

Philosophy 148 — Announcements & Such

- Overall, people did very well on the mid-term ($\mu = 90, \sigma = 16$).
- HW #2 graded will be posted very soon. Raul won't be able to give back the HW's until next week, and I will collect HW #3's for him today.
- Recall: we're using a straight grading scale for this course. Here it is:
 - A+ > 97, A (94,97], A- (90,94], B+ (87,90], B (84,87], B- (80,84], C+ (77,80], C (74,77], C- (70,74], D [50,70], F < 50.
- I've posted solutions for HW #1 (HW #2 solutions are coming soon).
- HW #3 is due today. HW #4 has been posted. We will have a discussion devoted to it in two weeks time. **Next week, I'm in Europe — so no office hours for me.** We'll have guest lecturers — so the class **will** meet!
- Today: More on "Logical" Probability and Inductive Logic
 - Review of Carnapian theory, and some new stuff on Carnap.
 - Another way of thinking about Inductive Logic.

Review of Carnapian "Logical" Probability 2

- *Exactly how c (i.e., Pr_\top) should be applicable to epistemology is not completely clear. But, Carnap says things which *constrain* applicability.*
- It should be possible for logically independent claims to be *correlated* with each other. Presumably, because it is possible for logically independent claims to *evidentially support* each other.
- Specifically, Carnap thinks Ga and Gb should be correlated "*a priori*":

$$\text{Pr}(Gb | Ga) > \text{Pr}(Gb) \quad \text{[this is called "instantial relevance"]}$$

- This *rules-out* m^\dagger . And, this leads Carnap to adopt m^* instead. But, m^* has "inapplicability problems" of its own. Carnap later came to think we should also be able to have the following chain of inequalities:

$$\text{Pr}(Gb | Ga) > \text{Pr}(Gb | Ga \& Fa \& \sim Fb) > \text{Pr}(Gb)$$

- But (HW #3!), *neither m^\dagger nor m^* are compatible with this.* This lead Carnap to continue his search. He moved to a more complex $m^{\lambda, \gamma}$.

Review of Carnapian "Logical" Probability 1

- Carnap endorses the *logicality desideratum* (\mathcal{D}_2) for c , and since Carnap defines c in terms of Pr , Carnap concludes this mandates a "logical" kind (theory/interpretation) of *probability* — the search for "logical Pr " is on.
- Carnap assumes that $c(C, P) = \text{Pr}(C | P)$. The principle of indifference (PI) follows from this assumption, since then " K does not *favor* any s_i over any s_j " \Rightarrow " K *confirms* each s_i to the same degree as K *confirms* each s_j " $\Rightarrow c(s_i, K) = c(s_j, K) \Rightarrow \text{Pr}(s_i | K) = \text{Pr}(s_j | K)$, for *all* i and j , *ergo* (PI).
- This leads Carnap, initially, to endorse the Wittgensteinian function m^\dagger , which assigns equal probability to each state description s_i of \mathcal{L} .
- But, m^\dagger implies that there can be no correlations between logically independent propositions. Carnap thinks this renders m^\dagger *inapplicable*.
- Here, Carnap is presupposing what I will call an *applicability desideratum*: (\mathcal{D}_3) Inductive logic (i.e., the theory of confirmation) should be *applicable* (presumably, to *epistemology*) — in *some substantive way*.

Carnap's Final Theory of "Logical" Probability ($m, n = 2, \lambda = \frac{1}{2}$)

