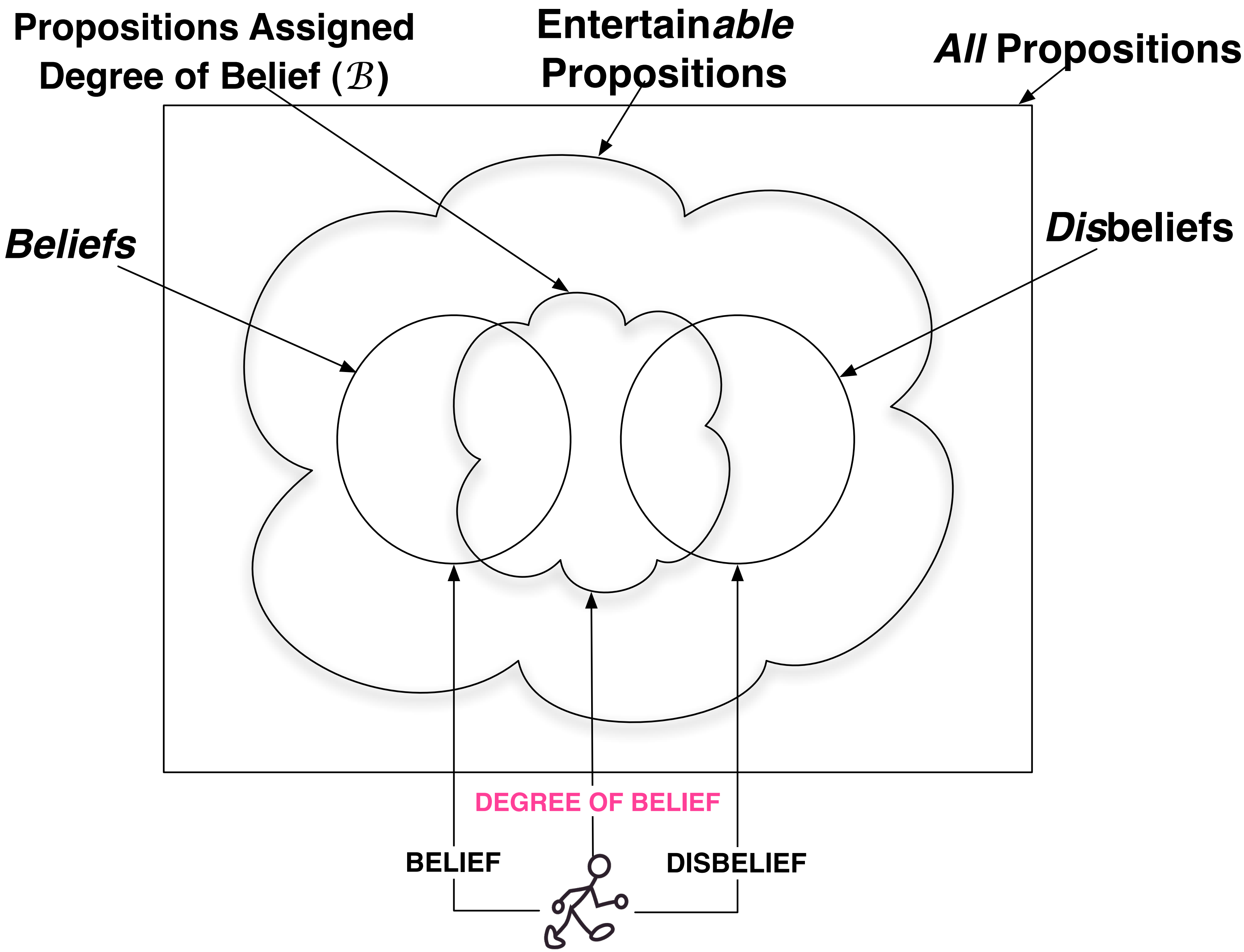
Philosophy 148 — Announcements & Such

- HW #1 returned (sections). Median: 93. SD: 22! "A Tale of Two Curves".
  - We will have a "homework discussion" before each HW is due. How about March 17 @ 6pm for a HW #2 discussion + mid-term review?
  - We will also allow you to *drop your lowest HW score* in the course.
- HW #2 is posted (due 3/**20**). There is also a *hints handout* for HW #2.
- **3 Schedule Changes: Mid-Term moved back 2 weeks (3/6 to 3/20), HW 3 moved back 1 week (3/13 to 3/20). HW 2 due 3/20 (not 3/13).**
- Setting up our evaluative doxastic framework:
  - The Big Picture: belief, disbelief, degree of belief, etc.
  - Evaluative *vs* Normative, States *vs* Processes.
  - Digression: Justified belief *vs* degree of belief — *Lotteries*.
- Then: Subjective Probability. First, *pragmatic* probability.
  - The Dutch Book Argument for Pragmatic Probabilism.


Propositions Assigned Degree of Belief ($\mathcal{B}$)
Entertain*able* Propositions
All Propositions
Beliefs
*Dis*beliefs
DEGREE OF BELIEF
BELIEF
DISBELIEF

An Evaluative Doxastic Framework: Review/Overview

- We will be interested in *evaluating* doxastic *states* of agents, against standards of ideal rationality — *not* advising, prescribing, blaming *etc*.
  - We are *not* making *normative* claims, and so questions like "can we *choose* to have beliefs/degrees of belief?" are sidestepped.
- We assume (without argument) that agents S (at times t) *have degrees of confidence/belief*, and that they are defined over a *Boolean algebra* $\mathcal{B}_S^t$.
- We will distinguish *pragmatic vs epistemic* rationality. Pragmatic rationality involves what is "best for the agent" (broadly). Epistemic rationality involves "epistemic values" (truth, justification, evidence).
- We will be looking mainly at *states* of agents, and *not processes* leading from one state to another (although, we will briefly discuss those).
- We will not assume any connection between the rationality of *beliefs* and the rationality of *degrees of belief*. *Why not?* Digression: *Lotteries*.

Digression: Lotteries, Justified Belief, and Justified Degrees of Belief I

- Context (C). S has entered a lottery with a huge number (n) of tickets, exactly one of which will be the winner (and each number was assigned to exactly one person, prior to the drawing). S's ticket number is s.
