Philosophy 148 — Announcements & Such

- We will have a “homework discussion” before each HW is due. For HW #2 and the Mid-Term, we have: **March 17 @ 6pm @ 223 Dwinelle.**
- Also: we will be *dropping your lowest HW score* in the course.
- HW #2 is posted (due 3/20). There is also a *hints handout* for HW #2.
- 3 Schedule Changes: Mid-Term moved back 2 weeks (3/6 to 3/20), HW 3 moved back 1 week (3/13 to 3/20), HW 2 due 3/20 (not 3/13).
- Brief Review from Last Time
  - Reviewing and motivating the set-up of the Dutch Book Argument.
  - Critically evaluating the DBA.
- Then: Epistemic Subjective Probability
  - Joyce’s Argument for Epistemic Probabilism
  - Critically evaluating Joyce’s argument

---

The Dutch Book Argument for (Pragmatic) Probabilism: Setup I

- The key assumptions/set-up of the Dutch Book argument are as follows:
  - For each proposition $p \in B'$ in our agent’s (Mr. B’s) doxastic state at $t$, Mr. B must announce a number $q(p)$ – called his *betting quotient* on $p$, at $t$ – and then Ms. A (the bookie) will choose the *stake* $s$ of the bet.
  - $|s|$ should be small in relation to Mr. B’s total wealth (more on this later). But, it can be positive or negative (so, she can “switch sides”).

  $\text{Mr. B’s payoff (in$ S \text{)} for a bet } \beta \text{ about } p = \begin{cases} s - q(p) \cdot s & \text{if } p \text{ is true.} \\ -q(p) \cdot s & \text{if } p \text{ is false.} \end{cases}$

  - NOTE: If $s > 0$, then the bet is *on* $p$, if $s < 0$, then the bet is *against* $p$.
  - $q(p)$ is taken to be a measure of Mr. B’s *degree of belief* in $p$ (at $t$).
  - If there is a sequence of multiple bets on multiple propositions, then Mr. B’s total payoff is the *sum* of the payoffs for each bet on each proposition. This is called “the package principle”. More on it later!

---

The Dutch Book Argument for (Pragmatic) Probabilism: Setup II

- “Fairness” of a bet is assumed to have the following meaning here:
  
  **Definition.** A bet $\beta$ about a proposition $p$ is said to be *fair* (for an agent $S$) if $\beta$ has an *expected utility* (for $S$) that is equal to zero.

- Let $q(\cdot)$ be $S$’s degree of belief function, $u(\cdot)$ be $S$’s utility function, $\beta_p$ be the outcome of bet $\beta$ if $p$ is true, and $\beta_{\neg p}$ be the outcome of $\beta$ if $p$ is false. Then, the expected utility (for $S$) of a bet $\beta$ about $p$ is defined as:

  **Definition.** $EU(\beta) \equiv q(p) \cdot u(\beta_p) + q(\neg p) \cdot u(\beta_{\neg p})$

- In the DBA setup, $u(\beta_p) \equiv s - q(p) \cdot s$, and $u(\beta_{\neg p}) \equiv -q(p) \cdot s$. Now, if we abbreviate $q(p)$ as $q$ and $q(\neg p)$ as $\neg q$, then, in the DBA setup, we have:

  $EU(\beta) = q \cdot (s - q \cdot s) + \neg q \cdot (\neg q \cdot s)$

  $= q \cdot s \cdot (1 - q - \neg q)$

- Therefore, if $\neg q = 1 - q$, then $EU(\beta) = 0$, and $\beta$ is a *fair bet* (by $S$’s lights).

- In this sense, the DBA setup is *fair* insofar as it *elicits probabilistic* $q$’s.
The Dutch Book Argument for (Pragmatic) Probabilism VII

- Here are some problems with/limitations of the Dutch Book Argument:
  - It assumes that bets (or gambles) which are severally acceptable are jointly acceptable. This “package principle” is not obvious (see below).
  - It is couched in terms of money. It tacitly assumes that utility is linear in money. But, money seems to have diminishing marginal utility.
  - More generally, DBAs involve betting behavior. One might wonder whether one’s betting behavior (when forced to post odds) is representative of rational behavior generally (gambling aversion?).
  - The DBA requires the converse DTB to be persuasive. And, even with the CDBT, it’s still not clear whether pragmatic probabilism follows from the DBA. Does it show that Pr is better than non-Pr?
  - It does not address process requirements, only state requirements, i.e., it does not constrain transitions from one doxastic state to another.
  - It presupposes P-rational agents are logically omniscient – that they can recognize all tautologies (in $B^t$). Does P-rationality require this?

