Philosophy 148 — Announcements & Such

- I hope you all enjoyed Mike & Kenny's lectures last week!
- HW #3 grades are posted. People did very well ($\mu = 93, \sigma = 8$).
- I've posted solutions (and common mistakes) for HW #2.
- HW #4 is due Thursday. I'll discuss its content today, and tonight in our HW #4 discussion session, which is at **6pm tonight @ 136 Barrows**.
- Today's Agenda
  - Finishing-up the "Carnapian Programme" stuff.
  - Inductive Logic and Inductive Epistemology (again)
  - Then: **Confirmation Theory** (also, for the rest of the semester)
    * Back to early theories of confirmation (Keynes, Nicod, Hempel)
    * Then: Contemporary (subjective) probabilistic approaches
    * The Paradoxes of Confirmation (Ravens and Grue)
    * Psychological Applications of Confirmation Theory

Carnap's Programme for Inductive Logic/Confirmation Theory

- Carnap's desiderata for inductive logic/confirmation theory:

($\mathcal{D}_1$) Confirmation theory aims to characterize a function $\mathfrak{c}(H, E)$, which *generalizes entailment*, in the sense that $\mathfrak{c}(H, E)$ should take on a *maximal* value when $E \vDash H$, and a *minimal* value when $E \vDash \sim H$.

($\mathcal{D}_2$) The relation $\mathfrak{c}$ should be *objective* and *logical*. [For Carnap, this was contrasted with *psychological* relations — *anti-psychologism*.]

($\mathcal{D}_3$) Confirmation theory/inductive logic should be *applicable to/connected with* epistemology in some (non-trivial) way. [For Carnap, this meant that some non-trivial *bridge principle* connecting $\mathfrak{c}$ and *evidential support* should hold. He suggested the (RTE), which has problems.]

($\mathcal{D}_4$) The relation $\mathfrak{c}$ should be defined in terms of *probability*. [For Carnap, ($\mathcal{D}_1$), ($\mathcal{D}_2$), and ($\mathcal{D}_4$) implied that there must be "logical" probabilities $\Pr_{\mathrm{T}}$. Later, I will explain an alternative way to satisfy these three $\mathcal{D}$'s.]

Inductive Logic and Inductive Epistemology (Applicability)

- Carnap originally proposed the following *bridge principle*:

(RTE_C) E evidentially supports H *for an agent S in an epistemic context C* $\Longleftrightarrow \Pr_{\mathrm{T}}(H \mid E \,\&\, K) > r$, where K is S's *total evidence in C*.

- Popperian (*e.g.*, "rare disease") examples lead to this alteration:

(RTE'_C) E evidentially supports H *for an agent S in an epistemic context C* $\Longrightarrow \Pr_{\mathrm{T}}(H \mid E \,\&\, K) > \Pr_{\mathrm{T}}(H \mid K)$, where K is S's *total evidence in C*.

- But, even this refinement of (RTE) has counterexamples. For instance, "old evidence" cases in which $K \vDash E$. We'll discuss another soon ("grue").
- This leads one to re-think the applicability desideratum ($\mathcal{D}_3$). Maybe it is misguided altogether, or maybe it's just really hard to satisfy.
- Last time, I talked about "bridge principles" in deductive logic (knowledge and $\vDash$). I pointed out that they are very difficult to articulate. Be that as it may, many still think there is *some* connection. I'll return to this later.

Confirmation Theory I: Keynes and Nicod (Roots)

- Keynes (1921) was the first to clearly articulate a *probabilistic relevance* conception of inductive support. Nicod (1930) continued this thread.
- Nicod's three basic tenets of (instantial) confirmation were as follows:
  - Instantial confirmation is a relation between singular and general propositions/statements (or, if you will, between "facts" and "laws").
  - Confirmation consists in *positive probabilistic relevance*, and disconfirmation consists in *negative probabilistic relevance* (where the salient probabilities are inductive / *a priori* in the Keynesian sense).
  - Universal generalizations are confirmed by their positive instances and disconfirmed by their negative instances. [*The Nicod Condition* (NC)]
- These tenets (especially NC) became the basic principles of early confirmation theory. Hempel (the father of modern confirmation theory) picked-up where Nicod left off, but in a rather strange (and different) way.