Fa	Ga	Fb	Gb	Carnap's $m^{\frac{1}{2}, \gamma}(s_i)$ [where $\gamma_F, \gamma_G \in (0, 1)$]
T	T	T	T	$\frac{1}{9}\gamma_F\gamma_G(\gamma_G + \gamma_F(5\gamma_G + 1) + 2)$
T	T	T	F	$-\frac{1}{9}\gamma_F(5\gamma_F + 1)(\gamma_G - 1)\gamma_G$
T	T	F	T	$-\frac{1}{9}(\gamma_F - 1)\gamma_F\gamma_G(5\gamma_G + 1)$
T	T	F	F	$\frac{5}{9}(\gamma_F - 1)\gamma_F(\gamma_G - 1)\gamma_G$
T	F	T	T	$-\frac{1}{9}\gamma_F(5\gamma_F + 1)(\gamma_G - 1)\gamma_G$
T	F	T	F	$\frac{1}{9}\gamma_F(\gamma_G - 1)(\gamma_G + \gamma_F(5\gamma_G - 6) - 3)$
T	F	F	T	$\frac{5}{9}(\gamma_F - 1)\gamma_F(\gamma_G - 1)\gamma_G$
T	F	F	F	$-\frac{1}{9}(\gamma_F - 1)\gamma_F(\gamma_G - 1)(5\gamma_G - 6)$
F	T	T	T	$-\frac{1}{9}(\gamma_F - 1)\gamma_F\gamma_G(5\gamma_G + 1)$
F	T	T	F	$\frac{5}{9}(\gamma_F - 1)\gamma_F(\gamma_G - 1)\gamma_G$
F	T	F	T	$\frac{1}{9}(\gamma_F - 1)\gamma_G(\gamma_F + (5\gamma_F - 6)\gamma_G - 3)$
F	T	F	F	$-\frac{1}{9}(\gamma_F - 1)(5\gamma_F - 6)(\gamma_G - 1)\gamma_G$
F	F	T	T	$\frac{5}{9}(\gamma_F - 1)\gamma_F(\gamma_G - 1)\gamma_G$
F	F	T	F	$-\frac{1}{9}(\gamma_F - 1)\gamma_F(\gamma_G - 1)(5\gamma_G - 6)$
F	F	F	T	$-\frac{1}{9}(\gamma_F - 1)(5\gamma_F - 6)(\gamma_G - 1)\gamma_G$
F	F	F	F	$\frac{1}{9}(\gamma_F - 1)(\gamma_G - 1)(-6\gamma_G + \gamma_F(5\gamma_G - 6) + 9)$

Carnapian Logical Probability: Analogy and Similarity 1

- More generally: if a has properties $P_1 \dots P_n$, and b has $P_1 \dots P_{n-2}$, but lacks P_{n-1} , that should be of *some* relevance to b 's having P_n ; $n = 3$ case:

$$\Pr(Hb | Ha) > \Pr(Hb | Ha \& Fa \& Ga \& Fb \& \sim Gb)$$

$$> \Pr(Hb | Ha \& Fa \& Ga \& \sim Fb \& \sim Gb) > \Pr(Hb)$$

- *I.e.*, Differing on 2 properties should be worse than 1, but neither should completely undermine instantial relevance. \Pr^\dagger and \Pr^* violate this.
- The “analogical” idea here involves certain judgments of “similarity” of objects, where “similarity” is measured by “counting shared predicates.”
- The slogan seems to be: The more properties a and b “share”, the more this *enhances* instantial relevance; and, the more properties they are known to *differ* on, the more this *undermines* instantial relevance.
- Unfortunately, any theory that satisfies this analogical principle will be *language-variant*, so long as the languages contain 3 or more predicates.

Carnapian Logical Probability: Analogy and Similarity 3

- That is, “the degree of similarity of a and b ” depends sensitively on the *syntax* of the language one uses to *describe* a and b . [Note: if $n = 2$, there is no language-variance — it requires 3 or more predicates.] Here’s why.

- The $ABCD$ language consists of four predicates A, B, C , and D . And, the $XYZU$ language also has four predicates X, Y, Z , and U such that $Xx \models Ax \equiv Bx, Yx \models Bx \equiv Cx, Zx \models Ax$, and $Ux \models Dx$.

☞ $ABCD$ and $XYZU$ are (extra-systematically) **expressively equivalent**.

Anything that can be said in $ABCD$ can be said in $XYZU$, and conversely — *intuitively, there is no semantic difference between the two languages.*

- Now, consider two objects a and b such that:

$$Aa \& Ba \& Ca \& Da$$

$$Ab \& \sim Bb \& Cb \& Db$$

- Question: How similar are a and b in our “predicate-sharing” sense?