Postscripts to DBA I: The Package Principle

- The Dutch Book argument for Additivity may appear as airtight as the Dutch Book arguments for Normality and Non-Negativity. But, Schick (and others) have spotted a possible flaw [see the Schick paper on website].
- In our Dutch Book argument for Additivity, we (implicitly) assumed:
  - Bets that are severally acceptable are jointly acceptable.
  - The value of a set of bets is the sum of values of its elements.
  - Might a rational agent be willing to accept each of the bets (1)-(3) without being willing to accept all three at once? And, if not why not?
  - Might they not value (1) at $Sa$, (2) at $Sb$, and (3) at $S−c$, without valuing the collection at $S[a + b − c]$? After all, they may see that, taken jointly, bets (1)-(3) lead to a sure loss, whereas no individual bet does.
  - It is important to note that the DBA for the ratio definition of conditional probability also presupposes this sort of “package principle”.
  - Are there counterexamples to the “package principle”? Maher thinks so:

Suppose that after a night on the town, you want to catch the bus home. Alas, you find that you only have 60 cents in your pocket, but the bus costs $1. A bookie, learning of your plight, offers you the following deal: If you give him your 60 cents, he will toss a coin, and if the coin lands heads then he will give you $1; otherwise, you have lost your 60 cents. If you accept the bookie’s proposal, then you stand a 50-50 chance of being able to take the bus home, while rejecting it means you will certainly have to walk. Under these conditions, you may well feel that the bookie’s offer is acceptable; let us suppose you do. Presumably, the offer would have been equally well acceptable if you were betting on tails rather than heads...

- As betting quotients were defined for the simple DBA, your quotient for heads in the above example is 0.6, and so is your quotient for tails.
- By additivity, $Pr(Heads ∨ Tails) = Pr(Heads) + Pr(Tails)$, since Heads and Tails are mutually exclusive. But, then, $q(Heads ∨ Tails) = 1.2 > 1$, which violates the axioms (recall, we have a theorem saying $Pr(p) ≤ 1$, for all $p$).
- Assumption: if you find a bet on Heads acceptable, and find a bet on Tails acceptable, then you should also find both bets acceptable, taken jointly.
- I have made a handout which shows exactly when the PP is required.

Postscripts to the DBA II: The Value of Money

- We have the caveat about $|s|$ being “small in comparison to the agent’s total wealth” for two reasons. First, if the agent could lose everything on a bet, this would undermine the probative value of the argument.
- Also, real agents (and, arguably, also rational agents!) marginally value money in a way that is non-linear, especially for larger sums of money.
- The difference in value between $1 and $1000 is pretty substantial. But, the difference in value between $1M + $1 and $1M + $1000 is not.
- This is called the diminishing marginal utility of money. Empirical studies show that actual agents have marginal utilities that are close to being linear only for $ amounts that are “small relative to their total wealth.”
- It’s not implausible to suppose that rational agents are like this too. So, if we want something with linear marginal value (for additivity purposes – think “package principle”), money is probably not the best thing to use.
**Postscripts to the DBA III: The Need for the Converse DBT**

- The DBT by itself cannot secure pragmatic probabilism. All the DBT establishes is that q’s coherence entails that q is a probability function. What if the converse of this were false? Would the DBA still be persuasive?
- If there were some probability functions that were also susceptible to Dutch Book, then pragmatic probabilism would not follow from the DBA.
- Remember, pragmatic probabilism says all probabilistic doxastic states are (pragmatically) better than all non-probabilistic doxastic states.
- The mere fact that (DBT) all non-probabilistic states are susceptible to Dutch Book is not sufficient to establish this. One also needs to show that (CDBT) all probabilistic doxastic states are immune from Dutch Book.
- Luckily, the (CDBT) is true. We have not proven this (and we will not). All we have shown is that particular Dutch Books for each of the axioms is blocked by satisfying that axiom. This does not establish (CDBT).
- See the Kemeny paper (on website), if you want to see the proof of (CDBT).

