Philosophy 148 — Announcements & Such

- Administrative Stuff
  - HW #1 is due on today (by the end of class).
  - HW #2 is posted (due 3/13). There is also a hints handout for HW #2.
    - Topics from arguments for pragmatic and epistemic probabilism — "Dutch Books" and "Epistemic Accuracy" Arguments.
  - **Two Schedule Changes: Mid-Term moved back 2 weeks (from 3/6 to 3/20), and HW #3 moved back one week (from 3/13 to 3/20).**
  - Today's Agenda
    - Brief review of Objective (physical) probabilities.
      - Actual frequency, hypothetical frequency, propensity/chance.
      - A context in which the objective chances aren't probative …
    - …Subjective Probability
      - An Evaluative Doxastic Framework.
      - Two arguments for probabilism (pragmatic and epistemic).

Objective Theories of Probability (Recap)

- Actual (finite) frequencies are *probabilities*. But, they are somewhat "odd":
  - They are *population-relative*.
  - They can only assume *finitely many values*. For instance, frequencies in a singleton set (a set with one member) must be zero or one, and frequencies in a set with two elements must be 0, 1, or $\frac{1}{2}$, *etc*.
- If frequencies "settle down" as **P** gets larger (*e.g.*, coin tossing), then a better candidate for "the" objective probability is whatever value the frequencies *converge to in the limit — were* **P** *to be extended indefinitely*.
- But, not all infinite extensions of **P** have convergent frequencies (it is not very helpful to say that convergence to the objective probability is "highly probable"!). And, *which* of those that *do* converge yield "the" probability?
- This leads to *propensities* or *chances* as *underlying physical properties*, which *manifest* themselves in frequencies, but which are *not identical* even to hypothetical limiting frequencies (*e.g.*, quantum probabilities).

Objective Theories of Probability (Some Further Issues)

- We proved that finite frequencies satisfy the probability axioms.
- Infinite frequencies *don't* satisfy the (classical) axioms of (infinite) probability calculus (see pp. 1–3 of the van Fraassen paper on website).
  - The algebra of limiting frequencies of events is non-Boolean.
  - Limiting frequencies do not satisfy the (infinite) additivity axiom.
- Some have claimed that QM-probabilities are also non-classical, owing to the fact that the underlying "quantum *logic*" is non-Boolean. [But, there are also interpretations of QM in terms of classical probabilities.]
- It is often *assumed* that objective chances satisfy the probability axioms, but it is not quite clear *why* (especially, in light of the above remarks).
- I won't be dwelling on objective (physical) probabilities, so I won't fuss about these puzzles (I think they have more to do with ∞ than Pr).
- Next, we'll discuss subjective probability, and we *will* dwell on that.

Subjective Theories of Probability (I)

- I'll begin by motivating subjective probability with an example/context:

  I'm holding a coin behind my back. It is either 2-headed or 2-tailed. You do not know which kind of coin it is (and you have no reason to favor one of these possibilities over the other). I'm about to toss it. What probability (or odds) would you assign to the proposition that it will land heads?

- Intuitively, $\frac{1}{2}$ (or even/50:50 odds) would not be an unreasonable answer to this question. But, $\frac{1}{2}$ — or *any non-extreme value* for that matter — *cannot* be the *objective* probability/chance of heads in this example.
- After all, we know that the coin is either 2-headed or 2-tailed. As such, the objective probability of heads is either 1 or 0 in this example.
- One might describe this as *epistemic* probability, because it seems *epistemically reasonable* to be *50% confident* that the coin will land heads.
- Also, *taking a bet at even odds on heads* seems *pragmatically* reasonable. This suggests a *pragmatic* theory of probability is also probative here.

Subjective Theories of Probability (II)