- Let $L_i \overset{\text{def}}{=}$ ticket i is a losing ticket. Plausibly, it would be rational/justified (in C) for S to have a very high *degree of belief* (perhaps $1 - \frac{1}{n}$) in L_s.
- Is S *justified in **believing*** L_s? [More generally, is X justified in believing L_x?]
- I will argue that this is not so clear, and that the connection between justified *belief* and justified *degree of belief* is therefore also not so clear.
- My argument will rest on the following two assumptions/claims:
  1. Justification is the solution to: "x + true belief = knowledge".
  2. In C, S does *not **know*** L_s (even if it turns out that s *is* a losing ticket).
- I will not argue for (1). But, I will argue for (2), below. This yields a conflict between justified belief (in *at least one sense*) *vs* degree of belief.

Digression: Lotteries, Justified Belief, and Justified Degrees of Belief II

- Here is an argument for (2). It is a *reductio*:

(2.1) S *knows* that their ticket s is a loser. [*reductio* assumption]

(2.2) There is *no epistemic asymmetry* between ticket-holders. That is to say, if *one* of them (S) knows that their own ticket (s) is a loser, then *every* ticket-holder (X) knows that their own ticket (x) is a loser.

(2.3) Knowledge is *factive*. If S *knows* that p, then p is *true* (for any S/p).

(2.4) ∴ *Every ticket is a loser*, which *contradicts* the lottery set-up. □

- If cogent, this argument shows that S does not know that their ticket is a loser, even if their ticket *really is* a loser. So, even if S's belief that L_s *happens to be* true, S does *not know* it. This is where (1) kicks-in.
- From (1), it then follows that S is not *justified* in believing L_s (assuming L_s happens to be true in C). On this sort of view, S may be justified in believing (or may even *know*) that L_s is (objectively) *highly probable*.
- But, on such a view, this is *distinct* from S being justified in *believing* L_s.

Digression: Lotteries, Justified Belief, and Justified Degrees of Belief III

- Last time, someone mentioned the following *prima facie* plausible principle, which connects justified belief and justified degree of belief:

  (†) If S is *justified in believing* p, then S is not (at the same time and in the very same context) justified in having a *low degree of belief* in p.

