Philosophy 148 — Announcements & Such

- HW #4 grades posted ($\mu = 75$). [This one was tougher than I thought.]
- New Plan for HW #5 (owing to my flu)
  - It will be due on the last day of class — next Thursday 5/8.
  - Our discussion for it will be next Tuesday 5/6 @ 6pm (room TBA).
- I will also be preparing some final extra-credit problems. They will be distributed next week, and due at the final exam (5/20 @ 8am).
- The final exam is **Tuesday, May 20 @ 8am @ 20 Barrows**.
  - I will hold a review session the day before the final (May 19). Would a time in the afternoon (say 4-6pm) work for people?

Today’s Agenda
- The Raven Paradox (cont’d)
- Next: The Grue Paradox

Digression on Bayesian Confirmation Theory

- As we have already seen for Carnap’s confirms, Bayesian confirmation theory will accept some of Hempel’s desiderata, and reject others.
- The EQC, the EC, and the NTC all seem quite intuitive, and they are satisfied by any probabilistic account of confirmation.
- CC is not intuitive. Typically, competing theories are not consistent (they’re mutually exclusive). Let $K$ describe the typical probability model of a standard deck of 52 cards. Then, consider the following examples.
  
  - $E = \text{card is black}, H = \text{card is the A}\spadesuit, H' = \text{card is the J}\spadesuit$. Intuitively, $E$ confirms both $H$ and $H'$ (rel. to $K$), even though they are inconsistent.
  
  - SCC is not intuitive either. Many intuitive counterexamples are out there. E.g., let $E = \text{card is black}, H = \text{card is the A}\spadesuit, H' = \text{card is an ace}.$

As for CCC, it is highly unintuitive (here, we agree with Hempel). E.g., let $E = \text{card is the A}\spadesuit, H = \text{card is card is an ace}, H' = \text{card is the A}\spadesuit.$

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Qualitative, Comparative, and Quantitative Confirmation

- $E$ confirms $H$, relative to $K$ iff
  - Nicod: $E$ is a syntactical instance of $H$ (only applies to universal $H$s).
  - Hempel: $E \& K \models dev(H) \Rightarrow I(E) = \text{individuals mentioned in } E$.
  - H-D: $H \& K = E$.
  - Absolute Bayes (confirms): $\Pr(H \mid E \& K) > r$, for some “threshold” $r$.
  - Incremental Bayes (confirms): $\Pr(H \mid E \& K) > \Pr(H \mid K)$.

- $E$ confirms $H$ rel. to $K$ more strongly than $E'$ confirms $H'$ rel. to $K'$ iff
  - Nicod, Hempel, H-D: No accounts (purely qualitative theories).
  - Absolute Bayes: $\Pr(H \mid E \& K) > \Pr(H' \mid E' \& K')$.
  - Incremental Bayes: $c(H, E \mid K) > c(H', E' \mid K')$ [some $\mathcal{X}$-measure $c$].

- $E$ confirms $H$ relative to $K$ to degree $\alpha$:
  - Nicod, Hempel, H-D: No accounts (purely qualitative theories).
  - Absolute Bayes: $f(\Pr(H \mid E \& K), r) = \alpha$ [some function $f$, thresh. $r$]
  - Incremental Bayes: $c(H, E \mid K) = \alpha$ [some $\mathcal{X}$-measure $c$].

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...back from our digression to Bayesian responses to the raven paradox ...

- Bayesians have said a great many (wildly different) things about Hempel’s paradox. Almost all Bayesian approaches fall into at least one of the following three categories [{“$H$” is short for “$\forall x(Rx \supset Bx)$”, “$E_1$” for “$Ra \& Ba$” (some $a$ drawn from the universe), and “$E_2$” for “$\sim Ba \& \sim Ra$”}):

  - **Qualitative**. Reject some precise, Bayesian rendition of (NC), on Bayesian grounds. Strictly speaking, this does not require the rejection of the corresponding rendition of (PC) [but this is often rejected too].
  
  - **Comparative**. Argue that $c(H, E_1 \mid K_\alpha) > c(H, E_2 \mid K_\alpha)$, for our actual background knowledge $K_\alpha$. Traditionally, these approaches accept (PC) and do not deny (NC) — they entail $c(H, E_1 \mid K_\alpha) > c(H, E_2 \mid K_\alpha) > 0$.
  