Confirmation Theory II: Hempel (The Father of 𝔠-Theory)

- Hempel wrote several seminal papers about confirmation theory in the 30's and 40's. This set the agenda for confirmation theory since.
- Hempel begins by discussing Nicod's views about instantial confirmation. Strangely, however, Hempel interprets Nicod's (NC) in the following way:

(NC_0) For all objects x (with names $\mathtt{x}$), and all predicate expressions ϕ and ψ:
x confirms $\ulcorner(\forall y)(\phi y \supset \psi y)\urcorner$ iff $\ulcorner\phi\mathtt{x} \,\&\, \psi\mathtt{x}\urcorner$ is true, and
x disconfirms $\ulcorner(\forall y)(\phi y \supset \psi y)\urcorner$ iff $\ulcorner\phi\mathtt{x} \,\&\, \sim\psi\mathtt{x}\urcorner$ is true.

- This is a somewhat puzzling way of reading Nicod, in several respects:
  - It interprets Nicod as describing a relation between *objects* and universal claims, not between *singular claims* and universal claims.
  - It abstracts away from (and does not mention) *probabilistic relevance*.
  - It understands the notion of "positive instance" in a *conjunctive* way.
  - It leads to an absurd confirmation relation in several respects.

Confirmation Theory II: Hempel (The Father of 𝔠-Theory)

- The most patent absurdity of Hempel's (NC_0)-reading of Nicod is that it leads to a 𝔠-relation that violates the *hypothetical equivalence condition*:

(EQC_H) If x confirms H, then x confirms anything logically equivalent to H.

- Hempel himself pointed this out, using the following example.
  - a confirms "$(\forall y)(Fy \supset Gy)$," provided a is such that $Fa \,\&\, Ga$.
  - *Nothing* can confirm "$(\forall y)[(Fy \,\&\, \sim Gy) \supset (Fy \,\&\, \sim Fy)]$," since *no object* a can be such that $Fa \,\&\, \sim Fa$.
  - But, $(\forall y)(Fy \supset Gy) \mathrel{=\!\mid\!\models} (\forall y)[(Fy \,\&\, \sim Gy) \supset (Fy \,\&\, \sim Fy)]$.
- This means that (NC_0) leads to a confirmation relation that depends on *how propositions are expressed*, which seems unintuitive.
- For Hempel, confirmation is a *logical* relation, and logical relations (for Hempel) do not depend on choice of description in this sensitive way.
- Hempel gives an alternative theory of confirmation that avoids this.

Hempelian Confirmation Theory I

- After giving-up on (NC_0), Hempel laid down the following *desiderata*, in addition to the Hypothetical Equivalence Condition (EQC_H).

  Entailment Condition (EC). If $E \models H$, then E confirms H.

  Special Consequence Condition (SCC). If E confirms H, and $H \models H'$, then E confirms H'.

  Consistency Condition (CC). If E confirms H, and E confirms H', then H and H' are logically consistent.

  Non-Triviality Condition (NTC). For all H, there exists an E which does *not* confirm H.

- Because Hempel accepts these desiderata, he *must* reject the following:

  Converse Consequence Condition (CCC). If E confirms H, and $H' \models H$, then E confirms H'.

- Otherwise, the desiderata would be *logically inconsistent*. HW #4!
- I will discuss these desiderata critically, below. But, first, let's look at the theory Hempel comes up with, which satisfies these desiderata.

