

Philosophy 148 — Announcements & Such

- I hope you all enjoyed Mike & Kenny's lectures last week!
- HW #3 grades are posted. People did very well ($\mu = 93, \sigma = 8$).
- I've posted solutions (and common mistakes) for HW #2.
- HW #4 is due Thursday. I'll discuss its content today, and tonight in our HW #4 discussion session, which is at **6pm tonight @ 136 Barrows**.
- Today's Agenda
 - Finishing-up the “Carnapian Programme” stuff.
 - Inductive Logic and Inductive Epistemology (again)
 - Then: **Confirmation Theory** (also, for the rest of the semester)
 - * Back to early theories of confirmation (Keynes, Nicod, Hempel)
 - * Then: Contemporary (subjective) probabilistic approaches
 - * The Paradoxes of Confirmation (Ravens and Grue)
 - * Psychological Applications of Confirmation Theory

Carnap's Programme for Inductive Logic/Confirmation Theory

- Carnap's desiderata for inductive logic/confirmation theory:
 - (\mathcal{D}_1) Confirmation theory aims to characterize a function $c(H, E)$, which *generalizes entailment*, in the sense that $c(H, E)$ should take on a *maximal* value when $E \models H$, and a *minimal* value when $E \models \sim H$.
 - (\mathcal{D}_2) The relation c should be *objective* and *logical*. [For Carnap, this was contrasted with *psychological* relations — *anti-psychologism*.]
 - (\mathcal{D}_3) Confirmation theory/inductive logic should be *applicable to/connected with* epistemology in some (non-trivial) way. [For Carnap, this meant that some non-trivial *bridge principle* connecting c and *evidential support* should hold. He suggested the (RTE), which has problems.]
 - (\mathcal{D}_4) The relation c should be defined in terms of *probability*. [For Carnap, (\mathcal{D}_1), (\mathcal{D}_2), and (\mathcal{D}_4) implied that there must be “logical” probabilities Pr_\top . Later, I will explain an alternative way to satisfy these three \mathcal{D} 's.]

Inductive Logic and Inductive Epistemology (Applicability)

- Carnap originally proposed the following *bridge principle*:
 (RTE_C) *E* evidentially supports *H* for an agent *S* in an epistemic context *C*
 $\iff \Pr_{\top}(H \mid E \ \& \ K) > r$, where *K* is *S*'s total evidence in *C*.
- Popperian (*e.g.*, “rare disease”) examples lead to this alteration:
 (RTE'_C) *E* evidentially supports *H* for an agent *S* in an epistemic context *C*
 $\implies \Pr_{\top}(H \mid E \ \& \ K) > \Pr_{\top}(H \mid K)$, where *K* is *S*'s total evidence in *C*.
- But, even this refinement of (RTE) has counterexamples. For instance, “old evidence” cases in which $K \models E$. We'll discuss another soon (“grue”).
- This leads one to re-think the applicability desideratum (\mathcal{D}_3). Maybe it is misguided altogether, or maybe it's just really hard to satisfy.
- Last time, I talked about “bridge principles” in deductive logic (knowledge and \models). I pointed out that they are very difficult to articulate. Be that as it may, many still think there is *some* connection. I'll return to this later.

Confirmation Theory I: Keynes and Nicod (Roots)

- Keynes (1921) was the first to clearly articulate a *probabilistic relevance* conception of inductive support. Nicod (1930) continued this thread.
- Nicod's three basic tenets of (instantial) confirmation were as follows:
 - Instantial confirmation is a relation between singular and general propositions/statements (or, if you will, between “facts” and “laws”).
 - Confirmation consists in *positive probabilistic relevance*, and disconfirmation consists in *negative probabilistic relevance* (where the salient probabilities are inductive / *a priori* in the Keynesian sense).
 - Universal generalizations are confirmed by their positive instances and disconfirmed by their negative instances. [*The Nicod Condition* (NC)]
- These tenets (especially NC) became the basic principles of early confirmation theory. Hempel (the father of modern confirmation theory) picked-up where Nicod left off, but in a rather strange (and different) way.

Confirmation Theory II: Hempel (The Father of c -Theory)

- Hempel wrote several seminal papers about confirmation theory in the 30's and 40's. This set the agenda for confirmation theory since.
- Hempel begins by discussing Nicod's views about instancial confirmation. Strangely, however, Hempel interprets Nicod's (NC) in the following way:
 (NC₀) For all objects x (with names x), and all predicate expressions ϕ and ψ :
 x confirms $\lceil (\forall y)(\phi y \supset \psi y) \rceil$ iff $\lceil \phi x \ \& \ \psi x \rceil$ is true, and
 x disconfirms $\lceil (\forall y)(\phi y \supset \psi y) \rceil$ iff $\lceil \phi x \ \& \ \sim \psi x \rceil$ is true.
- This is a somewhat puzzling way of reading Nicod, in several respects:
 - It interprets Nicod as describing a relation between *objects* and universal claims, not between *singular claims* and universal claims.
 - It abstracts away from (and does not mention) *probabilistic relevance*.
 - It understands the notion of “positive instance” in a *conjunctive* way.
 - It leads to an absurd confirmation relation in several respects.