- While (†) *seems* plausible, it may not be correct. Consider the following:

  (‡) If S is justified in believing p and S is justified in believing q, then S is justified in believing their conjunction $p \,\&\, q$.

- I think (‡) *also seems* plausible [particularly, on the (1)-conception of justification]. But, (†) and (‡) *cannot both be correct* evaluative norms.

- To see why (†) and (‡) come into conflict, consider any context in which S is justified in believing (but is *uncertain* about) a bunch of *probabilistically independent* propositions $p_1, \ldots, p_n$. By (‡), S is JIB $P \overset{\text{def}}{=} p_1 \,\&\, \cdots \,\&\, p_n$.

- But, by (†), S is *not* justified in believing P, since (plausibly) S should have a *low degree of belief* in P [think: multiplying n numbers on $(0, 1)$].

Examples of (Evaluative) Doxastic Norms

- Here are some examples of evaluative doxastic claims/principles. I'll call these "norms". Note: perhaps these should be read "*ceteris paribus*".

1. Logically *consistent* belief states are better than inconsistent states.
   - Why? Inconsistent sets must contain some falsehoods.
2. Degree-of-belief states that can be accurately represented as *probability models*, are better than those which cannot be.
   - Why? We'll see two arguments for this "probabilism norm" next.
3. *If* an ideally rational agent S satisfies the following two conditions:
   (*i*) S's doxastic state at t can be represented as a Pr-model $\langle \mathcal{B}_S^t, \Pr_S^t \rangle$,
   (*ii*) Between t and t', S learns q *and nothing else* (where q is in $\mathcal{B}_S^t$),
   then, (*iii*) the *ideal doxastic state* for S at t' is $\langle \mathcal{B}_S^t, \Pr_S^{t'} \rangle$, where $\Pr_S^{t'}(\bullet) = \Pr_S^t(\bullet \mid q)$. [d.o.b.–updating goes by *conditionalization*.]

- (1) and (2) are norms for *belief* states. (3) and (4) are norms for *degree-of-belief* states (or *sequences* of them). We'll focus mainly on (2).
- Next: our first argument for *probabilism* — The Dutch Book Argument.

The Dutch Book Argument for (Pragmatic) Probabilism I

- The key assumptions/set-up of the Dutch Book argument are as follows:
  - For each proposition $p \in \mathcal{B}^t$ in our agent's (Mr. B's) doxastic state at t, Mr. B must announce a number $q(p)$ – called his *betting quotient* on p, at t – and *then* Ms. A (the bookie) will choose the *stake* $\mathfrak{s}$ of the bet.
  - $|\mathfrak{s}|$ should be small in relation to Mr. B's total wealth (more on this later). But, it can be positive or negative (so, she can "switch sides").

$$\text{Mr. B's payoff (in \$) for a bet about } p = \begin{cases} \mathfrak{s} - q(p) \cdot \mathfrak{s} & \text{if } p \text{ is true.} \\ -q(p) \cdot \mathfrak{s} & \text{if } p \text{ is false.} \end{cases}$$

  - NOTE: If $\mathfrak{s} > 0$, then the bet is *on* p, if $\mathfrak{s} < 0$, then the bet is *against* p.
  - $q(p)$ is taken to be a measure of Mr. B's *degree of belief* in p (at t).
  - If there is a sequence of multiple bets on multiple propositions, then Mr. B's total payoff is the *sum* of the payoffs for each bet on each proposition. This is called "the package principle". More on *it* later!

The Dutch Book Argument for (Pragmatic) Probabilism II

- Mr. B's "degree of belief function" $q(\cdot)$ is *coherent* iff it is impossible for Ms. A to choose stakes $\mathfrak{s}$ such that she wins *no matter what happens*, *i.e.*, $q(\cdot)$ is *coherent* iff Ms. A cannot construct a "Dutch Book" against Mr. B.

  Theorem (DBT). $q(\cdot)$ is *coherent* **only if** $q(\cdot)$ is a *probability function*.

