Philosophy 148 — Announcements & Such

- Overall, people did very well on the mid-term ($\mu = 90, \sigma = 16$).
- HW #2 graded will be posted very soon. Raul won't be able to give back the HW's until next week, and I will collect HW #3's for him today.
- Recall: we're using a straight grading scale for this course. Here it is:
  - A+ > 97, A (94,97], A- (90,94], B+ (87,90], B (84,87], B- (80,84], C+ (77,80], C (74,77], C- (70,74], D [50,70], F < 50.
- I've posted solutions for HW #1 (HW #2 solutions are coming soon).
- HW #3 is due today. HW #4 has been posted. We will have a discussion devoted to it in two weeks time. **Next week, I'm in Europe — so no office hours for me.** We'll have guest lecturers — so the class **will** meet!
- Today: More on "Logical" Probability and Inductive Logic
  - Review of Carnapian theory, and some new stuff on Carnap.
  - Another way of thinking about Inductive Logic.

Review of Carnapian "Logical" Probability 1

- Carnap endorses the *logicality desideratum* ($\mathcal{D}_2$) for $\mathfrak{c}$, and since Carnap defines $\mathfrak{c}$ in terms of Pr, Carnap concludes this mandates a "logical" kind (theory/interpretation) of *probability* — the search for "logical Pr" is on.
- Carnap assumes that $\mathfrak{c}(C, P) = \Pr(C \mid P)$. The principle of indifference (PI) follows from this assumption, since then "K does not *favor* any s_i over any s_j" $\Rightarrow$ "K *confirms* each s_i to the same degree as K *confirms* each s_j" $\Rightarrow \mathfrak{c}(s_i, K) = \mathfrak{c}(s_j, K) \Rightarrow \Pr(s_i \mid K) = \Pr(s_j \mid K)$, for *all* i and j, *ergo* (PI).
- This leads Carnap, initially, to endorse the Wittgensteinian function $\mathfrak{m}^\dagger$, which assigns equal probability to each state description s_i of $\mathcal{L}$.
- But, $\mathfrak{m}^\dagger$ implies that there can be no correlations between logically independent propositions. Carnap thinks this renders $\mathfrak{m}^\dagger$ *inapplicable*.
- Here, Carnap is presupposing what I will call an *applicability desideratum*:

($\mathcal{D}_3$) Inductive logic (*i.e.*, the theory of $\mathfrak{c}$onfirmation) should be *applicable* (presumably, to *epistemology*) — in *some substantive way*.

Review of Carnapian "Logical" Probability 2

- *Exactly how* $\mathfrak{c}$ (*i.e.*, $\Pr_{\mathsf{T}}$) should be applicable to epistemology is not completely clear. But, Carnap says things which *constrain* applicability.
  - It should be possible for logically independent claims to be *correlated* with each other. Presumably, because it is possible for logically independent claims to *evidentially support* each other.
  - Specifically, Carnap thinks Ga and Gb should be correlated "*a priori*":

$$\Pr(Gb \mid Ga) > \Pr(Gb) \qquad \text{[this is called "instantial relevance"]}$$

  - This *rules-out* $\mathfrak{m}^{\dagger}$. And, this leads Carnap to adopt $\mathfrak{m}^*$ instead. But, $\mathfrak{m}^*$ has "inapplicability problems" of its own. Carnap later came to think we should also be able to have the following chain of inequalities:

$$\Pr(Gb \mid Ga) > \Pr(Gb \mid Ga \,\&\, Fa \,\&\, {\sim}Fb) > \Pr(Gb)$$

  - But (HW #3!), *neither* $\mathfrak{m}^{\dagger}$ *nor* $\mathfrak{m}^*$ are compatible with this. This lead Carnap to continue his search. He moved to a more complex $\mathfrak{m}^{\lambda,\gamma}$.

Carnap's Final Theory of "Logical" Probability ($m, n = 2, \lambda = \frac{1}{2}$)

