Philosophy 148 — Announcements & Such

- HW #3 grades are posted. People did very well ($\mu = 93, \sigma = 8$).
- I've posted solutions (and common mistakes) for HW #2.
- HW #4 is due **tomorrow** (by 3pm in Raul's box). I'm giving people an extra day to work on it, since we didn't get to it until yesterday.
  - I have posted a salient handout, which I'll be going over today.
- HW #5 has been posted — it's due in two weeks.
- Today's Agenda
  - Properties of Confirmation Relations, and 4 Theories of Confirmation
    * Hempelian, HD, and two Probabilistic Theories
  - Next: **The Paradoxes of Confirmation**
    * The Raven Paradox
    * The Grue Paradox
      · Then: some psychological applications of confirmation theory

Handout on Abstract Properties of $\mathfrak{c}$ & Four Theories of $\mathfrak{c}$

Some Abstract Properties of Confirmation Relations & Four Theories of Confirmation

04/16/07

Branden Fitelson

1 Some Properties of Confirmation Relations

Hempel (in his second installment) discusses various properties that confirmation relations might have. I will discuss a longer list of properties. Here are a bunch of properties that we'll discuss today. We will assume throughout most of our discussion that all of our E's, H's, and K's are logically contingent.

(M_E) If E confirms H relative to K, then $E \,\&\, E'$ confirms H relative to K (provided that E' does not contain any constant symbols not already contained in $\{H, E, K\}$).

(M_K) If E confirms H relative to K, then E confirms H relative to $K \,\&\, K'$ (provided that K' does not contain any constant symbols not already contained in $\{H, E, K\}$).

(NC) $\ulcorner \phi x \,\&\, \psi x \urcorner$ confirms $\ulcorner (\forall y)(\phi y \supset \psi y) \urcorner$ relative to (some/all/specific) K.

(SCC) If E confirms H relative to K and $H \vDash_K H'$, then E confirms H' relative to K.

(CCC) If E confirms H relative to K and $H' \vDash_K H$, then E confirms H' relative to K.

(CC) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash \sim(H \,\&\, H')$.

(CC′) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash \sim(H \equiv H')$.

(EC) If $E \vDash_K H$, then E confirms H relative to K.

(CEC) If $H \vDash_K E$, then E confirms H relative to K.

(EQC_E) If E confirms H relative to K and $K \vDash E \equiv E'$, then E' confirms H relative to K.

(EQC_H) If E confirms H relative to K and $K \vDash H \equiv H'$, then E confirms H' relative to K.

(EQC_K) If E confirms H relative to K and $K \dashv\vDash K'$, then E confirms H relative to K'.

(NT) For some E, H, and K, E confirms H relative to K.
And, for every E/K, there exists an H such that E does *not* confirm H relative to K.

(ST) If E confirms H relative to K and E confirms H relative to $\sim K$, then E confirms H relative to $\top$.

As exercises, let's think about some subsets of this large set of conditions. Consider the following triples:

- (NT), (CEC), (SCC)
  - Inconsistent. Pick an E. Then, by (NT), E does not confirm (some) H relative to (some) K. But, by (CEC), E confirms $H \,\&\, E$, relative to K. Then, by (SCC), E confirms H, relative to K. Contradiction.
- (NT), (EC), (CCC)
  - Inconsistent. Pick an E. Then, by (NT), E does not confirm H relative to K. By (CCC), E does not confirm $H \vee E$ relative to K (if it did, then, by (CCC), it would also confirm the logically stronger H, contrary to our initial assumption). By (EC), E confirms $H \vee E$ relative to K. Contradiction.
- (NT), (CCC), (SCC)
  - Inconsistent. By (NT), (some) E confirms (some) H relative to (some) K. By (CCC), E confirms $H \& H'$ relative to K. By (SCC), E confirms H' relative to K. But, H' was arbitrary here. So, we have found an E/K such that, for all H', E confirms H' relative to K, which contradicts (NT).

As an exercise, it is useful to think about the consistency of other subsets of this large set of conditions.

1

The Raven Paradox (aka., The Paradox of Confirmation)

- **Nicod Condition** (NC): For any object x and any properties F and G, the proposition that x has both F and G confirms the proposition that every F has G. Strong second-order condition:

  $(\forall F)(\forall G)(\forall x)[Fx \,\&\, Gx \text{ confirms } (\forall x)(Fx \supset Gx)]$

- **Equivalence Condition** (EC): For any propositions H_1, E, and H_2, if E confirms H_1 and H_1 is (*classically!*) logically equivalent to H_2, then E confirms H_2. Weak 2^{nd} order condition:

  $(\forall E)(\forall H_1)(\forall H_2)[E \text{ confirms } H_1 \text{ and } H_1 \mathrel{=\!\!|\!\vDash} H_2 \Rightarrow E \text{ confirms } H_2]$

- **Paradoxical Conclusion** (PC): The proposition that a is both nonblack and a nonraven confirms the proposition that every raven is black. This is a first-order condition (arbitrary a): $\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$.