- Note: the "if" part is also a theorem. That is the *converse* DBT. We won't prove it, but I'll discuss it. Advocates of the DBA think DBT justifies this:

  Claim. The doxastic state $\langle \mathcal{B}_S^t, q(\cdot)\rangle$ of an agent S who is faced with such a scenario is (pragmatically) rational ***only if*** $q(\cdot)$ is coherent (hence a probability function). Does this imply P-probabilism? More, below.

- Does Theorem justify Claim? There are worries about the relationship between gambling and P-rationality, and the "package principle" (and various other assumptions) will also be called into question.

- We'll discuss these worries after we look at the Theorem itself.

The Dutch Book Argument for (Pragmatic) Probabilism III

- The DBT has four parts: one for each of the three axioms, and one for the definition of conditional probability. In each case, we prove that if $q(\cdot)$ [or $q(\cdot \mid \cdot)$] *violates* the axioms (or defn.), then $q(\cdot)$ is *in*coherent.

- **If Mr. B violates Axiom 2, then his q is incoherent**. Proof:
  - If Mr. B assigns $q(\top) = a < 1$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
  - Similarly, if Mr. B assigns $q(\top) = a > 1$, then Ms. A sets $\mathfrak{s} > 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
    - ∗ NOTE: if $q(\top) = 1$, then Mr. B's payoff is always $\mathfrak{s} - \mathfrak{s} = 0$, which avoids *this* Book. I'll discuss the *converse* DBT further, below.

- **If Mr. B violates Axiom 1, then his q is incoherent**. Proof:
  - If $q(p) = a < 0$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is $\mathfrak{s} - a\mathfrak{s} < 0$ if p, and $-a\mathfrak{s} < 0$ if $\sim p$. [If $q(p) \geq 0$, then Mr. B's payoff is $\mathfrak{s} - q\mathfrak{s} \geq 0$ if $\mathfrak{s} > 0$ and p is true, and $-q\mathfrak{s} \geq 0$ if $\mathfrak{s} < 0$ and $\sim p$, avoiding *this* Book.]

The Dutch Book Argument for (Pragmatic) Probabilism IV

- Recall, our Axiom 3 requires that

$$\Pr(p \vee r) = \Pr(p) + \Pr(r)$$

  if p and r cannot both be true (*i.e.*, if they are mutually exclusive).

- The argument for this *additivity* axiom is more controversial. The main source of controversy is the "package principle". We'll just *assume* it for the proof of the *Theorem*. But, for the *Claim*, we will re-think it.

- I will now go through the additivity case of the *Theorem*, and then I will discuss Maher's (and Schick's) objection to the "package principle".

- **Setup**: Let p and r be some pair of mutually exclusive propositions in the agent's doxastic state at t. And, suppose Mr. B announces these b's:

$$q(p) = a,\ q(r) = b, \text{ and } q(p \vee r) = c, \text{ where } c \neq a + b.$$

- This will leave Mr. B susceptible to a Dutch Book. Here's why …

The Dutch Book Argument for (Pragmatic) Probabilism V

- **Case 1**: $c < a + b$. Ms. A asks Mr. B to make *all 3* of these bets ($\mathfrak{s} = +\$1$):
  1. Bet $\$a$ on p to win $\$(1 - a)$ if p, and to lose $\$a$ if $\sim p$.
  2. Bet $\$b$ on r to win $\$(1 - b)$ if r, and to lose $\$b$ if $\sim r$.
  3. Bet $\$(1 - c)$ *against* $p \vee r$ to win $\$c$ if $\sim(p \vee r)$, and lose $\$(1 - c)$ o.w.
- Since p and r are mutually exclusive (by assumption of the additivity axiom), the conjunction $p \,\&\, r$ cannot be true. $\therefore$ There are 3 cases:

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, \sim r$	$1 - a$	$-b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, r$	$-a$	$1 - b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, \sim r$	$-a$	$-b$	c	$c - (a + b)$

- Since $c < a + b$, $c - (a + b)$ is negative. So, Mr. B loses $\$[c - (a + b)]$.
- **Case 2**: $c > a + b$. Ms. A simply reverses the bets ($\mathfrak{s} = -\$1$), and a parallel argument shows that the total payoff for Mr. B is $\$-[c - (a + b)] < 0$.
- Note: he can avoid *this* Book, by setting $c = a + b$. More on CDBT, below.

