Philosophy 148 — Announcements & Such

• Branden will not be having office hours today (May 6).

• **New Plan for HW #5**
  - It will be due on the last day of class — this Thursday 5/8.
  - Our HW #5 discussion will be **Tonight 5/6 @ 6pm @ 110 Wheeler**.

• I will also be preparing some final extra-credit problems. They will be distributed Thursday, and due at the final exam (5/20 @ 8am).

• The final exam is **Tuesday, May 20 @ 8am @ 20 Barrows**.
  - I will hold a review session for the final exam — the day before the final (May 19). It will take place **May 19 @ 4–6pm @ 122 Wheeler**.
  - I will also be distributing a “practice final” later this week.

• Today’s Agenda (and next time too)
  - The Grue Paradox
Carnapian confirmation (i.e., later Carnapian theory [13] — see “Extras”) is based on **probabilistic relevance**, not entailment:

- $E$ confirms $H$, relative to $K$ iff $\Pr(H \mid E \& K) > \Pr(H \mid K)$, for some “suitable” conditional probability function $\Pr(\cdot \mid \cdot)$.

  - Note how this is an *explicitly* 3-place relation. Hempel’s was only 2-place. This is because $\Pr$ (unlike $\models$) is **non-monotonic**.

  - Carnap thought that “suitable $\Pr$” meant “logical $\Pr$” in a rather strong sense (see “Extras”). However, Goodman’s argument will work against *any* probability function $\Pr$.

**Carnap’s theory implies only 1 of our 3 Hempelian claims:**

(EQC). It does *not* imply (NC) or (M) (see “Extras” & [3]/[13]).

- This will allow Carnapian IL to avoid facing the full brunt of Goodman’s “grue” (but, it will still face a serious challenge).

  - For Carnap, confirmation is a *logical* relation (akin to entailment). Like entailment, confirmation can be *applied*, but this requires *epistemic bridge principles* [akin to (2)].

  - Carnap [1] discusses various bridge principles. The most well-known of these is the *requirement of total evidence*.
The Requirement of Total Evidence. In the application of IL to a given knowledge situation, the total evidence available must be taken as a basis for determining the degree of confirmation.

This *sounds* like a plausible principle. But, once it is made more precise, it will actually turn out to be subtly defective. More precisely, we have the following *bridge principle* connecting *confirmation* and *evidential support*:

\[(RTE)\] \( E \) evidentially supports \( H \) *for* \( S \) in \( C \) iff \( E \) confirms \( H \), relative to \( K \), where \( K \) is \( S \)'s *total evidence* in \( C \).

The (RTE) has often been (implicitly) presupposed by Bayesian epistemologists (both subjective and objective). However, as we will soon see, the (RTE) is not a tenable bridge principle, and for reasons independent of “grue”.

Moreover, Goodman’s “grue” argument will rely *more heavily* on (RTE) than the relevantists’ argument relies on (2). In this sense, Goodman’s argument will be even worse.

Before reconstructing the argument, a brief “grue” primer.
Let $Gx \overset{\text{def}}{=} x$ is green, $Ox \overset{\text{def}}{=} x$ is examined prior to $t$, and $Ex \overset{\text{def}}{=} x$ is an emerald. Goodman introduces a predicate “grue”

$Gx \overset{\text{def}}{=} x$ is grue $\overset{\text{def}}{=} Ox \equiv Gx$.

Consider the following two universal generalizations

$(H_1)$ All emeralds are green. $[(\forall x)(Ex \supset Gx)]$

$(H_2)$ All emeralds are grue. $[(\forall x)[Ex \supset (Ox \equiv Gx)]]$

And, consider the following instantial evidential statement

$(E) Ea \& Oa \& Ga$

Hempel’s confirmation theory $[(EQC) \& (NC) \& (M)]$ entails:

$(†) E$ confirms $H_1$, and $E$ confirms $H_2$. [▶proof]

As a result, his theory entails the following weaker claim

$(‡) E$ confirms $H_1$ if and only if $E$ confirms $H_2$.