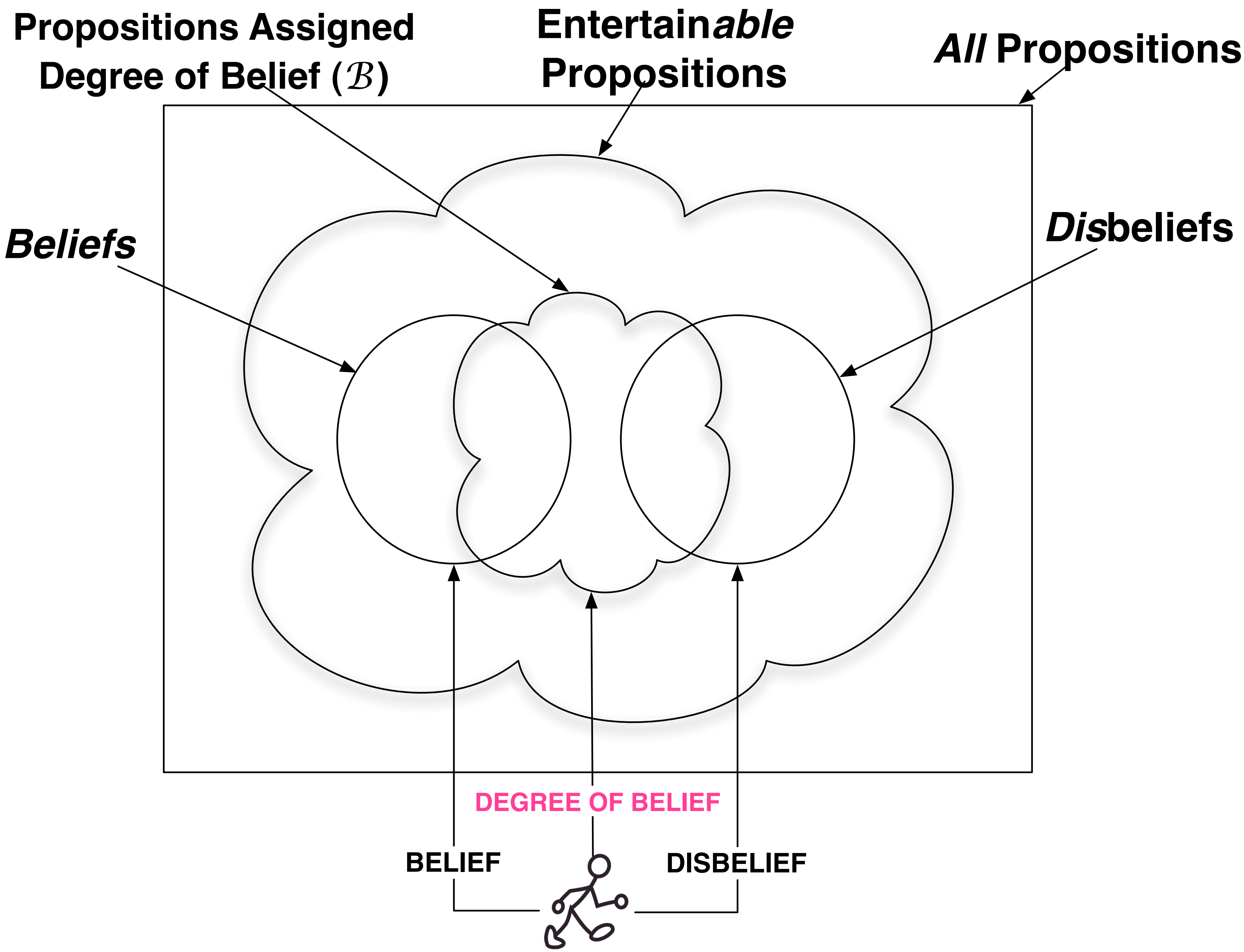
- It seems clear that there is such a thing as "degree of belief". And, it also seems clear that there are *some* sorts of constraints on such degrees.
- But, why should degrees of belief obey the *probability axioms*?
- There are arguments that epistemic and pragmatic probabilities should each be *probabilities*. We will examine examples of each type of argument.
- We will begin with *pragmatic* subjective probability.
- There are various arguments for *pragmatic probabilism*: that *pragmatically rational* degrees of belief obey the probability axioms.
- All such arguments must do (at least) two things:
  - Identify a *necessary condition* (N) for pragmatic rationality.
  - Show that having non-probabilistic degrees of belief *violates* N.
- The first argument we will examine is the Dutch Book Argument (DBA).
- But, first, a framework for *evaluating* agents' doxastic states.