Confirmation Theory II: Hempel (The Father of c -Theory)

- The most patent absurdity of Hempel's (NC₀)-reading of Nicod is that it leads to a c -relation that violates the *hypothetical equivalence condition*:
(EQC_H) If x confirms H , then x confirms anything logically equivalent to H .
- Hempel himself pointed this out, using the following example.
 - a confirms “ $(\forall y)(Fy \supset Gy)$,” provided a is such that $Fa \ \& \ Ga$.
 - *Nothing* can confirm “ $(\forall y)[(Fy \ \& \ \sim Gy) \supset (Fy \ \& \ \sim Fy)]$,” since *no object* a can be such that $Fa \ \& \ \sim Fa$.
 - But, $(\forall y)(Fy \supset Gy) \equiv (\forall y)[(Fy \ \& \ \sim Gy) \supset (Fy \ \& \ \sim Fy)]$.
- This means that (NC₀) leads to a confirmation relation that depends on *how propositions are expressed*, which seems unintuitive.
- For Hempel, confirmation is a *logical* relation, and logical relations (for Hempel) do not depend on choice of description in this sensitive way.
- Hempel gives an alternative theory of confirmation that avoids this.

Hempel's Confirmation Theory I

- After giving-up on (NC_0) , Hempel laid down the following *desiderata*, in addition to the Hypothetical Equivalence Condition (EQC_H).

Entailment Condition (EC). If $E \models H$, then E confirms H .

Special Consequence Condition (SCC). If E confirms H , and $H \models H'$, then E confirms H' .

Consistency Condition (CC). If E confirms H , and E confirms H' , then H and H' are logically consistent.

Non-Triviality Condition (NTC). For all H , there exists an E which does *not* confirm H .

- Because Hempel accepts these desiderata, he *must* reject the following:

Converse Consequence Condition (CCC). If E confirms H , and $H' \models H$, then E confirms H' .

- Otherwise, the desiderata would be *logically inconsistent*. HW #4!
- I will discuss these desiderata critically, below. But, first, let's look at the theory Hempel comes up with, which satisfies these desiderata.

Hempelian Confirmation Theory II

- Hempel advances an “instance” account satisfying his desiderata. The key definition behind his deductive theory of instancial confirmation is:
- The *development of a hypothesis H for a set of individuals I* [$dev_I(H)$] is (intuitively) “what H says (*extensionally*) about the members of I ”.
- $dev_I(H)$ is obtained by (i) *conjoining* all the I -instances of H , if H is a *universal* (\forall) claim, and (ii) *disjoining* all the I -instances of H , if H is an *existential* (\exists) claim. [I -instances of H are basic sentences that *satisfy* H .]
- Satisfaction is *semantical* (“*makes true*”) *not* syntactical (*contra* NC_0). If $H \models H'$, they have the same I -instances (say the same things about I).
- Let $I = \{a, b\}$, then we have:
 - If $H = (\forall x)Bx$, then $dev_I(H) = Ba \ \& \ Bb$.
 - If $H = (\exists x)Rx$, then $dev_I(H) = Ra \ \vee \ Rb$.
 - If $H = (\forall x)(\exists y)Lxy$, then (working from the outside-in):

$$dev_I(H) = (\exists y)Lay \ \& \ (\exists y)Lby = (Laa \ \vee \ Lab) \ \& \ (Lba \ \vee \ Lbb)$$

Hempelian Confirmation Theory III

- **Def.** E *directly-Hempel-confirms* H , just in case $E \models dev_I(H)$ for the class I of individuals mentioned in E . E *Hempel-confirms* H iff E directly Hempel confirms every member of a set of sentences S such that $S \models H$.
- Why the two definitions? $Ra \ \& \ Ba$ does *not directly* Hempel-confirm $Rb \supset Bb$, but $Ra \ \& \ Ba$ *does* Hempel-confirm $Rb \supset Bb$ (α -variants).
- Problematic Examples for Hempel's Theory:
 - Let $I = \{a, b\}$, $H = (\forall x)(\forall y)Rxy$, $E \stackrel{\text{def}}{=} Raa \ \& \ Rab \ \& \ Rbb \ \& \ Rba$, $E' \stackrel{\text{def}}{=} Raa \ \& \ Rab \ \& \ Rbb$, and $E'' \stackrel{\text{def}}{=} Raa$. Then, E'' confirms H , E' does not confirm H , and E does confirm H . Make sure you see why.
 - *No consistent* E can confirm the following, which is *true* on \mathbb{N} ,