What about (later) Carnapian theory? Does it entail even $(‡)$?

Interestingly, NO! There are (later) Carnapian Pr-models in which $E$ confirms $H_1$ but $E$ disconfirms $H_2$ (see “Extras”).

In this sense, Hempel was an easier target for Goodman than Carnap (Goodman claims to be attacking both).

Now, we’re ready to reconstruct Goodman’s argument.
A Proof of (†) From Hempel’s (NC), (M), and (EQC)

\[ (\forall x)(Ex \supset Gx) \]

\[ (\forall x)[Ex \supset (Ox \equiv Gx)] \]

\[ Ea \& Ga \]

\[ Ea \& (Oa \equiv Ga) \]

\[ (Ea \& (Oa \equiv Ga)) \& Oa \]

\[ Ea \& Oa \& Ga = E \]

(NC) \uparrow

(M) \uparrow

(EQC) \uparrow

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There is just one more ingredient in Goodman's argument:

- The agent $S$ who is assessing the evidential support that $E$ provides for $H_1$ vs $H_2$ in a Goodmanian “grue” context $C_G$ has $Oa$ as part of their total evidence in $C_G$. (e.g., [14].)

Now, we can run the following Goodmanian reductio:

1. $E$ confirms $H$, relative to $K$ iff $\Pr(H \mid E \& K) > \Pr(H \mid K)$.
2. $E$ evidentially supports $H$ for $S$ in $C$ iff $E$ confirms $H$, relative to $K$, where $K$ is $S$'s total evidence in $C$.
3. The agent $S$ who is assessing the evidential support $E$ provides for $H_1$ vs $H_2$ in a Goodmanian “grue” context $C_G$ has $Oa$ as part of their total evidence in $C_G$ [i.e., $K \models Oa$].
4. If $K \models Oa$, then—c.p.—$E$ confirms $H_1$ relative to $K$ iff $E$ confirms $H_2$ relative to $K$, for any $\Pr$ [i.e., ($\ddagger$) holds, $\forall \Pr$'s].
5. Therefore, $E$ evidentially supports $H_1$ for $S$ in $C_G$ if and only if $E$ evidentially supports $H_2$ for $S$ in $C_G$.

∴ (i)–(vi) lead to an absurdity. Hence, our initial assumption (i) must have been false. Carnapian inductive logic refuted?
Premise (vi) is based on Goodman’s *epistemic intuition* that, in “grue” contexts, \( E \) evidentially supports \( H_1 \) but not \( H_2 \).

Premise (v) follows logically from premises (i)-(iv).

Premise (iv) is a theorem of probability calculus (*any* \( \text{Pr}! \)).

The *c.p.* clause needed is \( \text{Pr}(Ea | H_1 & K) = \text{Pr}(Ea | H_2 & K) \), which is assumed in all probabilistic renditions of “grue”.

Premise (iii) is an assumption about the agent’s background knowledge \( K \) that’s implicit in Goodman’s set-up. See [14].

Premise (ii) is (RTE). It’s the *bridge principle*, akin to (2) in the relevantists’ *reductio*. This is the premise I will focus on.

Here are my two main points about Goodman’s argument:

(ii) must be rejected by Bayesians for independent reasons.

Carnapian confirmation theory *doesn't even entail* (‡).

[Hempel’s theory does, just as deductive logic entails (1).]

This suggests Goodman’s argument is *even less* a *reductio* of (i) than the relevantists’ argument is a *reductio* of (1).

Next, I will explain why Carnapians/Bayesians should reject (ii) on *independent* grounds: The Problem of Old Evidence.
As Tim Willimson points out [16, ch. 9], Carnap’s (RTE) must be rejected, because of the problem of old evidence [2].

If S’s total evidence in C (K) entails E, then, according to (RTE), E cannot evidentially support any H for S in C.

As a result, one cannot (in all contexts) use Pr(· | K) — for any Pr — when assessing the evidential import of E.