An Evaluative Doxastic Framework (I)

- We will assume that rational agents have attitudes toward propositions. One of these attitudes is *belief*. What is belief? This is not entirely clear.
- We will say that belief is a relation between an agent S and a proposition p. We needn't worry too much about the precise conditions under which S believes that p. Intuitively, belief is *dispositional* property.
- When S believes p, this will be accompanied by various dispositions to behave in certain ways: to provide arguments in favor of p should it be challenged, to act in accordance with the assumption that p is true, *etc*.
- We will not assume that an agent must actively (or consciously) *attend to* p in order to believe it. Thus, we can have *implicit* beliefs. But, all propositions believed by an agent must be *entertainable* for that agent.
- We'll also talk about *degrees of belief* (*degrees of confidence*, *credences*). This is a quantitative or comparative *generalization* of belief, which will have its own sorts of dispositional manifestations (*e.g.*, betting behavior).

An Evaluative Doxastic Framework (II)

- We'll assume that, at each time t, an ideally rational agent S has a *doxastic state*, which includes a *Boolean algebra of propositions* $\mathcal{B}_S^t$ — a subset of the p's that are *entertainable* for S at t (those S has "access" to at t).
- $\mathcal{B}_S^t$ will contain those p's in which S has *some degree of confidence* at t. S will only *believe some subset* of this set of propositions at t. But, each member of $\mathcal{B}_S^t$ is, in some sense, a "live option" as a belief for S at t.
- The idea here is that there are three kinds of propositions for an agent S at a time t: (1) entertain*able* (but not entertain*ed*) p's, (2) entertain*ed* p's ($p \in \mathcal{B}_S^t$ receive some degree of confidence), and (3) *un*-entertain*able* p's.
- Restricting the set of candidate beliefs (at t) to some proper subset of *all* the propositions seems right. But, why assume $\mathcal{B}_S^t$ is a *Boolean algebra*? [This implies (among other things) *closure* under Boolean operations.]
- For now, we'll assume this form of *logical omniscience*. Note: we're *not* assuming that the agent's *beliefs* are closed under the logical operations, or even that the set of all entertain*able* propositions is closed. Picture:


Propositions Assigned Degree of Belief ($\mathcal{B}$)
Entertain*able* Propositions
All Propositions
Beliefs
*Dis*beliefs
DEGREE OF BELIEF
BELIEF
DISBELIEF

An Evaluative Doxastic Framework (IV)

- What's the difference between the *pragmatic* rationality of doxastic states (including beliefs, degrees of belief, *etc.*) and *epistemic* rationality thereof?
- Example: there is psychological evidence that (actual) agents S tend to perform better at certain activities ϕ if they *believe* that (p_S^ϕ) S is very good at ϕ-ing. This can remain true even when the belief is *unjustified*.
- A case could be made that it would (in some cases) be *pragmatically* rational for (some) S to believe that p_S^ϕ, even when such a belief is not supported by S's evidence. But, this seems *epistemically ir*rational.
- Simpler example: I offer you \$1M to believe that the number of pebbles on Pebble Beach is *exactly* 10^{12}. You have no evidence for this claim, and otherwise no reason to believe it. But, you *really* value money, *etc., etc.*
- We'll bracket questions about whether beliefs (d.o.b.'s) are the *kinds* of things you *can choose* to have. We'll think of this in *evaluative* rather than *normative* terms, and (for now) as about *states* rather than *processes*.

An Evaluative Doxastic Framework (V)

- When we make judgments about rationality, we can take two "stances":
  - **Evaluative Stance**: Here, we're merely *evaluating* some state or process against some standard(s) of ideal rationality. We're not making any claims about what anyone *ought to do* (not *advising*, *blaming*, *etc*).
  - **Normative Stance**: Here, we are talking about what some agent(s) *ought to do*, from the point of view of some standard(s) of ideal (*normative*!) rationality. Here, we *do advise*, *prescribe*, *blame*, *etc*.
- And, we can be making judgments about *states* or *processes*:
  - **State Judgments**: These are judgments about the rationality ("goodness") of (some aspect of) the *doxastic state* of S at t.
  - **Process Judgments**: Judgments of the rationality ("goodness") of some *process* leading S from one doxastic state (at t) to another (at $t' > t$).
- We will be involved mainly with in *evaluation* of doxastic *states*. [We'll say a little about doxastic processes, but also from an *evaluative stance*.]