$$(H) \ (\forall x)(\exists y)x < y \ \& \ (\forall x)x \not< x \ \& \ (\forall x)(\forall y)(\forall z)[(x < y \ \& \ y < z) \supset x < z]$$
 since $dev_I(H)$ is *inconsistent*, for any finite I ! Exercise: Prove this!
- The Paradoxes of Confirmation pose deeper problems for Hempel.

Hempelian Confirmation Theory IV

- **The Paradox of the Ravens:** Consider the hypothesis that all ravens are black, $H: (\forall x)(Rx \supset Bx)$. Which of these 6 claims Hempel-confirms H ?

$E_1: Ra \ \& \ Ba$	$E_2: \sim Ra$	$E_3: Ba$
$E_4: \sim Ra \ \& \ \sim Ba$	$E_5: \sim Ra \ \& \ Ba$	$E_6: Ra \ \& \ \sim Ba$

Answer: *All but E_6 Hempel-confirm H !* Make sure you see why.

- **Goodman's New Riddle of Induction:** Consider the hypothesis that all ravens are "blite", where the predicate "blite" (B) is defined as follows:
 x is blite iff *either* (i) x is examined before (the end of) today, and x is black *or* (ii) x is examined after today, and x is white.
 $Ra \ \& \ Ba$ Hempel-confirms H . The observation of a black raven today confirms the hypothesis that ravens observed tomorrow will be white?!?
- We'll discuss these infamous historical paradoxes in great depth soon ...
- Also: $Ra \ \& \ Ba$ Hempel-confirms $(\forall x)[\phi x \supset Bx]$, for *any* ϕ .

Hempel's Desiderata — Critical Discussion

- Probabilistic accounts of confirmation will accept some of Hempel's desiderata, but they will reject others. And, these rejections are *intuitive*.
 - The EQC, the EC, and the NTC all seem quite intuitive, and they are satisfied by any probabilistic account of confirmation. Prove this!
 - CC is *not* intuitive. Typically, competing theories are *not* consistent. Still, it is often the case that evidence confirms several competing theories, though it may confirm some *more strongly* than others.
 - Example: E = card is black, H = card is the A♠, H' = card is the J♣. Intuitively, E confirms both H and H' , even though they are inconsistent.
 - SCC is not intuitive either. Many intuitive counterexamples are out there. *E.g.*, E = card is black, H = card is card is the A♠, and H' = card is an ace.
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- As for CCC, it is *highly unintuitive* (here, we agree with Hempel). *E.g.*, E = card is the A♠, H = card is card is an ace, and H' = card is the A♦.

A Catalogue of Properties of Confirmation Relations I

- (M_E) If E confirms H relative to K , then $E \& E'$ confirms H relative to K (provided that E' does not contain any constant symbols not contained in $\{H, E, K\}$).
- (M_K) If E confirms H relative to K , then E confirms H relative to $K \& K'$ (provided that K' does not contain any constant symbols not contained in $\{H, E, K\}$).
- (NC) ' $\phi x \& \psi x$ ' confirms ' $(\forall y)(\phi y \rightarrow \psi y)$ ' relative to (some/all/specific) K .
- (SCC) If E confirms H relative to K and $H \models_K H'$, then E confirms H' relative to K .
- (CCC) If E confirms H relative to K and $H' \models_K H$, then E confirms H' relative to K .
- (CC) If E confirms H relative to K and E confirms H' relative to K , then $K \neq \sim(H \& H')$.
- (CC') If E confirms H relative to K and E confirms H' relative to K , then $K \neq \sim(H \equiv H')$.
- (EC) If $E \models_K H$, then E confirms H relative to K .
- (CEC) If $H \models_K E$, then E confirms H relative to K .
- (EQC_E) If E confirms H relative to K and $K \models E \equiv E'$, then E' confirms H rel. to K .
- (EQC_H) If E confirms H relative to K and $K \models H \equiv H'$, then E confirms H' rel. to K .

(EQC_K) If E confirms H relative to K and $K \models K'$, then E confirms H relative to K' .

(NT) For some E, H , and K , E confirms H relative to K . And, for every E/K , there exists an H such that E does *not* confirm H relative to K .