  - **Quantitative**. Argue that $c(H, E_2 \mid K_\alpha)$ is “minute”, for our actual background knowledge $K_\alpha$. Traditionally, these approaches go hand in hand with the comparative approaches. They typically aim to show both that $c(H, E_1 \mid K_\alpha) \gg c(H, E_2 \mid K_\alpha) > 0$ and that $c(H, E_2 \mid K_\alpha) \approx 0$.

- Next, I’ll discuss the tradition, and then describe a new approach.
All Bayesian approaches begin by precisifying (NC) [and (PC)]. Since Bayesian confirmation is a three-place relation \( \mathcal{C}(H,E \mid K) \), we'll need a quantifier over the implicit K's in (NC). Four renditions:

\[
\begin{align*}
&\text{(NC}_w\text{)} \quad (\exists K)(\forall F)(\forall G)(\forall x)[\mathcal{C}(\forall x)(Fx \supset Gx), Fx & \& Gx \mid K] \\
&\text{(NC}_a\text{)} \quad (\forall F)(\forall G)(\forall x)[\mathcal{C}(\forall x)(Fx \supset Gx), Fx & \& Gx \mid K\alpha] \\
&\text{(NC}_T\text{)} \quad (\forall F)(\forall G)(\forall x)[\mathcal{C}(\forall x)(Fx \supset Gx), Fx & \& Gx \mid K_T] \\
&\text{(NC}_c\text{)} \quad (\forall K)(\forall F)(\forall G)(\forall x)[\mathcal{C}(\forall x)(Fx \supset Gx), Fx & \& Gx \mid K] \\
&\text{(NC}_w\text{)} \text{ is too weak} [\text{let } K = \text{“all instances confirm all generalizations”}].
\end{align*}
\]

Hempel's “explaining away” suggests (NCc) should be too strong.

So (NCa) and (NC_T) seem to be the salient renditions of (NC).

Qualitative Bayesians seek to refute some rendition of (NC).

The question for qualitative approaches is which (NC) to refute.

Early qualitative Bayesian approaches took aim at (NCa).

I.J. Good showed that the strong (Bayesian) rendition \((\text{NC}_s)\) of Nicod's condition is false. He gave the following counterexample:

\[ K: \text{Exactly one of the following two hypotheses is true: } (H) \text{ there are } 100 \text{ black ravens, no nonblack ravens, and } 1 \text{M other birds, or } (\sim H) \text{ there are } 1,000 \text{ black ravens, } 1 \text{ white raven, and } 1 \text{M other birds.} \]

\[ E: Ra \& Ba \quad (a \text{ is randomly sampled from the universe).} \]

\[ \Pr(E \mid H \& K) = \frac{100}{1000100} \ll \frac{1000}{1001001} = \Pr(E \mid \sim H \& K) \]

So, (NCa) is false, and even for “natural kinds” (pace Quine). Similar examples can be generated to show that (PCa) and Scheffler's \((^*\text{a})\) are false.

So? Hempel replies that (NC_T) not (NCc) is the salient rendition. That's plausible, but as we have seen it's incompatible with Hempel's theory!

Nonetheless, Good later tried to meet Hempel's (NC_T) challenge.

He gave the following example, which is known as “Good’s Baby”...

There have been many comparative Bayesian approaches. Here is the canonical, contemporary, comparative approach.

Assume that our actual background corpus \(K\alpha\) is such that:

\[
\begin{align*}
&\text{(1) } \Pr(\sim Ba \mid K\alpha) > \Pr(Ra \mid K\alpha) \\
&\text{(2) } \Pr(Ra \mid H \& K\alpha) = \Pr(Ra \mid K\alpha) \quad [\because \sim Ra \notin H \mid K\alpha (0)] \\
&\text{(3) } \Pr(\sim Ba \mid H \& K\alpha) = \Pr(\sim Ba \mid K\alpha) \quad [\because Ba \notin H \mid K\alpha (0)] \\
\end{align*}
\]

**Theorem.** Any \(\Pr\) satisfying (1), (2) and (3) will also be such that:

\[
\text{(4) } \Pr(H \mid Ra \& Ba \& K\alpha) > \Pr(H \mid \sim Ba \& \sim Ra \& K\alpha). \\
\]