The Dutch Book Argument for (Pragmatic) Probabilism VI

- We also need to show that an agent's conditional betting quotients are coherent only if they satisfy our ratio definition of conditional probability. There's a DBT for this too (and it also assumes the "package principle").
- Suppose Mr. B announces: $q(p \,\&\, r) = b$, $q(r) = c > 0$, and $q(p \mid r) = a$. Ms. A asks Mr. B to make *all 3* of these bets (stakes depend on quotients!):

1. Bet $\$(b \cdot c)$ on $p \,\&\, r$ to win $\$[(1-b) \cdot c]$ if $p \,\&\, r$, and lose $\$(b \cdot c)$ o.w. $[\mathfrak{s} = c]$
2. Bet $\$[(1-c) \cdot b]$ against r to win $\$(b \cdot c)$ if r, and lose $\$[(1-b) \cdot c]$ o.w. $[\mathfrak{s} = b]$
3. Bet $\$[(1-a) \cdot c]$ against p, conditional on r, to win $\$(a \cdot c)$ if $r \,\&\, p$, and lose $\$[(1-a) \cdot c]$ if $r \,\&\, {\sim}p$. If ${\sim}r$, then the bet is *called off*, and payoff is \$0. $[\mathfrak{s} = c]$

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, r$	$(1-b) \cdot c$	$-[(1-c) \cdot b]$	$-[(1-a) \cdot c]$	$(a \cdot c) - b$
${\sim}p \,\&\, r$	$-(b \cdot c)$	$-[(1-c) \cdot b]$	$a \cdot c$	$(a \cdot c) - b$
${\sim}r$	$-(b \cdot c)$	$b \cdot c$	0	0

- If $a < \frac{b}{c}$, then Mr. B loses *come what may*. If $a > \frac{b}{c}$, then Ms. A just asks Mr. B to take the other side on all three bets. So, coherence requires: $q(p \mid r) = \frac{q(p \& r)}{q(r)}$.

The Dutch Book Argument for (Pragmatic) Probabilism VII

- Here are some problems with/limitations of the Dutch Book *Argument*:
  - It assumes that bets (or gambles) which are severally acceptable are jointly acceptable. This "package principle" is not obvious (see below).
  - It is couched in terms of *money*. It tacitly assumes that *utility* is *linear* in *money*. But, money seems to have *diminishing marginal utility*.
    * More generally, DBAs involve *betting behavior*. One might wonder whether one's betting behavior (when *forced* to post odds) is representative of *rational* behavior *generally* (gambling aversion?).
  - The DBA requires the *converse* DBT to be persuasive. And, *even with* the CDBT, it's still not clear whether pragmatic probabilism follows from the DBA. Does it show that Pr is *better than* non-Pr?
  - It does not address *process* requirements, only state requirements, *i.e.*, it does not constrain *transitions* from one doxastic state to another.
  - It presupposes P-rational agents are *logically omniscient* – that they can *recognize all tautologies* (in $\mathcal{B}^t$). Does P-rationality require this?

Postscripts to DBA I: The Package Principle

- The Dutch Book argument for Additivity may appear as airtight as the Dutch Book arguments for Normality and Non-Negativity. But, Schick (and others) have spotted a possible flaw [see the Schick paper on website].
- In our Dutch Book argument for Additivity, we (implicitly) assumed:
  - Bets that are severally acceptable are jointly acceptable.
  - The value of a set of bets is the sum of values of its elements.
- Might a rational agent be willing to accept *each* of the bets (1)–(3) without being willing to accept *all three at once*? And, if not, why not?
- Might they not value (1) at $\$a$, (2) at $\$b$, and (3) at $\$-c$, *without* valuing the *collection* at $\$[a+b-c]$? After all, they may *see* that, *taken jointly*, bets (1)–(3) lead to a sure loss, whereas *no individual bet does*.
- It is important to note that the DBA for the ratio definition of conditional probability also presupposes this sort of "package principle".
- Are there counterexamples to the "package principle"? Maher thinks so:

Suppose that after a night on the town, you want to catch the bus home. Alas, you find that you only have 60 cents in your pocket, but the bust costs $1. A bookie, learning of your plight, offers you the following deal: If you give him your 60 cents, he will toss a coin, and if the coin lands heads then he will give you $1; otherwise, you have lost your 60 cents. If you accept the bookie's proposal, then you stand a 50-50 chance of being able to take the bus home, while rejecting it means you will certainly have to walk. Under these conditions, you may well feel that the bookie's offer is acceptable; let us suppose you do. Presumably, the offer would have been equally well acceptable if you were betting on tails rather than heads. . .