Carnapian Logical Probability: Analogy and Similarity 2

- Let x_1, \dots, x_n be the objects that fall under the predicate X [*i.e.*, those in $\text{Ext}(X)$]. And, let $s(x_1, \dots, x_n)$ be some measure of “the degree to which the objects falling under X are similar to each other”.
- Carnap doesn’t offer much in the way of a *theory* of $s(x_1, \dots, x_n)$. But, his discussion suggests the following account of $s(x_1, \dots, x_n)$:
- Let $\mathcal{P}(x)$ be the set of predicates that x falls under. Then, define:

$$s(x_1, \dots, x_n) = \left| \bigcap_i \mathcal{P}(x_i) \right|$$

- That is, $s(x_1, \dots, x_n)$ is the size (cardinality) of the intersection of all the $\mathcal{P}(x_i)$. This is “the size of the set of shared predicates of the x_i ”.
- There is a problem with this idea. Next, I will present an argument which shows that this measure of similarity is *language variant*.

Carnapian Logical Probability: Analogy and Similarity 4

- Answer: That depends on which of our expressively equivalent languages we use to describe a and b ! To see this, note that in $XYZU$ we have:

$$Xa \& Ya \& Za \& Ua$$

$$\sim Xb \& \sim Yb \& Zb \& Ub$$

- Therefore, in $ABCD$, a and b share three predicates. But, in $XYZU$, a and b share only two predicates. Or, to use a modified notation, we have:

$$s_{ABCD}(a, b) = 3 \neq 2 = s_{XYZU}(a, b)$$

- On the other hand, *probabilities* should *not* be language-variant. It shouldn’t matter which language you use to describe the world — equivalent statements should be *probabilistically indistinguishable*.
- One consequence is that if $p \models q, x \models y$ and $z \models u$, then we shouldn’t have both $\Pr(p | x) > \Pr(p | u)$ and $\Pr(q | y) < \Pr(q | z)$. Carnapian principles of analogy and similarity *contradict* this requirement ($n \geq 3$).

Carnapian Logical Probability: Analogy and Similarity 5

- Here's a concise way of stating the general Carnapian analogical principle:

(A) If $n > m$, then $\Pr(Xa | Xb \ \& \ s(a, b) = n) > \Pr(Xa | Xb \ \& \ s(a, b) = m)$

- Applying this principle to our example yields *both* of the following:

- (1) $\Pr(Da | Db \ \& \ Aa \ \& \ Ba \ \& \ Ca \ \& \ Ab \ \& \ \sim Bb \ \& \ Cb) > \Pr(Da | Db \ \& \ Aa \ \& \ Ba \ \& \ Ca \ \& \ Ab \ \& \ \sim Bb \ \& \ \sim Cb)$
- (2) $\Pr(Ua | Ub \ \& \ Xa \ \& \ Ya \ \& \ Za \ \& \ \sim Xb \ \& \ Yb \ \& \ Zb) > \Pr(Ua | Ub \ \& \ Xa \ \& \ Ya \ \& \ Za \ \& \ \sim Xb \ \& \ \sim Yb \ \& \ Zb)$

- Now, let $p \stackrel{\text{def}}{=} Da$, $q \stackrel{\text{def}}{=} Ua$, and

$x \stackrel{\text{def}}{=} Db \ \& \ Aa \ \& \ Ba \ \& \ Ca \ \& \ Ab \ \& \ \sim Bb \ \& \ Cb$	$y \stackrel{\text{def}}{=} Ub \ \& \ Xa \ \& \ Ya \ \& \ Za \ \& \ \sim Xb \ \& \ \sim Yb \ \& \ Zb$
$z \stackrel{\text{def}}{=} Ub \ \& \ Xa \ \& \ Ya \ \& \ Za \ \& \ \sim Xb \ \& \ Yb \ \& \ Zb$	$u \stackrel{\text{def}}{=} Db \ \& \ Aa \ \& \ Ba \ \& \ Ca \ \& \ Ab \ \& \ \sim Bb \ \& \ \sim Cb$

- Then, $p \models q$, $x \models y$ and $z \models u$, but the Carnapian principle (A) implies *both* (1) $\Pr(p | x) > \Pr(p | u)$, and (2) $\Pr(q | y) < \Pr(q | z)$. Bad.
- It seems that principle (A) must go. Otherwise, some restriction on the choice of language is required to block inferring both (1) and (2) from it.