There are (basically) two kinds of strategies for revising (RTE). Carnap [1, p. 472] & Williamson [16, ch. 9] propose:

\[(\text{RTE}_\top) \quad E \text{ evidentially supports } H \text{ for } S \text{ in } C \text{ iff } S \text{ possesses } E \text{ as evidence in } C \text{ and } \Pr_\top(H \mid E \& K_\top) > \Pr_\top(H \mid K_\top). \]

[K_\top is a priori, Pr_\top is “inductive” [13]/“evidential” [16]/“logical” [1].]

Note: Hempel explicitly required that confirmation be taken “relative to K_\top” in all treatments of the paradoxes [9, 10].

(RTE_\top) is a charitable Carnapian reconstruction of Hempel.

A more “standard” way to revise (RTE) is [(RTE')] to use \( \Pr_{S'}(\cdot \mid K') \), where \( K \models K' \neq E \), and \( \Pr_{S'} \) is the credence function of a “counterpart” \( S' \) of \( S \) with total evidence \( K' \).
Carnap never re-wrote the part of LFP [1] that discusses the (RTE), in light of a probabilistic relevance (“increase in firmness” [1]) notion of confirmation. This is too bad.

If Carnap had discussed this (“old evidence”) issue, I suspect he would have used something like Williamson’s (RTEiations) as his bridge principle connecting confirmation and evidence.

Various other philosophers have proposed similar accounts of “support” as some probabilistic relation, taken relative to an “informationless” or “a priori” background/probability.

- Richard Fumerton (who, unlike Williamson, is an epistemological internalist) proposes such a view in his [4].
- Patrick Maher [13] applies such relations extensively in his recent (neo-Carnapian) work on confirmation theory.

So, many Bayesians already reject (RTE). [Of course, “grue” gives Bayesians another important reason to reject (RTE).]
So far, I have left open (precisely) what I think Bayesian confirmation theorists should say (logically & epistemically) in light of Goodman’s “grue” paradox (but, see “Extras”).

Clearly, BCTs will need to revise (RTE) in light of “grue”. But, the standard (RTE’) way of doing this to cope with “old evidence” isn’t powerful enough to avoid both problems.

Williamson’s (RTE ⊤) revision of (RTE) — also suggested by Carnap — avoids both problems, from a logical point of view (if “inductive”/“logical”/“evidential” probabilities exist!). But, what should BCTs say on the epistemic side?

I don’t have a fully satisfactory answer to this question (yet). But, I remain unconvinced that the epistemic problem (if there is one) is caused by the “non-naturalness” of “grue”.

The problem, I suspect, may involve an observation selection effect: we know something about the “grue” observation process that undermines (or defeats) evidence it produces.

I hope we can discuss this (and IL) in the Q&A (see “Extras”).


“Carnapian” Counterexamples to (NC) and (M)

(K) Either: (H) there are 100 black ravens, no nonblack ravens, and 1 million other things, or (∼H) there are 1,000 black ravens, 1 white raven, and 1 million other things.

Let \( E \equiv Ra \& Ba \) (\( a \) randomly sampled from universe). Then:

\[
\Pr(E \mid H \& K) = \frac{100}{1000100} \ll \frac{1000}{1001001} = \Pr(E \mid \sim H \& K)
\]

∴ This \( K/Pr \) constitute a counterexample to (NC), assuming a “Carnapian” theory of confirmation. This model can be emulated in the later Carnapian \( \lambda/\gamma \)-systems [13].

Let \( Bx \equiv x \) is a black card, \( Ax \equiv x \) is the ace of spades, \( Jx \equiv x \) is the jack of clubs, and \( K \equiv \) a card \( a \) is sampled at random from a standard deck (where \( \Pr \) is also standard):

- \( \Pr(Aa \mid Ba \& K) = \frac{1}{26} > \frac{1}{52} = \Pr(Aa \mid K) \).
- \( \Pr(Aa \mid Ba \& Ja \& K) = 0 < \frac{1}{52} = \Pr(Aa \mid K) \).
A "Carnapian" Counterexample to (‡)

(K) Either: (H₁) there are 1000 green emeralds 900 of which have been examined before t, no non-green emeralds, and 1 million other things in the universe, or (H₂) there are 100 green emeralds that have been examined before t, no green emeralds that have not been examined before t, 900 non-green emeralds that have not been examined before t, and 1 million other things.