(ST) If E confirms H relative to K and E confirms H relative to $\sim K$, then E confirms H relative to T .

- The monotonicity properties (M_E) and (M_K) are satisfied by Hempel's theory of confirmation. It is worth examining why this is the case.
- As it turns out, the monotonicity properties are not desirable for confirmation relations — even by Hempel's own lights, as we'll soon see.
- This is an important (and embarrassing) fact about Hempel's theory — one which is not shared by Pr-accounts (probability is *non-monotonic*!).
- (CC') is something that *is* satisfied by probabilistic accounts. Why?
- Interestingly, (ST) is *violated* by probabilistic relevance (PR) accounts. But, it is satisfied by the conditional-probability-threshold (CPT) account. Why?
- Does Hempel's theory satisfy (ST)? Why/why not? How about CPT?

Hypothetico-Deductive (HD) Confirmation

- The general form of a deductive (*i.e.*, H-D) prediction is:

H. Hypothesis under test.

K. Background assumptions (initial conditions, *etc.*).

E. Observational (deductive) prediction.

- We can also look at the “reverse inference”, *from* the observation *E* to the hypothesis *H* (*given K*). NOTE: this direction is *inductive* (double-line)!

E. Observational (deductive) prediction.

K. Background assumptions (initial conditions, *etc.*).

H. Hypothesis under test.

- The basic idea: *E* HD-confirms *H*, relative to *K* iff $H \models_K E$. Note how this has *H* on the left of a \models , whereas Hempel’s has *E* on the left. This is a crucial difference. Think about (SCC) and (CCC) for the two theories.
- This is merely a *qualitative* claim, that *E* confirms *H*, relative to *K* (to some positive degree or other — like Hempel’s qualitative theory).

A Catalogue of Properties of Confirmation Relations II

	Does Concept Satisfy Condition?										
Concept	EQC	EC	CC	NT	SCC	CCC	CEC	M	NC	CC'	ST
Hempel	YES	YES	YES ^a	YES	YES	NO	NO	YES	YES	YES ^a	YES
HD	YES	NO	NO	YES	NO	YES	YES	NO	NO	YES ^e	YES
CPT	YES	YES ^b	NO	YES	YES	NO	NO	NO	NO	YES ^d	YES
PR	YES	YES ^c	NO	YES	NO	NO	YES ^c	NO	NO	YES	NO

^aAssuming that $E \& K$ is not self-contradictory.

^bAssuming that $\Pr(E | K) \neq 0$.

^cAssuming that $\Pr(H | K) \in (0, 1)$, and $\Pr(E | K) \in (0, 1)$.

^dAssuming that $t \geq \frac{1}{2}$.

^eAssuming that $K \neq E$.

The Raven Paradox (aka., The Paradox of Confirmation)

- **Nicod Condition (NC):** For any object x and any properties F and G , the proposition that x has both F and G confirms the proposition that every F has G . Strong second-order condition:

$$(\forall F)(\forall G)(\forall x)[Fx \ \& \ Gx \text{ confirms } (\forall x)(Fx \supset Gx)]$$

- **Equivalence Condition (EC):** For any propositions H_1 , E , and H_2 , if E confirms H_1 and H_1 is (*classically!*) logically equivalent to H_2 , then E confirms H_2 . Weak 2nd order condition:

$$(\forall E)(\forall H_1)(\forall H_2)[E \text{ confirms } H_1 \text{ and } H_1 \models H_2 \Rightarrow E \text{ confirms } H_2]$$

- **Paradoxical Conclusion (PC):** The proposition that a is both nonblack and a nonraven confirms the proposition that every raven is black. This is a first-order condition (arbitrary a): $\sim Ba \ \& \ \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$.

Proof. (1) By (NC), $\sim Ba \ \& \ \sim Ra$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$.

(2) By Classical Logic, $(\forall x)(\sim Bx \supset \sim Rx) \models (\forall x)(Rx \supset Bx)$.

\therefore (PC) By (1), (2), (EC), $\sim Ba \ \& \ \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$.