Hempelian Confirmation Theory II

- Hempel advances an "instance" account satisfying his desiderata. The key definition behind his deductive theory of instantial confirmation is:
- The *development of a hypothesis H for a set of individuals I* [$dev_I(H)$] is (intuitively) "what H says (*extensionally*) about the members of I".
- $dev_I(H)$ is obtained by (*i*) *conjoining* all the I-instances of H, if H is a *universal* ($\forall$) claim, and (*ii*) *disjoining* all the I-instances of H, if H is an *existential* ($\exists$) claim. [I-instances of H are basic sentences that *satisfy* H.]
- Satisfaction is *semantical* ("*makes true*") *not* syntactical (*contra* NC_0). If $H \mathrel{=\!\mid\!\models} H'$, they have the same I-instances (say the same things about I).
- Let $I = \{a, b\}$, then we have:
  - If $H = (\forall x)Bx$, then $dev_I(H) = Ba \,\&\, Bb$.
  - If $H = (\exists x)Rx$, then $dev_I(H) = Ra \vee Rb$.
  - If $H = (\forall x)(\exists y)Lxy$, then (working from the outside-in):

$$dev_I(H) = (\exists y)Lay \,\&\, (\exists y)Lby = (Laa \vee Lab) \,\&\, (Lba \vee Lbb)$$

Hempelian Confirmation Theory III

- **Def.** E *directly-Hempel-confirms* H, just in case $E \vDash dev_I(H)$ for the class I of individuals mentioned in E. E *Hempel-confirms* H iff E directly Hempel confirms every member of a set of sentences S such that $S \vDash H$.
- Why the two definitions? $Ra \,\&\, Ba$ does *not directly* Hempel-confirm $Rb \supset Bb$, but $Ra \,\&\, Ba$ *does* Hempel-confirm $Rb \supset Bb$ (α-variants).
- Problematic Examples for Hempel's Theory:
  - Let $I = \{a, b\}$, $H = (\forall x)(\forall y)Rxy$, $E \stackrel{\text{def}}{=} Raa \,\&\, Rab \,\&\, Rbb \,\&\, Rba$, $E' \stackrel{\text{def}}{=} Raa \,\&\, Rab \,\&\, Rbb$, and $E'' \stackrel{\text{def}}{=} Raa$. Then, E'' confirms H, E' does not confirm H, and E does confirm H. Make sure you see why.
  - *No consistent* E can confirm the following, which is *true* on $\mathbb{N}$,

    (H) $(\forall x)(\exists y)x < y \,\&\, (\forall x)x \nless x \,\&\, (\forall x)(\forall y)(\forall z)[(x < y \,\&\, y < z) \supset x < z]$

    since $\text{dev}_I(H)$ *is inconsistent, for any finite* I! Exercise: Prove this!
- The Paradoxes of Confirmation pose deeper problems for Hempel.

Hempelian Confirmation Theory IV

- **The Paradox of the Ravens**: Consider the hypothesis that all ravens are black, H: $(\forall x)(Rx \supset Bx)$. Which of these 6 claims Hempel-confirms H?

E_1: $Ra \,\&\, Ba$	E_2: $\sim Ra$	E_3: Ba
E_4: $\sim Ra \,\&\, \sim Ba$	E_5: $\sim Ra \,\&\, Ba$	E_6: $Ra \,\&\, \sim Ba$

  Answer: *All but* E_6 *Hempel-confirm* H! Make sure you see why.
- **Goodman's New Riddle of Induction**: Consider the hypothesis that all ravens are "blite", where the predicate "blite" (B) is defined as follows:

  x is blite iff *either* (*i*) x is examined before (the end of) today, and x is black *or* (*ii*) x is examined after today, and x is white.

  $Ra \,\&\, Ba$ Hempel-confirms H. The observation of a black raven today confirms the hypothesis that ravens observed tomorrow will be white?!?
- We'll discuss these infamous historical paradoxes in great depth soon …
- Also: $Ra \,\&\, Ba$ Hempel-confirms $(\forall x)[\phi x \supset Bx]$, for *any* ϕ.