Imagine an urn containing true descriptions of each object in the universe (Pr ≡ urn model). Let \( E \equiv "Ea \& Oa \& Ga" \) be drawn. \( E \) confirms \( H₁ \) but \( E \) disconfirms \( H₂ \), relative to \( K \):

\[
\Pr(E \mid H₁ \& K) = \frac{900}{1001000} > \frac{100}{1001000} = \Pr(E \mid H₂ \& K)
\]

This \( K/\Pr \) constitute a counterexample to (‡), assuming a "Carnapian" theory of confirmation. This probability model can be emulated in the later Carnapian \( \lambda/\gamma \)-systems [13].
Is “Grue” an Observation Selection Effect? Part I

- **Canonical Example of an OSE:** I use a fishing net to capture samples of fish from various (randomly selected) parts of a lake. Let $E$ be the claim that all of the sampled fish were over one foot in length. Let $H$ be the hypothesis that all the fish in the lake are over one foot \[ ((\forall x)((Fx \& Lx) \supset Ox)) \].

  Intuitively, one might think $E$ should evidentially support $H$. This may be so for an agent who knows only the above information ($K$) about the observation process. That is, it seems plausible that $\Pr(E \mid H \& K) > \Pr(E \mid \sim H \& K)$, where $\Pr$ is taken to be “evidential” (or “epistemic”) probability.

- But, what if I also tell you that ($D$) the net I used to sample the fish from the lake (which generated $E$) has holes that are all over one foot in diameter? It seems that $D$ defeats the support $E$ provides for $H$ (relative to $K$), because $D$ ensures $O$. Thus, intuitively, $\Pr(E \mid H \& D \& K) = \Pr(E \mid \sim H \& D \& K)$. 

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Note: the “grue” hypothesis \( H_2 \) entails the following claim, which is not entailed by the green hypothesis \( H_1 \):

\[ (H') \text{ All green emeralds have been (or will have been) examined prior to } t. \]  
\[ [(\forall x)(Ex \& Gx) \supset Ox)] \]

Now, consider the following two observation processes:

- **Process 1.** For each green emerald in the universe, a slip of paper is created, on which is written a true description of that object as to whether it has property \( O \). All the slips are placed in an urn, and one slip is sampled at random from the urn. By *this* process, we learn \( E \) that \( Ea \& Ga \& Oa \).

- **Process 2.** Suppose all the green emeralds in the universe are placed in an urn. We sample an emerald \( a \) at random from this urn, and we examine it. [We know *antecedently* that the examination of \( a \) will take place prior to \( t \), *i.e.*, that \( Oa \) is true.] By *this* process, we learn \( E \) that \( Ea \& Ga \& Oa \).

Goodman seems to presuppose Process 2 in his set-up.
What Could “Carnapian” Inductive Logic Be? Part I

- The early Carnap dreamt that probabilistic inductive logic (confirmation theory) could be formulated in such a way that it *supervenes* on deductive logic in a *very strong* sense.
  - **Strong Supervenience** (SS). All confirmation relations involving sentences of a first-order language $\mathcal{L}$ supervene on the deductive relations involving sentences *of* $\mathcal{L}$.
- Hempel clearly saw (SS) as a *desideratum* for confirmation theory. The early Carnap also seems to have (SS) in mind.
- I think it is fair to say that Carnap’s project — understood as requiring (SS) — was unsuccessful. [I think *this* is true for reasons that are *independent* of “grue” considerations.]
- The later Carnap seems to be aware of this. Most commentators interpret this shift as the later Carnap simply *giving up* on inductive logic (*qua logic*) altogether.
- I want to resist this “standard” reading of the history.
I propose a different reading of the later Carnap, which makes him much more coherent with the early Carnap.