---

**Postscripts to the DBA IV: Is the Converse DBT Enough?**

- Even if we use both DBT and CDBT (which are both theorems), does the Dutch Book Argument establish pragmatic probabilism? Maybe not.
- Alan Hájek thinks there is still a gap in the DBA, even with both directions of the DBT, and even if we grant the truth of the “package principle”.
- As Hájek explains in his paper “Scotching Dutch Books” (now on website), the structure of the DBA is more-or-less something like the following:
  1. S’s q is coherent ↔ S’s q is a probability function. [DBT + CDBT]
  2. S is susceptible to Dutch Book ↔ S’s q is incoherent. [definition]
  3. Susceptibility to DB is “bad” and immunity from DB is “good”. [ass.]
  4. ∴ S’s q is better if it is probabilistic than if it is non-Pr. [P-probabilism]
- Does (4) follow from (1)–(3)? Clearly, (1)–(3) entail that there is some sense in which q is guaranteed to be “better” in virtue of being probabilistic.
- What if there is also some sense in which q is guaranteed to be “worse” in virtue of being probabilistic. Is there a symmetric “Good Book” argument?

---

**Postscripts to the DBA V: Hájek’s “Good Book” Argument**

- A Good Book is a sequence of bets that wins money come what may. Betting quotients q are good if they are susceptible to a Good Book.
- It turns out that the following as also a theorem (we won’t prove it, but it shouldn’t be surprising, given the Dutch Book theorem and it Converse):
  
  **Good Book Theorem** (GBT). q is good ↔ q is not a probability function.

- So, why not the following, symmetric argument against P-probabilism?
  1. S’s q is good ↔ S’s q is not a probability function. [GBT]
  2. S is susceptible to Good Book ↔ S’s q is good. [definition]
  3. Susceptibility to GB is “good” and immunity from GB is “bad”. [ass.]
  4. ∴ S’s q is worse if it is probabilistic than if it is non-Pr. [¬P-probabilism]

- The question now becomes: Is the DBA more compelling than the GBA? Which is pragmatically better: immunity from DB or susceptibility to GB?
- And, are there yet other dimensions of “goodness” we have overlooked?

---

**Postscripts to the DBA VI: States vs. Processes Again 1**

- The Dutch Book Argument we have seen only aims to establish that rationality requires an agent’s doxastic state at a particular time to be representable as a probability model. This is a state requirement.
- These kinds of requirements are also sometimes called requirements of synchronic rationality. One might wonder: what about processes that lead us from one probability model to another — diachronic rationality?
- Traditional subjective probabilists (like Ramsey and de Finetti) didn’t think there were any process requirements in this sense. So long as you are “coherent” at each time, that’s all there is to pragmatic rationality.
- Contemporary subjective probabilists (often called “Bayesians”) do offer diachronic Dutch Book arguments in support of what I called norm (3), which is sometimes called the “rule of conditionalization” (ROC).
- We won’t look at diachronic Dutch Book arguments in this course. But, there are various arguments of this kind in the literature.
Postscripts to the DBA VI: States vs. Processes Again 2

- [The “accuracy arguments” we will look at next in our epistemic probability unit do not seem to have any diachronic analogues. This is an interesting asymmetry in the literature on subjective probability.]
- There are other kinds of process requirements in the literature on pragmatic subjective probability. One of these is (roughly) as follows:
  - When an agent goes from one doxastic state to another, they should do so in a way that constitutes a “minimal change”— the new state should be “closest” to their old one, subject to some “constraints”.
- This rough idea can be made more precise, and it can lead to answers that diverge from the (ROC), depending on how one precisifies the terms “learning”, “minimal change”, “closest”, and “constraints”.
- E.g., if “learning” does not require assigning probability 1 to what is learned, then this sort of “minimal change” approach leads to a more general form of conditionalization known as Jeffrey Conditionalization. [We may touch on this later, when we discuss Bayesian confirmation.]

Epistemic Probability: Joyce's Argument for Epistemic Probabilism I

Joyce’s argument is based on the following assumptions and set-up:
- Each agent S has a finite (or countable) set of propositions B (it need not be a Boolean algebra) that they are capable of entertaining and assigning some degree of credence (i.e., epistemically rational d.o.b) to.
- We are concerned with the accuracy of an agent’s credence function q. We use the notation I(q, w) to denote the inaccuracy of q in world w.
- \[ I(q, w) = \sum_{X \in B} \rho(q(X), w(X)) \], where q(X) is S’s credence in X, w(X) is the truth-value of X at w (0 if X is false in w, 1 if X is true in w), and \( \rho(q(X), w(X)) \) is a measure of “distance” between q(X) and w(X).
- Note two things about I. First, I is defined piecewise. Second, I is an additive function of the “distances from the truth” of each \( p \in B \).
- The Norm of Gradational Accuracy (NGA) says that an epistemically rational agent strives to have credences that are as close to the truth as possible, i.e., to minimize I(q, w) in some suitable sense of “minimize”.