- As betting quotients were defined for the simple DBA, your quotient for heads in the above example is 0.6, and so is your quotient for tails.
- By additivity, $\Pr(\text{Heads} \vee \text{Tails}) = \Pr(\text{Heads}) + \Pr(\text{Tails})$, since Heads and Tails are mutually exclusive. But, then, $q(\text{Heads} \vee \text{Tails}) = 1.2 > 1$, which violates the axioms (recall, we have a theorem saying $\Pr(p) \leq 1$, for all p).
- Assumption: if you find a bet on Heads acceptable, and find a bet on Tails acceptable, then you should also find both bets acceptable, *taken jointly*.
- I have made a handout which shows exactly when the PP is required.

Postscripts to the DBA II: The Value of Money

- We have the caveat about $\$|\mathfrak{s}|$ being "small in comparison to the agent's total wealth" for two reasons. First, if the agent could lose *everything* on a bet, this would undermine the probative value of the argument.
- Also, real agents (and, arguably, also rational agents!) marginally value money in a way that is *non-linear*, especially for larger sums of money.
- The difference in value between \$1 and \$1000 is pretty substantial. But, the difference in value between \$1M + \$1 and \$1M + \$1000 is not.
- This is called the *diminishing marginal utility of money*. Empirical studies show that actual agents have marginal utilities that are close to being linear only for \$ amounts that are "small relative to their total wealth."
- It's not implausible to suppose that rational agents are like this too. So, if we want something with *linear* marginal value (for *additivity* purposes – think "package principle"!), money is probably not the best thing to use.

Postscripts to the DBA III: The Need for the *Converse* DBT

- The DBT *by itself* cannot secure pragmatic probabilism. All the DBT establishes is that q's coherence entails that q is a probability function. What if the *converse* of this were *false*? Would the DBA still be persuasive?
- If there were some probability functions that were *also* susceptible to Dutch Book, then pragmatic probabilism would *not follow* from the DBA.
- Remember, pragmatic probabilism says *all* probabilistic doxastic states are (pragmatically) better than *all* non-probabilistic doxastic states.
- The mere fact that (DBT) *all non*-probabilistic states are *susceptible* to Dutch Book is *not* sufficient to establish this. One *also* needs to show that (CDBT) *all probabilistic* doxastic states are *immune* from Dutch Book.
- Luckily, the (CDBT) is *true*. We have *not* proven this (and we will not). All we have shown is that *particular* Dutch Books for each of the axioms is *blocked* by satisfying *that* axiom. This does *not* establish (CDBT).
- See the Kemeny paper (on website), if you want to see the proof of (CDBT).

Postscripts to the DBA IV: Is the *Converse* DBT *Enough*?

- *Even if* we use both DBT and CDBT (which *are* both theorems), does the Dutch Book *Argument* establish pragmatic probabilism? Maybe not.
- Alan Hájek thinks there is still a gap in the DBA, *even with* both directions of the DBT, *and even if* we grant the truth of the "package principle".
- As Hájek explains in his paper "Scotching Dutch Books" (now on website), the structure of the DBA is more-or-less something like the following:
  1. S's q is coherent $\Longleftrightarrow$ S's q is a probability function. [DBT + CDBT]
  2. S is susceptible to Dutch Book $\Longleftrightarrow$ S's q is incoherent. [definition]
  3. Susceptibility to DB is "bad" and immunity from DB is "good". [ass.]
  4. $\therefore$ S's q is *better* if it is probabilistic than if it is *non*-Pr. [P-probabilism]
- Does (4) follow from (1)–(3)? Clearly, (1)–(3) entail that there is *some sense* in which q is *guaranteed* to be "better" *in virtue of* being probabilistic.
- What if there is *also* some sense in which q is *guaranteed* to be "*worse*" *in virtue of* being probabilistic. Is there a symmetric "Good Book" argument?