I propose *weakening* the supervenience requirement in such a way that it (a) ensures this coherence, and (b) maintains the “logicality” of confirmation relations in Carnap’s sense.

Let $\mathcal{L}$ be a formal language strong enough to express the fragment of probability theory Carnap needs for his later, more sophisticated confirmation-theoretic framework.

- **Weak Supervenience** (WS). All confirmation relations involving sentences of a first-order language $\mathcal{L}$ supervene on the deductive relations involving sentences of $\mathcal{L}$.

As it turns out, $\mathcal{L}$ needn’t be very strong (in fact, one can get away with PRA!). So, even by early (logicist) Carnapian lights, satisfying (WS) is all that is *really* required for “logicality”.

The specific (WS) approach I propose takes confirmation to be a *four*-place relation: between $E$, $H$, $K$, and a function $\Pr$. 
Consequences of moving to a 4-place confirmation relation:

- We need not try to “construct” “logical” probability functions from the syntax of $\mathcal{L}$. This is a dead-end anyhow.
- Indeed, on this view, inductive logic has nothing to say about the interpretation/origin of Pr. That is not a logical question, but a question about the application of logic.
  - Analogy: Deductive logicians don’t owe us a “logical interpretation” of the truth value assignment function $v$.
- Moreover, this leads to a vast increase in the generality of inductive logic. Carnap was stuck with an impoverished set of “logical” probability functions (in his $\lambda/\gamma$-continuum).
  - On my approach, any probability function can be part of a confirmation relation. Which functions are “suitable” or “appropriate” or “interesting” will depend on applications.
  - So, some confirmation relations will not be “interesting”, etc. But, this is (already) true of entailments, as Harman showed.
- Questions: Now, what is the job of the inductive logician, and how (if at all) do they interact with epistemologists?
What Could “Carnapian” Inductive Logic Be? Part IV

- The inductive logician must explain how it is that inductive logic can satisfy the following Carnapian desiderata.
  - The confirmation function \( c(H, E \mid K) \) quantifies a logical (in a Carnapian sense) relation among statements \( E, H, \) and \( K \).
    - (\( D_1 \)) One aspect of “logicality” is ensured by moving from (SS) to (WS) [from an \( \mathcal{L} \)-determinate to an \( \mathcal{L} \)-determinate concept].
    - (\( D_2 \)) Another aspect of “logicality” insisted upon by Carnap is that \( c(H, E \mid K) \) should generalize the entailment relation.
      - This means (at least) that we need \( c(H, E \mid K) \) to take a maximum (minimum) value when \( E \& K \models H \) (\( E \& K \models \sim H \)).
      - Very few relevance measures \( c \) satisfy this “generalizing \( \models \)” requirement. That’s another job for the inductive logician.
    - (\( D_3 \)) There must be some interesting “bridge principles” linking \( c \) and some relations of evidential support, in some contexts.
      - (\( D_2 \)) implies that if there are any such bridge principles linking entailment and conclusive evidence, these should be inherited by \( c \). This brings us back to Harman’s problem!
Three Salient Quotes from Goodman [7]

The “new riddle” is about inductive logic (not epistemology).

Quote #1 (page 67): “Just as deductive logic is concerned primarily with a relation between statements — namely the consequence relation — that is independent of their truth or falsity, so inductive logic …is concerned primarily with a comparable relation of confirmation between statements. Thus the problem is to define the relation that obtains between any statement $S_1$ and another $S_2$ if and only if $S_1$ may properly be said to confirm $S_2$ in any degree.”

Quote #2 (73): “Confirmation of a hypothesis by an instance depends …upon features of the hypothesis other than its syntactical form”.

But, Goodman’s methodology appeals to epistemic intuitions.

Quote #3 (page 73): “…the fact that a given man now in this room is a third son does not increase the credibility of statements asserting that other men now in this room are third sons, and so does not confirm the hypothesis that all men now in this room are third sons.”