Fa	Ga	Fb	Gb	Carnap's $\mathfrak{m}^{\frac{1}{2},\gamma}(s_i)$ [where $\gamma_F, \gamma_G \in (0,1)$]
T	T	T	T	$\frac{1}{9}\gamma_F\gamma_G\left(\gamma_G+\gamma_F\left(5\gamma_G+1\right)+2\right)$
T	T	T	F	$-\frac{1}{9}\gamma_F\left(5\gamma_F+1\right)\left(\gamma_G-1\right)\gamma_G$
T	T	F	T	$-\frac{1}{9}\left(\gamma_F-1\right)\gamma_F\gamma_G\left(5\gamma_G+1\right)$
T	T	F	F	$\frac{5}{9}\left(\gamma_F-1\right)\gamma_F\left(\gamma_G-1\right)\gamma_G$
T	F	T	T	$-\frac{1}{9}\gamma_F\left(5\gamma_F+1\right)\left(\gamma_G-1\right)\gamma_G$
T	F	T	F	$\frac{1}{9}\gamma_F\left(\gamma_G-1\right)\left(\gamma_G+\gamma_F\left(5\gamma_G-6\right)-3\right)$
T	F	F	T	$\frac{5}{9}\left(\gamma_F-1\right)\gamma_F\left(\gamma_G-1\right)\gamma_G$
T	F	F	F	$-\frac{1}{9}\left(\gamma_F-1\right)\gamma_F\left(\gamma_G-1\right)\left(5\gamma_G-6\right)$
F	T	T	T	$-\frac{1}{9}\left(\gamma_F-1\right)\gamma_F\gamma_G\left(5\gamma_G+1\right)$
F	T	T	F	$\frac{5}{9}\left(\gamma_F-1\right)\gamma_F\left(\gamma_G-1\right)\gamma_G$
F	T	F	T	$\frac{1}{9}\left(\gamma_F-1\right)\gamma_G\left(\gamma_F+\left(5\gamma_F-6\right)\gamma_G-3\right)$
F	T	F	F	$-\frac{1}{9}\left(\gamma_F-1\right)\left(5\gamma_F-6\right)\left(\gamma_G-1\right)\gamma_G$
F	F	T	T	$\frac{5}{9}\left(\gamma_F-1\right)\gamma_F\left(\gamma_G-1\right)\gamma_G$
F	F	T	F	$-\frac{1}{9}\left(\gamma_F-1\right)\gamma_F\left(\gamma_G-1\right)\left(5\gamma_G-6\right)$
F	F	F	T	$-\frac{1}{9}\left(\gamma_F-1\right)\left(5\gamma_F-6\right)\left(\gamma_G-1\right)\gamma_G$
F	F	F	F	$\frac{1}{9}\left(\gamma_F-1\right)\left(\gamma_G-1\right)\left(-6\gamma_G+\gamma_F\left(5\gamma_G-6\right)+9\right)$

Carnapian Logical Probability: Analogy and Similarity 1

- More generally: if a has properties $P_1 \ldots P_n$, and b has $P_1 \ldots P_{n-2}$, but lacks P_{n-1}, that should be of *some* relevance to b's having P_n; $n = 3$ case:

$$\Pr(Hb \mid Ha) > \Pr(Hb \mid Ha \,\&\, Fa \,\&\, Ga \,\&\, Fb \,\&\, {\sim}Gb)$$
$$> \Pr(Hb \mid Ha \,\&\, Fa \,\&\, Ga \,\&\, {\sim}Fb \,\&\, {\sim}Gb) > \Pr(Hb)$$

- *I.e.*, Differing on 2 properties should be worse than 1, but neither should completely undermine instantial relevance. $\Pr^{\dagger}$ and $\Pr^{*}$ violate this.
- The "analogical" idea here involves certain judgments of "similarity" of objects, where "similarity" is measured by "counting shared predicates."
- The slogan seems to be: The more properties a and b "share", the more this *enhances* instantial relevance; and, the more properties they are known to *differ* on, the more this *undermines* instantial relevance.
- Unfortunately, any theory that satisfies this analogical principle will be *language-variant*, so long as the languages contain 3 or more predicates.

Carnapian Logical Probability: Analogy and Similarity 2

- Let $x_1, \ldots, x_n$ be the objects that fall under the predicate X [*i.e.*, those in $\mathrm{Ext}(X)$]. And, let $\mathfrak{s}(x_1, \ldots, x_n)$ be some measure of "the degree to which the objects falling under X are similar to each other".
- Carnap doesn't offer much in the way of a *theory* of $\mathfrak{s}(x_1, \ldots, x_n)$. But, his discussion suggests the following account of $\mathfrak{s}(x_1, \ldots, x_n)$:
- Let $\mathcal{P}(x)$ be the set of predicates that x falls under. Then, define:

$$\mathfrak{s}(x_1, \ldots, x_n) = \left| \bigcap_i \mathcal{P}(x_i) \right|$$

- That is, $\mathfrak{s}(x_1, \ldots, x_n)$ is the size (cardinality) of the intersection of all the $\mathcal{P}(x_i)$. This is "the size of the set of shared predicates of the x_i".
- There is a problem with this idea. Next, I will present an argument which shows that this measure of similarity is *language variant*.