Postscripts to the DBA V: Hájek's "Good Book" Argument

- A *Good Book* is a sequence of bets that *wins* money *come what may*. Betting quotients q are *good* if they are susceptible to a Good Book.
- It turns out that the following as *also* a theorem (we won't prove it, but it shouldn't be surprising, given the Dutch Book theorem and it Converse):

  Good Book Theorem (GBT). q is good $\Longleftrightarrow$ q is *not* a probability function.

- So, why not the following, symmetric argument *against* P-probabilism?
  1. S's q is good $\Longleftrightarrow$ S's q is not a probability function. [GBT]
  2. S is susceptible to Good Book $\Longleftrightarrow$ S's q is good. [definition]
  3. Susceptibility to GB is "good" and immunity from GB is "bad". [ass.]
  4. $\therefore$ S's q is *worse* if it is probabilistic than if it is *non*-Pr. [$\sim$P-probabilism]
- The question now becomes: Is the DBA more compelling than the GBA? Which is pragmatically *better*: immunity from DB or susceptibility to GB?
- And, are there yet *other dimensions* of "goodness" we have overlooked?

Postscripts to the DBA VI: States *vs.* Processes Again 1

- The Dutch Book Argument we have seen only aims to establish that rationality requires an agent's doxastic state *at a particular time* to be representable as a probability model. This is a *state* requirement.
- These kinds of requirements are also sometimes called requirements of *synchronic* rationality. One might wonder: what about *processes* that lead us from one probability model to another — *diachronic* rationality?
- Traditional subjective probabilists (like Ramsey and de Finetti) didn't think there were any process requirements in this sense. So long as you are "coherent" at each time, that's all there is to pragmatic rationality.
- Contemporary subjective probabilists (often called "Bayesians") do offer *diachronic* Dutch Book arguments in support of what I called (4) above, which is sometimes called the "rule of conditionalization" (ROC).
- We won't look at diachronic Dutch Book arguments in this course. But, there are various arguments of this kind in the literature.

Postscripts to the DBA VI: States *vs*. Processes Again 2

- [The "accuracy arguments" we will look at next in our *epistemic* probability unit do not seem to have any diachronic analogues. This is an interesting asymmetry in the literature on subjective probability.]
- There are other kinds of process requirements in the literature on pragmatic subjective probability. One of these is (roughly) as follows:
  - When an agent goes from one doxastic state to another, they should do so in a way that constitutes a "minimal change"— the new state should be "closest" to their old one, subject to some "constraints".
- This rough idea can be made more precise, and it can lead to answers that diverge from the (ROC), depending on how one precisifies the terms "learning", "minimal change", "closest", and "constraints".
- *E.g.*, if "learning" does not require *assigning probability 1* to what is learned, then this sort of "minimal change" approach leads to a more general form of conditionalization known as *Jeffrey Conditionalization*. [We may touch on this later, when we discuss Bayesian confirmation.]

Postscripts to the DBA VII: Logical Omniscience

- If the DBA is sound, then *pragmatically* rational agents are *logically omniscient* — they can demarcate the logical truths from the non-logical-truths in their doxastic state $\mathcal{B}^t$. This is a strong requirement!
- Several authors (*e.g.*, Hacking, Harman) have objected that this is simply too strong a requirement to place on a pragmatically rational agent.
- One could try to weaken this requirement in some way. Hacking's "Slightly More Realistic Personal Probability" (website) is a good example.
- It is not so easy to weaken this requirement and still ensure that the resulting doxastic states are *probability models*. Consider two proposals:
  - S is required to have $\Pr(p) = 1$ *only* for p's S *knows* to be logical $\top$'s.
  - S is required to have $\Pr(p) = 1$ *only* for p's that are expressible (say, in some language S uses) with *at most* some degree k of *complexity*.
- Problem: how will we ensure that $\mathcal{B}^t$ is a *Boolean algebra* on these proposals? We'll return to this issue in the Bayesian confirmation unit.