A Closer Look at Inductive Logic and Applicability 2

- Popper discusses examples of the following kind (we've seen an example like this in the class before), which involve testing for a rare disease.
 - Let E report a positive test result for a very rare disease (for someone named John), and let H be the (null) hypothesis that John does *not* have the disease in question. We assume further that John knows (his K entails) the test is highly reliable, and that the disease is very rare.
- In such an example, it is plausible (and Carnap should agree) that (to the extent that \Pr_T is *applicable* to modeling the *epistemic* relations here):
 - (1) $\Pr_T(H | E \ \& \ K)$ is very high.
 - (2) But, $\Pr_T(H | E \ \& \ K) < \Pr_T(H | K)$.
- Because of (2), it would be odd to say that E *supports* H (for John) in this context. (2) suggests that E is (intuitively) evidence *against* H here.
- But, because of (1), Carnap's (RTE_C) implies that E *supports* H (for John) here. This looks like a counterexample to [the \Leftarrow of] Carnap's (RTE_C) .

A Closer Look at Inductive Logic and Applicability 1

- Carnap suggested various *bridge principles* for connecting inductive logic and inductive epistemology. The most well-known of these was:
 - The Requirement of Total Evidence.** In the application of inductive logic to a given knowledge situation, the total evidence available must be taken as a basis for determining the degree of evidential support.
- A more precise way of putting this principle is:

(RTE) E evidentially supports H for an agent S in an epistemic context C
 $\iff c(H, E | K) > r$, where K is S 's total evidence in C .
- For Carnap, $c(H, E | K) = \Pr_T(H | E \ \& \ K)$, where \Pr_T is a suitable "logical" probability function. So, we can restate Carnap's (RTE) as follows:

(RTE_C) E evidentially supports H for an agent S in an epistemic context C
 $\iff \Pr_T(H | E \ \& \ K) > r$, where K is S 's total evidence in C .
- Carnap's version of (RTE) faces a challenge (first articulated by Popper) involving *probabilistic relevance vs high conditional probability*.

A Closer Look at Inductive Logic and Applicability 3

- This suggests the following refinement of Carnap's (RTE_C):

(RTE'_C) E evidentially supports H for an agent S in an epistemic context C
 $\implies \Pr_T(H | E \ \& \ K) > \Pr_T(H | K)$, where K is S 's total evidence in C .
- In other words, (RTE'_C) says that *evidential support in (for S in C) implies probabilistic relevance, conditional upon K* (for a suitable \Pr_T function).
- Note: this only states a *necessary* condition for evidential support.
- While (RTE'_C) avoids Popper's objection, it faces serious challenges of its own (e.g., Goodman's "Grue" example — more on that later). Here's one:
- Consider any context in which S already knows (*with certainty*) that E is true. That is, S 's total evidence in the context *entails* E ($K \models E$).
- In such a case, $\Pr(H | E \ \& \ K) = \Pr(H | K)$, for *any* probability function \Pr . Thus, (RTE'_C) implies that E *cannot support anything* (for S , in any such C). This shows that (RTE'_C) isn't a correct principle either. ["Old Evidence"]