By (4), the observation of a black raven \(\vdash\text{ confirms that all ravens are black more strongly than}\) the observation of a white shoe, provided:

\[
\begin{align*}
\text{(1) there are (proportionally) fewer ravens than non-black things} \\
\text{(2)/(3) whether something } a \text{ (sampled at random from the universe) is a} \\
\text{raven/black is independent of whether all ravens are black } H. \\
\end{align*}
\]

As it turns out, assumptions (1)-(3) entail a lot more than just (4) . . .
• Assumptions (1)–(3) also entail:
  (5) \( \Pr(H | Ra & Ba & K_\alpha) > \Pr(H | K_\alpha) \)
  (6) \( \Pr(H | \sim Ba & \sim Ra & K_\alpha) > \Pr(H | K_\alpha) \)
  (7) \( \Pr(H | Ba & \sim Ra & K_\alpha) < \Pr(H | K_\alpha) \)

  • I.e., the canonical comparative Bayesian assumptions entail the qualitative claims that \( Ra & Ba \) and (PC) \( \sim Ba & \sim Ra \) confirm \( H \), which is inconsistent with denying the salient instances of \( \langle NC_\alpha \rangle \). And, (7) is pretty crazy, no?

  • So (i) this cannot undergird a purely comparative approach – one that is consistent with a qualitative approach, and (ii) it entails (7), which is bad.

  • A canonical quantitative approach is had by strengthening (1) to:
    \( \Pr(\sim Ba | K_\alpha) \gg \Pr(Ra | K_\alpha) \)
    \( (1') \sim (3) \Rightarrow \Pr(H | Ra & Ba & K_\alpha) \gg \Pr(H | \sim Ba & \sim Ra & K_\alpha) > \Pr(H | K_\alpha) \), and
    \( \Pr(H | \sim Ba & \sim Ra & K_\alpha) \approx \Pr(H | K_\alpha) \), i.e., \( c(H, \sim Ba & \sim Ra | K_\alpha) \approx 0 \).

  • That’s the traditional comparative/quantitative Bayesian story.

  • It would be nice to have a purely comparative approach …

• Most contemporary Bayesians are “Hempelian” in that they accept (PC).

• Hempel appeals to “tautological” vs “nontautological” C in his “explanation away.” But, this contradicts his (monotonic) theory of C.

• This distinction can be theoretically grounded by Bayesians, via:
  \( \star \) \( c(H, \sim Ba & \sim Ra | T) > c(H, \sim Ba | \sim Ra) \approx 0 \)

• Maher shows \( \star \) is true relative to a class of Carnapian \( \Pr_T \) models.

• Maher is in the minority. Most Bayesians try to distinguish qualitative vs comparative/quantitative. But, their strong assumptions conflate them.

• Assumptions (1)–(3) entail that \( \star \) is false! As a result, the traditional Bayesian approach can’t recover Hempel’s central intuition! Bad.

Our (1) + (\( \ddagger \)) approach \( \Rightarrow \star \) is false! Thus, our approach:
  - is consistent with qualitative approaches.
  - avoids independencies and other bad consequences like (7).
  - is consistent with the truth of Hempel’s (\( \star \)).

• Most people seem to think that (1) — and even (1’) — is true. That is, most people accept that there are (proportionally) fewer (even far fewer) ravens than non-black things in the universe (and this seems right to me).

• The controversial assumptions are the independences: (2) and (3).

• Vranas (on website) provides compelling objections to (2) and (3), and their standard rationales. He also argues that (3) is “for all practical purposes necessary” for the traditional quantitative conclusions.

• Pace Vranas, it turns out that assumptions much weaker than (1)–(3) will suffice both for comparative and for quantitative approaches.

• (2) and (3) can be replaced by the following, weaker assumption:
  \( \ddagger \) \( \Pr(H | Ra & K_\alpha) > \Pr(H | \sim Ba & K_\alpha) \) [i.e., \( \Pr(Ra | K_\alpha) \gg \Pr(\sim Ba | K_\alpha) \)]

  \( \ddagger \) \( (1) \) and \( \ddagger \) \( \Rightarrow \Pr(H | Ra & Ba & K_\alpha) > Pr(H | \sim Ba & \sim Ra & K_\alpha) \).