- Hempel & Goodman *embraced* (NC), (EC) *and* (PC). They saw **no paradox**. Hempel *explains away* the paradoxical *appearance* (Goodman does same):
 - ... in the seemingly paradoxical cases of confirmation, we are often not judging the relation of the given evidence E *alone* to the hypothesis H
 - ... instead, we tacitly introduce a comparison of H with ... E in conjunction with ... additional ... information we ... have at our disposal.
- Hempel's Idea: $E [\sim Ra \ \& \ \sim Ba]$ confirms $H [(\forall x)(Rx \supset Bx)]$ *relative to* T , but E doesn't confirm H relative to some (nontautological) $K \neq T$.
- *Which* $K \neq T$? Later, Hempel discusses $K = \sim Ra$. Intuition: if you already know that a is a nonraven, then observing its color will not tell you anything about the color of ravens. Hempel: (PC) is true, but (PC*) is false:
 - (PC) $\sim Ra \ \& \ \sim Ba$ confirms $(\forall x)(Rx \supset Bx)$, relative to T .
 - (PC*) $\sim Ra \ \& \ \sim Ba$ confirms $(\forall x)(Rx \supset Bx)$, relative to $\sim Ra$.
- This is a good insight! Unfortunately, it is *logically incompatible* with the (deductive) confirmation *theories* that Hempel and Goodman accept.
- Specifically, this possibility contradicts the *K-monotonicity* property:

(M_K) E confirms H , relative to $T \Rightarrow E$ confirms H relative to *any* K (provided that K does not mention any individuals not already mentioned in E).

- Because Hempel's theory of confirmation satisfies (M), his theory implies that (PC) entails (PC*). So, it is logically impossible for Hempel's theory to undergird his suggestion that (PC) is true, while (PC*) is false.
- This is bad news for Hempel/Goodman. Surprisingly, nobody noticed this inconsistency in the Hempel/Goodman approach to the paradox.
- As we will see shortly, *Bayesians* can better accommodate Hempel's intuitions here, since *their* theory of confirmation does *not* satisfy (M).
- Interestingly, later in this very same passage, Hempel offers an argument for premise (1) which, itself, *depends on* this very monotonicity property!
 If ... E consists *only* of one ... nonraven [$\sim Ra$], then E ... confirm[s] that all objects are nonravens [$(\forall x) \sim Rx$], and *a fortiori* E supports the weaker assertion that all nonblack objects are nonravens [$(\forall x)(\sim Bx \supset \sim Rx)$].
- The dependence on (M) is almost invisible here! My conjecture: (M) is a vestige of “objectual” ways of thinking about confirmation (like NC₀).

- This independent argument for (1) presupposes not only (M), but (SCC):
(SCC) $(\forall E)(\forall H_1)(\forall H_2)[E \text{ confirms } H_1 \text{ and } H_1 \models H_2 \Rightarrow E \text{ confirms } H_2]$.
- To see this, take a closer look at the reasoning of the argument:
 - (1.1) $\sim Ra$ confirms $(\forall x)\sim Rx$. (Nicod)
 - (1.2) $(\forall x)\sim Rx \models (\forall x)(\sim Bx \supset \sim Rx)$ (Logic)
 - (1.3) $\sim Ra$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$ (SCC)
 - (1) $\sim Ra$ & $\sim Ba$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$ (M?!)
- (M) and (SCC) are consequences of Hempel's (and Goodman's) confirmation theory, which says E confirms H iff $E \models \text{dev}_E(H)$.
- Modern Bayesians *reject both* (M) *and* (SCC). And, as a result, Bayesians are able to say what Hempel wanted to say (but can't!).
- Before Bayesianism, we'll look briefly at Scheffler [who accepts (NC), but denies (EC)], and Quine [who accepts (EC), but denies (NC)].
- Then, we'll look at Bayesian resolutions of several different kinds. Some of these will reject (NC), while others will take a different tack.

- Scheffler rejects (PC), but accepts (1). He denies (EC). He proposes:
 (*) A Hempelian positive instance (E) of a \forall -hypothesis (H) confirms H ,
unless E is also a positive instance of the contrary (H') of H .
- Let $H: (\forall x)[Rx \rightarrow Bx]$. The contrary of H is $H': (\forall x)[Rx \rightarrow \sim Bx]$. Let
 $E: \sim Ra \ \& \ \sim Ba$. E is a Hempelian positive instance of H , *and* H' .
- Thus, according to Scheffler's (*), E does not confirm H after all.
- Scheffler accepts (1) [and (NC)]. E confirms $H^*: (\forall x)[\sim Bx \rightarrow \sim Rx]$ —
even according to ()*. This is because E is *not* a Hempelian positive
 instance of the *contrary* of H^* , $H^{*'}: (\forall x)[\sim Bx \rightarrow Rx]$.
- This leads to a violation of (EC), of course, since – according to (*) – E
 confirms H^* , but E does not confirm H — even though $H \models H^*$.
- Is Scheffler's (*) true? **Exercise:** show that Scheffler's (*), and (NC) are
 both *false* from the point of view of PR-theory. I'll return to this when
 we discuss I.J. Good and (NC). This will be one of the many subtle (and
 non-Hempelian) aspects of probabilistic relevance accounts of c .