Hempel's Desiderata — Critical Discussion

- Probabilistic accounts of confirmation will accept some of Hempel's desiderata, but they will reject others. And, these rejections are *intuitive*.
- The EQC, the EC, and the NTC all seem quite intuitive, and they are satisfied by any probabilistic account of confirmation. Prove this!
- CC is *not* intuitive. Typically, competing theories are *not* consistent. Still, it is often the case that evidence confirms several competing theories, though it may confirm some *more strongly* than others.
- Example: E = card is black, H = card is the A♠, H' = card is the J♣. Intuitively, E confirms both H and H', even though they are inconsistent.
- SCC is not intuitive either. Many intuitive counterexamples are out there. *E.g.*, E = card is black, H = card is card is the A♠, and H' = card is an ace.

- As for CCC, it is *highly un*intuitive (here, we agree with Hempel). *E.g.*, E = card is the A♠, H = card is card is an ace, and H' = card is the A♦.

A Catalogue of Properties of Confirmation Relations I

(M_E) If E confirms H relative to K, then $E \,\&\, E'$ confirms H relative to K (provided that E' does not contain any constant symbols not contained in $\{H, E, K\}$).

(M_K) If E confirms H relative to K, then E confirms H relative to $K \,\&\, K'$ (provided that K' does not contain any constant symbols not contained in $\{H, E, K\}$).

(NC) $\ulcorner \phi x \,\&\, \psi x \urcorner$ confirms $\ulcorner (\forall y)(\phi y \to \psi y) \urcorner$ relative to (some/all/specific) K.

(SCC) If E confirms H relative to K and $H \vDash_K H'$, then E confirms H' relative to K.

(CCC) If E confirms H relative to K and $H' \vDash_K H$, then E confirms H' relative to K.

(CC) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash \sim(H \,\&\, H')$.

(CC′) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash \sim(H \equiv H')$.

(EC) If $E \vDash_K H$, then E confirms H relative to K.

(CEC) If $H \vDash_K E$, then E confirms H relative to K.

(EQC_E) If E confirms H relative to K and $K \vDash E \equiv E'$, then E' confirms H rel. to K.

(EQC_H) If E confirms H relative to K and $K \vDash H \equiv H'$, then E confirms H' rel. to K.

(EQC_K) If E confirms H relative to K and $K \dashv\vDash K'$, then E confirms H relative to K'.

(NT) For some E, H, and K, E confirms H relative to K. And, for every E/K, there exists an H such that E does *not* confirm H relative to K.

(ST) If E confirms H relative to K and E confirms H relative to $\sim K$, then E confirms H relative to $\top$.

- The monotonicity properties (M_E) and (M_K) are satisfied by Hempel's theory of confirmation. It is worth examining why this is the case.
- As it turns out, the monotonicity properties are not desirable for confirmation relations — even by Hempel's own lights, as we'll soon see.
- This is an important (and embarrassing) fact about Hempel's theory — one which is not shared by Pr-accounts (probability is *non-monotonic*!).
- (CC′) is something that *is* satisfied by probabilistic accounts. Why?
- Interestingly, (ST) is *violated* by probabilistic relevance (PR) accounts. But, it is satisfied by the conditional-probability-threshold (CPT) account. Why?
- Does Hempel's theory satisfy (ST)? Why/why not? How about CPT?

Hypopthetico-Deductive (HD) Confirmation

- The general form of a deductive (*i.e.*, H-D) prediction is:

  H. Hypothesis under test.
  K. Background assumptions (initial conditions, *etc.*).
  ———
  E. Observational (deductive) prediction.

- We can also look at the "reverse inference", *from* the observation E *to* the hypothesis H (*given* K). NOTE: this direction is *inductive* (double-line)!

  E. Observational (deductive) prediction.
  K. Background assumptions (initial conditions, *etc.*).
  ═══
  H. Hypothesis under test.