Postscripts to the DBA VII: Logical Omniscience

- If the DBA is sound, then pragmatically rational agents are logically omniscient — they can demarcate the logical truths from the non-logical-truths in their doxastic state \( B' \). This is a strong requirement!
- Several authors (e.g., Hacking, Harman) have objected that this is simply too strong a requirement to place on a pragmatically rational agent.
- One could try to weaken this requirement in some way. Hacking’s “Slightly More Realistic Personal Probability” (website) is a good example.
- It is not so easy to weaken this requirement and still ensure that the resulting doxastic states are probability models. Consider two proposals:
  - S is required to have \( \Pr(p) = 1 \) only for p’s S knows to be logical \( \top \)'s.
  - S is required to have \( \Pr(p) = 1 \) only for p’s that are expressible (say, in some language S uses) with at most some degree k of complexity.
- Problem: how will we ensure that \( B' \) is a Boolean algebra on these proposals? We’ll return to this issue in the Bayesian confirmation unit.
1. **Dominance** (WC). If \( q_1(y) = q_2(y) \), for all \( y \neq x \), then \( I(q_1, w) > I(q_2, w) \) iff \( \rho(q_1(x), w(x)) > \rho(q_2(x), w(x)) \). [Nobody objects to this one!]

- This ensures the appropriateness of the linear, piecewise way of thinking about \( I(q, w) = \sum_{X \in B} \rho(q(X), w(X)). \) No Holism!

2. **Normality**. If, for all \( x, |w(x) - q_1(x)| = |w'(x) - q_2(x)| \), then \( I(q_1, w) = I(q_2, w'). \) [Allan Gibbard objects to this. We won't discuss.]

- Overestimating a proposition's \( p \)'s truth-value by \( \epsilon \) is counted as exactly as inaccurate as underestimating its truth-value by \( \epsilon \).

3. **Weak Convexity** (WC). If \( I(q_1, w) = I(q_2, w) \), then

\[
I(q_1, w) \geq I(\frac{1}{2} q_1 + \frac{1}{2} q_2, w) \text{ with identity only if } q_1 = q_2.
\]

- Averaging two equally inaccurate \( q \)'s \( q_1, q_2 \) can't yield a strictly more inaccurate \( q \). And, \( q, q_1, q_2 \) are equally inaccurate only if \( q_1 = q_2 \).

4. **Symmetry** (S). If \( I(q_1, w) = I(q_2, w) \) then, for any \( \lambda \in [0, 1] \),

\[
I(\lambda \cdot q_1 + (1 - \lambda) \cdot q_2, w) = I((1 - \lambda) \cdot q_1 + \lambda \cdot q_2, w).
\]

- Symmetric weighted averages of equally inaccurate \( q \)'s remain so.

- (WC) and (S) are the really controversial (and non-obvious) ones. To wit . . .

---

**Epistemic Probability: Joyce's Argument for Epistemic Probabilism IV**

- \( \rho^* \) leads to an \( I \) that violates both (WC) and (S). To find a counterexample to (WC) for \( \rho^* \), one needs at least 2 propositions in \( B \) (HW!). Here's one:

  - Assume \( A \) is true, and \( B \) is false, in \( w \). So, \( w(A) = 1 \) and \( w(B) = 0 \).

  - Let \( q_1(A) = q_1(B) = 1 \), and \( q_2(A) = q_2(B) = 0 \).

  - Therefore, \( q_3(A) = \frac{q_1(A) + q_2(A)}{2} = \frac{1 + 0}{2} = \frac{1}{2} \).

  - And, \( q_3(B) = \frac{q_1(B) + q_2(B)}{2} = \frac{1 + 0}{2} = \frac{1}{2} \).

  - Thus, \( I(q_1, w) = \rho^*(q_1(A), w) + \rho^*(q_1(B), w) = |1 - 1| + |1 - 0| = 1 \).

  - And, \( I(q_2, w) = \rho^*(q_2(A), w) + \rho^*(q_2(B), w) = |1 - 0| + |0 - 0| = 1 \).

  - And, \( I(q_3, w) = \rho^*(q_3(A), w) + \rho^*(q_3(B), w) = |\frac{1}{2} - 1| + |\frac{1}{2} - 0| = 1 \).

  - So, we have \( I(q_1, w) = I(q_2, w) = I(q_3, w) \), but \( q_1 \neq q_2 \), violating (WC).