Carnapian Logical Probability: Analogy and Similarity 3

- That is, "the degree of similarity of a and b" depends sensitively on the *syntax* of the language one uses to *describe* a and b. [Note: if $n = 2$, there is no language-variance — it requires 3 or more predicates.] Here's why.

- The $ABCD$ language consists of four predicates A, B, C, and D. And, the $XYZU$ language also has four predicates X, Y, Z, and U such that $Xx \mathrel{=\!\mid\!\mid\!\mid\!\models} Ax \equiv Bx$, $Yx \mathrel{=\!\mid\!\mid\!\mid\!\models} Bx \equiv Cx$, $Zx \mathrel{=\!\mid\!\mid\!\mid\!\models} Ax$, and $Ux \mathrel{=\!\mid\!\mid\!\mid\!\models} Dx$.

☞ *$ABCD$ and $XYZU$ are (extra-systematically)* ***expressively equivalent***. Anything that can be said in $ABCD$ can be said in $XYZU$, and conversely — *intuitively, there is no semantic difference between the two languages.*

- Now, consider two objects a and b such that:

$$Aa \,\&\, Ba \,\&\, Ca \,\&\, Da$$
$$Ab \,\&\, {\sim}Bb \,\&\, Cb \,\&\, Db$$

- Question: How similar are a and b in our "predicate-sharing" sense?

Carnapian Logical Probability: Analogy and Similarity 4

- Answer: That depends on which of our expressively equivalent languages we use to describe a and b! To see this, note that in $XYZU$ we have:

$$Xa \,\&\, Ya \,\&\, Za \,\&\, Ua$$
$$\sim\! Xb \,\&\, \sim\! Yb \,\&\, Zb \,\&\, Ub$$

- Therefore, in $ABCD$, a and b share three predicates. But, in $XYZU$, a and b share only two predicates. Or, to use a modified notation, we have:

$$\mathfrak{s}_{ABCD}(a,b) = 3 \neq 2 = \mathfrak{s}_{XYZU}(a,b)$$

- On the other hand, *probabilities* should *not* be language-variant. It shouldn't matter which language you use to describe the world — equivalent statements should be *probabilistically indistinguishable*.
- One consequence is that if $p \dashv\Vdash\vDash q$, $x \dashv\Vdash\vDash y$ and $z \dashv\Vdash\vDash u$, then we shouldn't have both $\Pr(p \mid x) > \Pr(p \mid u)$ and $\Pr(q \mid y) < \Pr(q \mid z)$. Carnapian principles of analogy and similarity *contradict* this requirement ($n \geqslant 3$).

Carnapian Logical Probability: Analogy and Similarity 5

- Here's a concise way of stating the general Carnapian analogical principle:

($\mathcal{A}$) If $n > m$, then $\Pr(Xa \mid Xb \,\&\, \mathfrak{s}(a,b) = n) > \Pr(Xa \mid Xb \,\&\, \mathfrak{s}(a,b) = m)$

- Applying this principle to our example yields *both* of the following:

(1) $\Pr(Da \mid Db \& Aa \& Ba \& Ca \& Ab \& {\sim}Bb \& Cb) > \Pr(Da \mid Db \& Aa \& Ba \& Ca \& Ab \& {\sim}Bb \& {\sim}Cb)$

(2) $\Pr(Ua \mid Ub \& Xa \& Ya \& Za \& {\sim}Xb \& Yb \& Zb) > \Pr(Ua \mid Ub \& Xa \& Ya \& Za \& {\sim}Xb \& {\sim}Yb \& Zb)$

- Now, let $p \overset{\text{def}}{=} Da$, $q \overset{\text{def}}{=} Ua$, and

$x \overset{\text{def}}{=} Db \,\&\, Aa \,\&\, Ba \,\&\, Ca \,\&\, Ab \,\&\, {\sim}Bb \,\&\, Cb$	$y \overset{\text{def}}{=} Ub \,\&\, Xa \,\&\, Ya \,\&\, Za \,\&\, {\sim}Xb \,\&\, {\sim}Yb \,\&\, Zb$
$z \overset{\text{def}}{=} Ub \,\&\, Xa \,\&\, Ya \,\&\, Za \,\&\, {\sim}Xb \,\&\, Yb \,\&\, Zb$	$u \overset{\text{def}}{=} Db \,\&\, Aa \,\&\, Ba \,\&\, Ca \,\&\, Ab \,\&\, {\sim}Bb \,\&\, {\sim}Cb$

- Then, $p \mathrel{=\!\!|\!|\!|\!\!\vDash} q$, $x \mathrel{=\!\!|\!|\!|\!\!\vDash} y$ and $z \mathrel{=\!\!|\!|\!|\!\!\vDash} u$, but the Carnapian principle ($\mathcal{A}$) implies *both* (1) $\Pr(p \mid x) > \Pr(p \mid u)$, *and* (2) $\Pr(q \mid y) < \Pr(q \mid z)$. Bad.

