$ at $t$ (those $S$ has "access" to at $t$).
- $\mathcal{B}_S^t$ includes all the propositions that are *candidates* for belief by $S$ at $t$. $S$ will only believe *some subset* of this set of propositions at $t$. But, each member of $\mathcal{B}_S^t$ is, in some sense, a "live option" as a belief for $S$ at $t$.
- The idea here is that some propositions are not even in the realm of *possible* belief for $S$ at $t$. $S$ may not possess the requisite concepts to form a belief in $p$ at $t$. Or, if $p$'s possibility *couldn't occur to $S$* at $t$.
- Restricting the set of candidate beliefs (at $t$) to some proper subset of *all* the propositions seems right. But, why assume $\mathcal{B}_S^t$ is a *Boolean algebra*? [This implies (among other things) *closure* under Boolean operations.]
- For now, we'll assume this form of *logical omniscience*. Note: we're *not* assuming that the agent's *beliefs* are closed under the logical operations!

# A Doxastic Framework (III)

- So far, we've talked about *full* belief. We're thinking of this as a *relation* between the agent $S$ and propositions $p \in \mathcal{B}_S^t$ in their doxastic state. What about *degrees* of belief (*degrees* of confidence, *degrees* of credence, *etc.*)?
- We can think of degree of belief as a *quantitative* (or *comparative*) generalization of full belief. It is a *quantitative* (or *comparative*) relation between $S$ and propositions $p \in \mathcal{B}_S^t$ in their doxastic state. Examples:
  - The *degree* to which $S$ believes $p$ at $t$ is $x$.
  - $S$ believes $p$ *more strongly than* $S$ believes $q$ at $t$.
- Intuitively, we do say things like this. I am more confident that Bush is president than I am that it will rain tomorrow (even if I don't have *degrees* of belief here). I'd place (roughly) even odds on the toss of a fair coin.
- What we'll be talking about next are these kinds of claims. Probabilism is the view that *degrees of belief should be probabilities*. That is, *that the degrees of belief of a rational agent $S$ at $t$ are **probabilities** over $\mathcal{B}_S^t$*.

## A Doxastic Framework (IV)

- What's the difference between the *pragmatic* rationality of doxastic states (including beliefs, degrees of belief, *etc.*) and *epistemic* rationality thereof?
- There is psychological evidence that (actual) agents $S$ tend to perform better at certain activities $\phi$ if they *believe* that ($p_S^\phi$) $S$ is very good at $\phi$-ing. This can remain true even when the belief is *unjustified*.
- A case could be made that it would (in some cases) be *pragmatically* rational for (some) $S$ to believe that $p_S^\phi$, even when such a belief is not supported by $S$'s evidence. But, this seems *epistemically ir*rational.
- Simpler example: I offer you $1M to believe that the number of pebbles on Pebble Beach is *exactly* $10^{12}$. You have no evidence for this claim, and otherwise no reason to believe it. But, you *really* value money, *etc., etc*.
- We'll bracket questions about whether beliefs (d.o.b.'s) are the *kinds* of things you *can choose* to have. We'll think of this in *evaluative* rather than *normative* terms, and (for now) as about *states* rather than *processes*.

## A Doxastic Framework (V)

- When we make judgments about rationality, we can take two “stances”:
  - **Evaluative Stance**: Here, we’re merely *evaluating* some state or process against some standard(s) of ideal rationality. We’re not making any claims about what anyone *ought to do* (not *advising*, *blaming*, *etc*).
  - **Normative Stance**: Here, we are talking about what some agent(s) *ought to do*, from the point of view of some standard(s) of ideal (*normative*!) rationality. Here, we *do advise*, *prescribe*, *blame*, *etc*.
- And, we can be making judgments about *states* or *processes*:
  - **State Judgments**: These are judgments about the rationality (“goodness”) of (some aspect of) the *doxastic state* of $S$ at $t$.
  - **Process Judgments**: Judgments of the rationality (“goodness”) of some *process* leading $S$ from one doxastic state (at $t$) to another (at $t' > t$).
- We will be involved mainly with in *evaluation* of doxastic *states*. [We’ll say a little about doxastic processes, but also from an *evaluative stance*.]

## A Doxastic Framework (VI)

- Examples of some evaluative doxastic claims/principles ("norms"):
  1. Logically *consistent* belief states are better than inconsistent states.
  2. If a belief state includes $\ulcorner p \urcorner$ and $\ulcorner p \to q \urcorner$, then it would be better if it contained $\ulcorner q \urcorner$ and did not contain $\ulcorner {\sim}q \urcorner$ (than it would otherwise be).
  3. Degree-of-belief states that can be accurately represented as *probability models*, are better than those which cannot be.
  4. *If* an ideally rational agent $S$ satisfies the following two conditions:
     (*i*) $S$'s doxastic state at $t$ can be represented as a Pr-model $\langle \mathcal{B}_S^t, \Pr_S^t \rangle$,
     (*ii*) Between $t$ and $t'$, $S$ learns $q$ *and nothing else* (where $q$ is in $\mathcal{B}_S^t$),

     *then*, (*iii*) the *ideal doxastic state* for $S$ at $t'$ is $\langle \mathcal{B}_S^{t'}, \Pr_S^{t'} \rangle$, where $\Pr_S^{t'}(\bullet) = \Pr_S^t(\bullet \mid q)$. [d.o.b.–updating goes by *conditionalization*.]
- (1) and (2) are evaluative norms for (full) *belief* states. (3) and (4) are evaluative norms for *degree-of-belief* states (or *sequences* of them). We'll focus on (3) [and (4)], which will require also thinking about (1) and (2).