  - Note: \( \rho^*(q_3(A), w) = \rho^*(q_3(B), w) = (\frac{1}{2} - 1)^2 = \frac{1}{4} \), which blocks the counterexample. [HW: there are no 2-proposition counterexamples for \( \rho^+ \).]

---

**Epistemic Probability: Joyce's Argument for Epistemic Probabilism V**

- To find a counterexample to (S) for \( \rho^* \), one also needs \( \geq 2 \) propositions:

  - Assume \( A \) is true, and \( B \) is true, in \( w \). So, \( w(A) = 1 \) and \( w(B) = 1 \).

  - Let \( q_1(A) = 1 \) and \( q_1(B) = 3 \). And, let \( q_2(A) = q_2(B) = 0 \).

  - Thus, \( I(q_1, w) = \rho^*(q_1(A), w) + \rho^*(q_1(B), w) = |1 - 1| + |1 - 3| = 2 \).

  - And, \( I(q_2, w) = \rho^*(q_2(A), w) + \rho^*(q_2(B), w) = |1 - 0| + |1 - 0| = 2 \).

  - And, \( I(q_3, w) = \rho^*(q_3(A), w) + \rho^*(q_3(B), w) = |\frac{1}{2} - 1| + |\frac{1}{2} - 0| = 1 \).

  - So, we have \( I(q_1, w) = I(q_2, w) = I(q_3, w) \), but \( q_1 \neq q_2 \), violating (WC).

  - Note: \( \rho^+(q_3(A), w) = \rho^+(q_3(B), w) = (\frac{1}{2} - 1)^2 = \frac{1}{4} \), which blocks the counterexample. [HW: there are no 2-proposition counterexamples for \( \rho^+ \).]

---
**Epistemic Probability: Joyce’s Argument for Epistemic Probabilism VI**

- **Joyce’s Reply**: define expected distance of \( q’ \) (according to \( q \)) from the truth concerning \( X \), relative to \( \rho \), \( EDT(q’,q,X,\rho) \), in the following way:

\[
EDT(q’,q,X,\rho) = q(X) \cdot \rho(q’(X),1) + (1 - q(X)) \cdot \rho(q’(X),0)
\]

\[
= q \cdot \rho(q’,1) + (1 - q) \cdot \rho(q’,0)
\]

- **Theorem.** If \( \rho = \rho^* \), then \( EDT(q’,q,X,\rho) \) can be minimized only by assigning \( q’(X) = 0 \), \( q’(X) = \frac{1}{2} \), or \( q’(X) = 1 \). If \( \rho = \rho^\dagger \), \( EDT(q’,q,X,\rho) \) will be minimized when \( q’(X) = q(X) \), which can lie anywhere on \([0,1]\).

- If you aim to minimize your expected inaccuracy of your own \( q \), then you shouldn’t use \( \rho^* \), since this will lead you to view your current degrees of belief as inferior to one that assigns \( 0,1 \) or \( \frac{1}{2} \) to every \( X \) in \( B \).

- But, it’s OK to use \( \rho^\dagger \), since this leads to a “stable” \( q \). See plots, below.

- Questions: why minimize \( EDT \)? Wasn’t the goal to minimize \( DT \)? Also, why do we get to assume here that \( q(\sim X) = 1 - q(X) \)? Question begging?

---

**Review of Joyce’s Epistemic Argument for Probabilism**

- Joyce aims to show that, unless \( q \) is a probability function, there will exist a \( q’ \) which is closer to the truth - no matter what the truth turns out to be.

- The “closeness to the truth” of \( q \) in world \( w \) is based on some measure \( \rho \) of the distance between \( q(X) \) and the truth-value \( (w(X) = 0 \text{ or } 1) \) of \( X \) in \( w \), summed over all of the \( X \)’s in \( S \)’s set of entertainable propositions \( B \).

- Joyce’s argument rests on four assumptions \( A \) about the inaccuracy measure \( I \). \( A \) is consistent with \( \rho^\dagger(q(X),w(X)) = |q(X) - w(X)|^2 \), but not \( \rho^*(q(X),w(X)) = |q(X) - w(X)|^2 \), or various other distance measures.

- Maher shows that \( \rho^* \) violates two of Joyce’s axioms: (WC) and (S), and he argues that \( \rho^* \) is nonetheless a reasonable measure of “distance from truth”. Maher also shows that Joyce’s Theorem fails if one uses \( \rho^* \).

- Joyce’s response appeals to “Expected Distance from Truth”. But, this undermines one of the nice things about Joyce’s original argument — that it made no appeal to Expected Utility Theory (which seems pragmatic).