A Closer Look at Inductive Logic and Applicability 4

- I think this whole way of approaching inductive logic is wrongheaded.
- First, why must Pr itself be logical, if c (which is defined in terms of Pr) is to be logical? Analogy: must the truth-value assignment function v itself be logical, if \models (which is defined in terms of v) is to be logical?
 - But: there is a crucial disanalogy here, which I will discuss below.
- Second, Carnap's proposal $c(H, E | K) = \text{Pr}_T(H | E \& K)$ is suspect, because (as Popper pointed out) it is not sensitive to *probabilistic relevance*.
 - Note: this undermines Carnap's argument for the "logicality" of (PI).
- Third, the applicability desideratum (\mathcal{D}_3) may be fundamentally misguided. The search for logic/epistemology "bridge principles" is fraught with danger, even in the *deductive* case. And, since IL is supposed to *generalize* DL, it will also face these dangers *and new ones* (as above).
 - I think this is the true (but, surprisingly, unappreciated) lesson of Goodman's "grue" example. I will explain why in the confirmation unit.

An Alternative Conception of Inductive Logic 2

- Here's the idea. Confirmation is a *four*-place relation, between E, H, K , and a *probability function* Pr . The resulting relation is still *logical* in Carnap's sense, since, *given* a choice of Pr , c is logically (mathematically, if you prefer) determined, provided only that c is defined in terms of Pr .
- So, on this conception, desiderata (\mathcal{D}_1) and (\mathcal{D}_2) are satisfied.
- As usual, the subtle questions involve the applicability desideratum (\mathcal{D}_3).
- What do we say about that? Well, I think any naive "bridge principle" like (RTE or RTE') is doomed to failure. But, perhaps there is *some* connection.
- Thinking back to the deductive case, there may be *some* connection between deductive *logic* and deductive *inference*. But, what is it?
- This is notoriously difficult to say. The best hope seems to be that there is *some* connection between *knowledge* and entailment. [Note: connecting or bridging *justified belief* and *entailment* seems much more difficult.]

An Alternative Conception of Inductive Logic 1

- In light of the above considerations, we might seek a measure c satisfying the following (provided that E, H , and K are *logically contingent*):

$$c(H, E | K) \text{ should be } \begin{cases} \text{Maximal } (> 0, \text{ constant}) & \Leftarrow E \& K \models (\text{or } \models) H. \\ > 0 \text{ (confirmation rel. to } K) & \Rightarrow \text{Pr}(H | E \& K) > \text{Pr}(H | K). \\ = 0 \text{ (irrelevance rel. to } K) & \Rightarrow \text{Pr}(H | E \& K) = \text{Pr}(H | K). \\ < 0 \text{ (disconfirmation rel. to } K) & \Rightarrow \text{Pr}(H | E \& K) < \text{Pr}(H | K). \\ \text{Minimal } (< 0, \text{ constant}) & \Leftarrow E \& K \models (\text{or } \models) \sim H. \end{cases}$$

- Carnap would add: "and Pr should be a 'logical' probability function Pr_T ". But, I suggested that this was a mistake. OK, but then what do I say about the Pr 's above? There is an implicit quantifier over the Pr 's above...
 - \exists is *too weak* a quantifier here, since there will *always* be *some* such Pr .
 - \forall is *too strong* a quantifier here, because that is *demonstrably false!*
 - What's the alternative? The alternative is that Pr is a *parameter* in c itself. That is, perhaps Pr is simply an *argument* of the function c .

An Alternative Conception of Inductive Logic 3

- Many people think there is *some* connection between knowledge and entailment. But, simple/naive version of bridge principles don't work. Here is a progression of increasingly subtle "bridge principles":
 1. If S knows p and $p \models q$, then S knows q .
 - What if S doesn't *know* that $p \models q$?
 2. If S knows p and S knows $p \models q$, then S knows q .
 - What if S doesn't even *believe* q ? [Discuss Dretske's Zebra Example.]
 3. If S knows p and S knows $p \models q$ and S believes q , then S knows q .
 - What if S believes q for reasons *other than* p ?
 4. If S knows p and S knows $p \models q$ and S comes to believe q because of their (initial) belief that p , then S knows q .
 - What if S no longer believes p , while/after they infer q ?
 5. If S knows p & S knows $p \models q$ & S competently deduces q from p (thus coming to believe q) while maintaining their belief in p , then S knows q .