  (1) and \( \ddagger \) are consistent with denying each of the three qualitative claims (5)–(7). Thus, we have a purely comparative approach, which is (i) consistent with qualitative approaches, and (ii) avoids (7) as well.

Overview: Does \( \sim Ba & \sim Ra \) [E] confirm \( (\forall x)(\sim Bx \supset \sim Rx) [H] \)?

  - **Hempel**: Yes, relative to T. But, don’t conflate this with the claim (PC*) of confirmation relative to \( \sim Ra \), which is intuitively false. That’s a nice intuition, but, unfortunately, it contradicts Hempel’s theory [(M_K)].

  - **Scheffler**: Yes, but this does not imply that E confirms \( (\forall x)(Rx \supset Bx) \), since E is also a positive instance of the contrary of \( (\forall x)(Rx \supset Bx) \).

  - **Quine**: No [relative to T] because \( (NC_\Gamma) \) does not apply to \( \sim Ba, \sim Ra \), since they are not “natural kinds,” despite the fact that \( Ra \) and \( Ba \) are “NKs”. For Quine, “NKs” must have “sufficiently similar instances”.

  - **Maher** (2004): No, not even relative to T, since \( (NC_\Gamma) \) is demonstrably false within a Carnapian theory of “confirmation relative to T”. Note: the falsity of \( (NC_\Gamma) \) does not depend on “naturalness” of \( F \) and \( G \).

  - **Bayesians**: Depends on whether \( \Pr(H | E & K_\alpha) > \Pr(H | K_\alpha) \), where \( K_\alpha \) is our actual background knowledge. Bayesians think that \( (NC_\Gamma) \) is irrelevant, epistemically, and so they don’t care whether it’s true. And, even if the \( K_\alpha \) version is true, we can still give a comparative account.
Goodman’s “Grue” Paradox: Basic Linguistic Structures and Facts I

- Let $Ox \equiv x$ is observed prior to $t$, $Gx \equiv x$ is green, and $Bx \equiv x$ is blue.
- “Grue”: $Gx \equiv x$ is either observed prior to $t$ and green or $x$ is not observed prior to $t$ and blue. That is, $Gx \equiv (Ox \land Gx) \lor (\neg Ox \land Bx)$.
- We can also define “Bleen” as: $Bx \equiv (Ox \land Bx) \lor (\neg Ox \land Gx)$.

Two Facts.
- $Gx$ is logically equivalent to $(Ox \land Gx) \lor (\neg Ox \land Bx)$.
- $Bx$ is logically equivalent to $(Ox \land Bx) \lor (\neg Ox \land Gx)$.

So, from the point of view of the Green/Blue language, “Grue” and “Bleen” are “gerrymandered” or “positional” or “non-qualitative”.

But, from the point of view of the Grue/Bleen language, “Green” and “Blue” are “gerrymandered” or “positional” or “non-qualitative”.

So, no appeal to syntax will forge an asymmetry here, unless one assumes a privileged language. Note: the languages are expressively equivalent.

Goodman’s “Grue” Paradox: Basic Linguistic Structures and Facts II

- I’m going to simplify things by re-defining “grue” using green and non-green. Quine wouldn’t have liked this, but Goodman/Hempel wouldn’t have minded. It will make the subsequent discussion easier.
- Thus, “Grue” becomes: $Gx \equiv Ox \equiv Gx$. Now, consider the following two universal generalizations, and three singular evidential claims:
  - $H_1$: All emeralds are green. $(\forall x)(Ex \Rightarrow Gx)$.
  - $H_2$: All emeralds are grue. $(\forall x)(Ex \Rightarrow Gx)$. I.e., $(\forall x)(Ex \Rightarrow (Ox \equiv Gx))$.
  - $E_1$: $a$ is a green emerald. $Ea \land Ga$.
  - $E_2$: $a$ is a grue emerald. $Ea \land Ga$. I.e., $Ea \land (Oa \equiv Ga)$.
  - $E$: $a$ is a grue and green emerald. $Ea \land (Oa \land Ga)$.

The first part of Goodman’s argument involves identifying an evidential claim that Hempel-confirms $H_1$ and $H_2$. $E_1/E_2$ do not fit the bill. Why?

As Goodman points out (more detail later), $E$ Hempel-confirms both $H_1$ and $H_2$. Goodman thinks this is “bad news” for Hempel’s theory. Why?