Proof. (1) By (NC), $\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$.

(2) By Classical Logic, $(\forall x)(\sim Bx \supset \sim Rx) \mathrel{=\!\!|\!\vDash} (\forall x)(Rx \supset Bx)$.

$\therefore$ (PC) By (1), (2), (EC), $\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$.

- Hempel & Goodman *embraced* (NC), (EC) *and* (PC). They saw **no paradox**. Hempel *explains away* the paradoxical *appearance* (Goodman does same):

  > ... in the seemingly paradoxical cases of confirmation, we are often not judging the relation of the given evidence E *alone* to the hypothesis H ... instead, we tacitly introduce a comparison of H with ... E in conjunction with ... additional ... information we ... have at our disposal.

- Hempel's Idea: E [$\sim Ra \,\&\, \sim Ba$] confirms H [$(\forall x)(Rx \supset Bx)$] *relative to* T, but E doesn't confirm H relative to some (nontautological) $K \neq T$.

- *Which* $K \neq \mathsf{T}$? Later, Hempel discusses $K = \sim Ra$. Intuition: if you already know that a is a nonraven, then observing its color will not tell you anything about the color of ravens. Hempel: (PC) is true, but (PC*) is false:

(PC) $\sim Ra \,\&\, \sim Ba$ confirms $(\forall x)(Rx \supset Bx)$, relative to T.

(PC*) $\sim Ra \,\&\, \sim Ba$ confirms $(\forall x)(Rx \supset Bx)$, relative to $\sim Ra$.

- This is a good insight! Unfortunately, it is *logically incompatible* with the (deductive) confirmation *theories* that Hempel and Goodman accept.

- Specifically, this possibility contradicts the K-*monotonicity* property:

(M_K) E confirms H, relative to $\top \Rightarrow E$ confirms H relative to *any* K (provided that K does not mention any individuals not already mentioned in E).

- Because Hempel's theory of confirmation satisfies (M), his theory implies that (PC) entails (PC*). So, it is logically impossible for Hempel's theory to undergird his suggestion that (PC) is true, while (PC*) is false.
- This is bad news for Hempel/Goodman. Surprisingly, nobody noticed this inconsistency in the Hempel/Goodman approach to the paradox.
- As we will see shortly, *Bayesians* can better accommodate Hempel's intuitions here, since *their* theories of confirmation do *not* satisfy (M).
- Interestingly, later in this very same passage, Hempel offers an argument for premise (1) which, itself, *depends on* (M)! [See my handout.]

  If ... E consists *only* of one ... nonraven [$\sim Ra$], then E ... confirm[s] that all objects are nonravens [$(\forall x)\sim Rx$], and *a fortiori* E supports the weaker assertion that all nonblack objects are nonravens [$(\forall x)(\sim Bx \supset \sim Rx)$].

- The dependence on (M) is almost invisible here! My conjecture: (M) is a vestige of "objectual" ways of thinking about confirmation (like NC_0).

- Scheffler rejects (PC), but accepts (1). He denies (EC). He proposes:

(*) A Hempelian positive instance (E) of a $\forall$-hypothesis (H) confirms H, *unless E is also a positive instance of the contrary (H') of H*.

- Let H: $(\forall x)[Rx \to Bx]$. The contrary of H is H': $(\forall x)[Rx \to \sim Bx]$. Let E: $\sim Ra \,\&\, \sim Ba$. E is a Hempelian positive instance of H, *and* H'.
- Thus, according to Scheffler's (*), E does not confirm H after all.
- Scheffler accepts (1) [and (NC)]. E confirms H^*: $(\forall x)[\sim Bx \to \sim Rx]$ — *even according to* (*). This is because E is *not* a Hempelian positive instance of the *contrary* of H^*, $H^{*\prime}$: $(\forall x)[\sim Bx \to Rx]$.
- This leads to a violation of (EC), of course, since – according to (*) – E confirms H^*, but E does not confirm H — even though $H \mathrel{=}\!\mid\!\models H^*$.
- Is Scheffler's (*) true? **Exercise**: show that Scheffler's (*), and (NC) are both *false* from the point of view of PR-theory. I'll return to this when we discuss I.J. Good and (NC). This will be one of the many subtle (and non-Hempelian) aspects of of probabilistic relevance accounts of $\mathfrak{c}$.