- The basic idea: E HD-confirms H, relative to K iff $H \vDash_K E$. Note how this has H on the left of a $\vDash$, whereas Hempel's has E on the left. This is a crucial difference. Think about (SCC) and (CCC) for the two theories.
- This is merely a *qualitative* claim, that E confirms H, relative to K (to some positive degree or other — like Hempel's qualitative theory).

A Catalogue of Properties of Confirmation Relations II

	Does Concept Satisfy Condition?										
Concept	EQC	EC	CC	NT	SCC	CCC	CEC	M	NC	CC′	ST
Hempel	YES	YES	YES[a]	YES	YES	NO	NO	YES	YES	YES[a]	YES
HD	YES	NO	NO	YES	NO	YES	YES	NO	NO	YES[e]	YES
CPT	YES	YES[b]	NO	YES	YES	NO	NO	NO	NO	YES[d]	YES
PR	YES	YES[c]	NO	YES	NO	NO	YES[c]	NO	NO	YES	NO

[a] Assuming that $E \,\&\, K$ is not self-contradictory.
[b] Assuming that $\Pr(E \mid K) \neq 0$.
[c] Assuming that $\Pr(H \mid K) \in (0,1)$, and $\Pr(E \mid K) \in (0,1)$.
[d] Assuming that $t \geqslant \frac{1}{2}$.
[e] Assuming that $K \nvDash E$.

The Raven Paradox (aka., The Paradox of Confirmation)

- **Nicod Condition** (NC): For any object x and any properties F and G, the proposition that x has both F and G confirms the proposition that every F has G. Strong second-order condition:

  $(\forall F)(\forall G)(\forall x)[Fx \,\&\, Gx \text{ confirms } (\forall x)(Fx \supset Gx)]$

- **Equivalence Condition** (EC): For any propositions H_1, E, and H_2, if E confirms H_1 and H_1 is (*classically!*) logically equivalent to H_2, then E confirms H_2. Weak 2^{nd} order condition:

  $(\forall E)(\forall H_1)(\forall H_2)[E \text{ confirms } H_1 \text{ and } H_1 \dashv\vDash H_2 \Rightarrow E \text{ confirms } H_2]$

- **Paradoxical Conclusion** (PC): The proposition that a is both nonblack and a nonraven confirms the proposition that every raven is black. This is a first-order condition (arbitrary a): $\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$.

Proof. (1) By (NC), $\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$.

(2) By Classical Logic, $(\forall x)(\sim Bx \supset \sim Rx) \dashv\vDash (\forall x)(Rx \supset Bx)$.

$\therefore$ (PC) By (1), (2), (EC), $\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$.

- Hempel *&* Goodman *embraced* (NC), (EC) *and* (PC). They saw **no paradox**. Hempel *explains away* the paradoxical *appearance* (Goodman does same):

  > ... in the seemingly paradoxical cases of confirmation, we are often not judging the relation of the given evidence E *alone* to the hypothesis H ... instead, we tacitly introduce a comparison of H with ... E in conjunction with ... additional ... information we ... have at our disposal.

- Hempel's Idea: E $[\sim Ra \,\&\, \sim Ba]$ confirms H $[(\forall x)(Rx \supset Bx)]$ *relative to* T, but E doesn't confirm H relative to some (nontautological) $K \neq T$.
- *Which* $K \neq \mathsf{T}$? Later, Hempel discusses $K = \sim Ra$. Intuition: if you already know that a is a nonraven, then observing its color will not tell you anything about the color of ravens. Hempel: (PC) is true, but (PC*) is false:

(PC) $\sim Ra \,\&\, \sim Ba$ confirms $(\forall x)(Rx \supset Bx)$, relative to T.

(PC*) $\sim Ra \,\&\, \sim Ba$ confirms $(\forall x)(Rx \supset Bx)$, relative to $\sim Ra$.