- It seems that principle ($\mathcal{A}$) must go. Otherwise, some restriction on the choice of language is required to block inferring both (1) and (2) from it.

A Closer Look at Inductive Logic and Applicability 1

- Carnap suggested various *bridge principles* for connecting inductive logic and inductive epistemology. The most well-known of these was:
  - **The Requirement of Total Evidence**. In the application of inductive logic to a given knowledge situation, the total evidence available must be taken as a basis for determining the degree of evidential support.
- A more precise way of putting this principle is:

(RTE) E evidentially supports H *for an agent* S *in an epistemic context* C $\iff \mathfrak{c}(H, E \mid K) > r$, where K is S's *total evidence in* C.

- For Carnap, $\mathfrak{c}(H, E \mid K) = \Pr_{\mathsf{T}}(H \mid E \,\&\, K)$, where $\Pr_{\mathsf{T}}$ is a suitable "logical" probability function. So, we can restate *Carnap's* (RTE) as follows:

(RTE_C) E evidentially supports H *for an agent* S *in an epistemic context* C $\iff \Pr_{\mathsf{T}}(H \mid E \,\&\, K) > r$, where K is S's *total evidence in* C.

- Carnap's version of (RTE) faces a challenge (first articulated by Popper) involving *probabilistic relevance vs high conditional probability*.

A Closer Look at Inductive Logic and Applicability 2

- Popper discusses examples of the following kind (we've seen an example like this in the class before), which involve testing for a rare disease.
  - Let E report a positive test result for a very rare disease (for someone named John), and let H be the (null) hypothesis that John does *not* have the disease in question. We assume further that John knows (his K entails) the test is highly reliable, and that the disease is very rare.
- In such an example, it is plausible (and Carnap should agree) that (to the extent that $\Pr_{\mathsf{T}}$ is *applicable* to modeling the *epistemic* relations here):

  (1) $\Pr_{\mathsf{T}}(H \mid E \,\&\, K)$ is very high.

  (2) But, $\Pr_{\mathsf{T}}(H \mid E \,\&\, K) < \Pr_{\mathsf{T}}(H \mid K)$.
- Because of (2), it would be odd to say that E *supports* H (for John) in this context. (2) suggests that E is (intuitively) evidence *against* H here.
- But, because of (1), Carnap's (RTE_C) implies that E supports H (for John) here. This looks like a counterexample to [the $\Leftarrow$ of] Carnap's (RTE_C).

A Closer Look at Inductive Logic and Applicability 3

- This suggests the following refinement of Carnap's (RTE$_C$):

(RTE$'_C$) E evidentially supports H *for an agent* S *in an epistemic context* C $\Longrightarrow \Pr_{\mathsf{T}}(H \mid E \,\&\, K) > \Pr_{\mathsf{T}}(H \mid K)$, where K is S's *total evidence in* C.

- In other words, (RTE$'_C$) says that *evidential support in (for* S *in* C*) implies probabilistic relevance, conditional upon* K (for a suitable $\Pr_{\mathsf{T}}$ function).
- Note: this only states a *necessary* condition for evidential support.
- While (RTE$'_C$) avoids Popper's objection, it faces serious challenges of its own (e.g., Goodman's "Grue" example — more on that later). Here's one:
- Consider any context in which S already knows (*with certainty*) that E is true. That is, S's total evidence in the context *entails* E ($K \Vdash E$).
- In such a case, $\Pr(H \mid E \,\&\, K) = \Pr(H \mid K)$, for ***any*** probability function Pr. Thus, (RTE$'_C$) implies that E *cannot support anything* (for S, in any such C). This shows that (RTE$'_C$) isn't a correct principle either. ["Old Evidence"]