- Quine rejects (PC) but accepts (EC). As a result, Quine rejects (1), and he argues that $\forall F$ and $\forall G$ in (NC) must be *restricted in scope*:

(NC′) $(\forall F' \in N)(\forall G' \in N)(\forall x)[F'x \,\&\, G'x$ confirms $(\forall x)(F'x \supset G'x)]$

- Quine calls properties F', G' satisfying (NC′) "*projectible.*" And, he says that *natural kinds* are distinctively projectible in this sense.

- Many (*e.g.*, H & G) are inclined to follow Quine in restricting (NC) to "natural kinds" (*e.g.*, "GRUE"). But, many (*e.g.*, H & G) *reject* Quine's classification of $\sim R$ and $\sim B$ *in particular* as "unnatural".

- Quine thinks R and B *are* "natural" (hence "projectible"). This may seem odd, but there is a history [esp. in metaphysics] of denying the "naturalness" of "negative properties" (denials of "naturals").

- Some have followed Quine (Kim Quines the confirmation of psychological laws). Why should "non-naturalness" rule-out *confirmation*? And, what's Quine's *theory* of confirmation?

- This is another lecture! Meanwhile, let's look at Bayesianism …

- All Bayesian approaches begin by *precisifying* (NC) [and (PC)].
- Since Bayesian confirmation is a *three-place* relation [$C(H, E \mid K)$], we'll need a *quantifier* over the *implicit K's* in (NC). Four renditions:

(NC$_w$) $(\exists K)(\forall F)(\forall G)(\forall x)[C((\forall x)(Fx \supset Gx), Fx \,\&\, Gx \mid K)]$

(NC$_\alpha$) $(\forall F)(\forall G)(\forall x)[C((\forall x)(Fx \supset Gx), Fx \,\&\, Gx \mid K_\alpha)]$

(NC$_\top$) $(\forall F)(\forall G)(\forall x)[C((\forall x)(Fx \supset Gx), Fx \,\&\, Gx \mid K_\top)]$

(NC$_s$) $(\forall K)(\forall F)(\forall G)(\forall x)[C((\forall x)(Fx \supset Gx), Fx \,\&\, Gx \mid K)]$

- (NC$_w$) is *too weak* [let K = "all instances confirm all generalizations"].
- Hempel's "explaning away" *suggests* (NC$_s$) should be *too strong*.
- So (NC$_\alpha$) and (NC$_\top$) seem to be the salient renditions of (NC).
- *Qualitative* Bayesians seek to refute *some* rendition of (NC).
- The question for qualitative approaches is *which* (NC) to refute.
- Early qualitative Bayesian approaches took aim at (NC$_s$).

- I.J. Good showed that the strong (Bayesian) rendition (NC_s) of Nicod's condition is false. He gave the following counterexample:

  K: Exactly one of the following two hypotheses is true: (H) there are 100 black ravens, no nonblack ravens, and 1M other birds, or ($\sim H$) there are 1,000 black ravens, 1 white raven, and 1M other birds.

  E: $Ra \,\&\, Ba$ (a is randomly sampled from the universe).

  So, $H = (\forall x)(Rx \supset Bx)$, and $E = Ra \,\&\, Ba$. And, we have:

$$\Pr(E \mid H \,\&\, K) = \frac{100}{1000100} \ll \frac{1000}{1001001} = \Pr(E \mid \sim H \,\&\, K)$$

- So, (NC_s) is false, and *even for "natural kinds"* (*pace* Quine). Similar examples can be generated to show that (PC_s) and Scheffler's ($*_s$) are false.
- So? Hempel replies that (NC_T) *not* (NC_s) is the salient rendition. That's plausible, but as we have seen *it's incompatible with Hempel's theory*!
- Nonetheless, Good later tried to meet Hempel's (NC_T) challenge.
- He gave the following example, which is known as "Good's Baby"...

- Here's Good's attempt to meet Hempel's Challenge about (NC_{T}):

  ...imagine an infinitely intelligent newborn baby having built-in neural circuits enabling him to deal with formal logic, English syntax, and subjective probability. He might now argue, after defining a crow in detail, that it is initially extremely unlikely that there are any crows, and therefore that it is extremely likely that all crows are black. ...On the other hand, if there are crows, then there is a reasonable chance that they are a variety of colours. Therefore, if I were to discover that even a black crow exists I would consider [H] to be less probable than it was initially.

- Even Good wasn't so confident about this "counterexample" to (NC_{T}). Maher argues this is *not* a counterexample to (NC_{T}).
- However, Maher has recently provided a very compelling (Carnapian) counterexample to (NC_{T}), which is beyond our scope.
- Most modern Bayesians don't *understand* (NC_{T}). Unlike Carnap, they have *no theory of* "$\Pr_{\mathsf{T}}$." They've nothing to say about (NC_{T}). This is why they *abandon qualitative* approaches in favor of the *comparative/quantiative*.