- This is a good insight! Unfortunately, it is *logically incompatible* with the (deductive) confirmation *theories* that Hempel and Goodman accept.
- Specifically, this possibility contradicts the *K-monotonicity* property:

(M_K) E confirms H, relative to $\mathsf{T} \Rightarrow E$ confirms H relative to *any* K (provided that K does not mention any individuals not already mentioned in E).

- Because Hempel's theory of confirmation satisfies (M), his theory implies that (PC) entails (PC*). So, it is logically impossible for Hempel's theory to undergird his suggestion that (PC) is true, while (PC*) is false.
- This is bad news for Hempel/Goodman. Surprisingly, nobody noticed this inconsistency in the Hempel/Goodman approach to the paradox.
- As we will see shortly, *Bayesians* can better accommodate Hempel's intuitions here, since *their* theory of confirmation does *not* satisfy (M).
- Interestingly, later in this very same passage, Hempel offers an argument for premise (1) which, itself, *depends on* this very monotonicity property!

  > If ... E consists *only* of one ... nonraven $[\sim Ra]$, then E ... confirm[s] that all objects are nonravens $[(\forall x)\sim Rx]$, and *a fortiori* E supports the weaker assertion that all nonblack objects are nonravens $[(\forall x)(\sim Bx \supset \sim Rx)]$.

- The dependence on (M) is almost invisible here! My conjecture: (M) is a vestige of "objectual" ways of thinking about confirmation (like NC_0).

- This independent argument for (1) presupposes not only (M), but (SCC):

(SCC) $(\forall E)(\forall H_1)(\forall H_2)[E$ confirms H_1 and $H_1 \vDash H_2 \Rightarrow E$ confirms $H_2]$.

- To see this, take a closer look at the reasoning of the argument:

(1.1) $\sim Ra$ confirms $(\forall x)\sim Rx$. (Nicod)

(1.2) $(\forall x)\sim Rx \vDash (\forall x)(\sim Bx \supset \sim Rx)$ (Logic)

(1.3) $\sim Ra$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$ (SCC)

(1) $\sim Ra \,\&\, \sim Ba$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$ (M?!)

- (M) and (SCC) are consequences of Hempel's (and Goodman's) confirmation theory, which says E confirms H iff $E \vDash \mathrm{dev}_E(H)$.
- Modern Bayesians *reject both* (M) *and* (SCC). And, as a result, Bayesians are able to say what Hempel wanted to say (but can't!).
- Before Bayesianism, we'll look briefly at Scheffler [who accepts (NC), but denies (EC)], and Quine [who accepts (EC), but denies (NC)].
- Then, we'll look at Bayesian resolutions of several different kinds. Some of these will reject (NC), while others will take a different tack.

- Scheffler rejects (PC), but accepts (1). He denies (EC). He proposes:

  (*) A Hempelian positive instance (E) of a $\forall$-hypothesis (H) confirms H, *unless E is also a positive instance of the contrary (H') of H.*

- Let H: $(\forall x)[Rx \to Bx]$. The contrary of H is H': $(\forall x)[Rx \to \sim Bx]$. Let E: $\sim Ra \,\&\, \sim Ba$. E is a Hempelian positive instance of H, *and* H'.
- Thus, according to Scheffler's (*), E does not confirm H after all.
- Scheffler accepts (1) [and (NC)]. E confirms H^*: $(\forall x)[\sim Bx \to \sim Rx]$ — *even according to* (*). This is because E is *not* a Hempelian positive instance of the *contrary* of H^*, $H^{*\prime}$: $(\forall x)[\sim Bx \to Rx]$.
- This leads to a violation of (EC), of course, since – according to (*) – E confirms H^*, but E does not confirm H — even though $H \mathrel{=}\!\mathrel{\mid}\!\vDash H^*$.
- Is Scheffler's (*) true? **Exercise**: show that Scheffler's (*), and (NC) are both *false* from the point of view of PR-theory. I'll return to this when we discuss I.J. Good and (NC). This will be one of the many subtle (and non-Hempelian) aspects of of probabilistic relevance accounts of $\mathfrak{c}$.