A Closer Look at Inductive Logic and Applicability 4

- I think this whole way of approaching inductive logic is wrongheaded.
- First, why must Pr *itself* be logical, if $\mathfrak{c}$ (which is defined in terms of Pr) is to be logical? Analogy: must the truth-value assignment function v *itself* be logical, if $\vDash$ (which is defined in terms of v) is to be logical?
  - But: there is a crucial disanalogy here, which I will discuss below.
- Second, Carnap's proposal $\mathfrak{c}(H, E \mid K) = \Pr_{\mathsf{T}}(H \mid E \,\&\, K)$ is suspect, because (as Popper pointed out) it is not sensitive to *probabilistic relevance*.
  - Note: this undermines Carnap's argument for the "logicality" of (PI).
- Third, the applicability desideratum ($\mathcal{D}_3$) may be fundamentally misguided. The search for logic/epistemology "bridge principles" is fraught with danger, even in the *deductive* case. And, since IL is supposed to *generalize* DL, it will also face these dangers *and new ones* (as above).
  - I think this is the true (but, surprisingly, un-appreciated) lesson of Goodman's "grue" example. I will explain why in the confirmation unit.

An Alternative Conception of Inductive Logic 1

- In light of the above considerations, we might seek a measure $\mathfrak{c}$ satisfying the following (provided that E, H, and K are *logically contingent*):

$$\mathfrak{c}(H, E \mid K) \text{ should be } \begin{cases} \text{Maximal } (> 0, \text{ constant}) & \Leftarrow E \,\&\, K \vDash (\text{or} \Vdash)\ H. \\ > 0 \text{ (confirmation rel. to } K) & \Rightarrow \Pr(H \mid E \,\&\, K) > \Pr(H \mid K). \\ = 0 \text{ (irrelevance rel. to } K) & \Rightarrow \Pr(H \mid E \,\&\, K) = \Pr(H \mid K). \\ < 0 \text{ (disconfirmation rel. to } K) & \Rightarrow \Pr(H \mid E \,\&\, K) < \Pr(H \mid K). \\ \text{Minimal } (< 0, \text{ constant}) & \Leftarrow E \,\&\, K \vDash (\text{or} \Vdash) \sim H. \end{cases}$$

- Carnap would add: "and Pr should be a 'logical' probability function $\Pr_{\mathsf{T}}$". But, I suggested that this was a mistake. OK, but then what do *I* say about the Pr's above? There is an implicit quantifier over the Pr's above...
  - $\exists$ is *too weak* a quantifier here, since there will *always* be *some* such Pr.
  - $\forall$ is *too strong* a quantifier here, because that is *demonstrably false*!
  - What's the alternative? The alternative is that Pr is a *parameter* in $\mathfrak{c}$ itself. That is, perhaps Pr is simply an *argument* of the function $\mathfrak{c}$.

An Alternative Conception of Inductive Logic 2

- Here's the idea. Confirmation is a *four*-place relation, between E, H, K, *and a probability function* Pr. The resulting relation is still *logical* in Carnap's sense, since, *given* a choice of Pr, $\mathfrak{c}$ is logically (mathematically, if you prefer) determined, provided only that $\mathfrak{c}$ is defined in terms of Pr.
- So, on this conception, desiderata ($\mathcal{D}_1$) and ($\mathcal{D}_2$) are satisfied.
- As usual, the subtle questions involve the applicability desideratum ($\mathcal{D}_3$).
- What do we say about that? Well, I think any naive "bridge principle" like (RTE or RTE′) is doomed to failure. But, perhaps there is *some* connection.
- Thinking back to the deductive case, there may be *some* connection between deductive *logic* and deductive *inference*. But, what is it?
- This is notoriously difficult to say. The best hope seems to be that there is *some* connection between *knowledge* and entailment. [Note: connecting or bridging *justified belief* and *entailment* seems much more difficult.]

An Alternative Conception of Inductive Logic 3

- Many people think there is *some* connection between knowledge and entailment. But, simple/naive version of bridge principles don't work. Here is a progression of increasingly subtle "bridge principles":

1. If S knows p and $p \vDash q$, then S knows q.
   – What if S doesn't *know* that $p \vDash q$?
2. If S knows p *and* S *knows* $p \vDash q$, then S knows q.
   – What if S doesn't even *believe* q? [Discuss Dretske's Zebra Example.]
3. If S knows p and S knows $p \vDash q$ *and* S *believes* q, then S knows q.
   – What if S believes q for reasons *other than* p?
4. If S knows p and S knows $p \vDash q$ *and* S *comes to believe* q *because of their (initial) belief that* p, then S knows q.
   – What if S *no longer* believes p, *while/after* they infer q?
5. If S knows p & S knows $p \vDash q$ & S *competently deduces* q *from* p *(thus coming to believe* q*) while maintaining their belief in* p, then S knows q.