An Evaluative Doxastic Framework (VI)

- Examples of some evaluative doxastic claims/principles ("norms"):
  1. Logically *consistent* belief states are better than inconsistent states.
  2. If a belief state includes $\ulcorner p \urcorner$ and $\ulcorner p \to q \urcorner$, then it would be better if it contained $\ulcorner q \urcorner$ and did not contain $\ulcorner {\sim} q \urcorner$ (than it would otherwise be).
  3. Degree-of-belief states that can be accurately represented as *probability models*, are better than those which cannot be.
  4. *If* an ideally rational agent S satisfies the following two conditions:
     (*i*) S's doxastic state at t can be represented as a Pr-model $\langle \mathcal{B}_S^t, \Pr_S^t \rangle$,
     (*ii*) Between t and t', S learns q *and nothing else* (where q is in $\mathcal{B}_S^t$),

     then, (*iii*) the *ideal doxastic state* for S at t' is $\langle \mathcal{B}_S^{t'}, \Pr_S^{t'} \rangle$, where $\Pr_S^{t'}(\bullet) = \Pr_S^t(\bullet \mid q)$. [d.o.b.–updating goes by *conditionalization*.]
- (1) and (2) are norms for *belief* states. (3) and (4) are norms for *degree-of-belief* states (or *sequences* of them). We'll focus mainly on (3).
- Next: our first argument for *probabilism* — The Dutch Book Argument.

The Dutch Book Argument for (Pragmatic) Probabilism I

- The key assumptions/set-up of the Dutch Book argument are as follows:
  - For each proposition $p \in \mathcal{B}^t$ in our agent's (Mr. B's) doxastic state at t, Mr. B must announce a number $q(p)$ – called his *betting quotient* on p, at t – and *then* Ms. A (the bookie) will choose the *stake* $\mathfrak{s}$ of the bet.
  - $|\mathfrak{s}|$ should be small in relation to Mr. B's total wealth (more on this later). But, it can be positive or negative (so, she can "switch sides").

$$\text{Mr. B's payoff (in \$) for a bet about } p = \begin{cases} \mathfrak{s} - q(p) \cdot \mathfrak{s} & \text{if } p \text{ is true.} \\ -q(p) \cdot \mathfrak{s} & \text{if } p \text{ is false.} \end{cases}$$

  - NOTE: If $\mathfrak{s} > 0$, then the bet is *on* p, if $\mathfrak{s} < 0$, then the bet is *against* p.
  - $q(p)$ is taken to be a measure of Mr. B's *degree of belief* in p (at t).
  - If there is a sequence of multiple bets on multiple propositions, then Mr. B's total payoff is the *sum* of the payoffs for each bet on each proposition. This is called "the package principle". More on *it* later!

The Dutch Book Argument for (Pragmatic) Probabilism II

- Mr. B's "degree of belief function" $q(\cdot)$ is *coherent* iff it is impossible for Ms. A to choose stakes $\mathfrak{s}$ such that she wins *no matter what happens*, *i.e.*, $q(\cdot)$ is *coherent* iff Ms. A cannot construct a "Dutch Book" against Mr. B.

  Theorem (DBT). $q(\cdot)$ is *coherent* **only if** $q(\cdot)$ is a *probability function*.

- Note: the "if" part is also a theorem. That is the *converse* DBT. We won't prove it, but I'll discuss it. Advocates of the DBA think DBT justifies this:

  Claim. The doxastic state $\langle \mathcal{B}_S^t, q(\cdot) \rangle$ of an agent S who is faced with such a scenario is (pragmatically) rational ***only if*** $q(\cdot)$ is coherent (hence a probability function). Does this imply P-probabilism? More, below.

- Does Theorem justify Claim? There are worries about the relationship between gambling and P-rationality, and the "package principle" (and various other assumptions) will also be called into question.

- We'll discuss these worries after we look at the Theorem itself.

The Dutch Book Argument for (Pragmatic) Probabilism III

- The DBT has four parts: one for each of the three axioms, and one for the definition of conditional probability. In each case, we prove that if $q(\cdot)$ [or $q(\cdot \mid \cdot)$] *violates* the axioms (or defn.), then $q(\cdot)$ is *in*coherent.
- **If Mr. B violates Axiom 2, then his q is incoherent**. Proof:
  - If Mr. B assigns $q(\top) = a < 1$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
  - Similarly, if Mr. B assigns $q(\top) = a > 1$, then Ms. A sets $\mathfrak{s} > 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
    - ∗ NOTE: if $q(\top) = 1$, then Mr. B's payoff is always $\mathfrak{s} - \mathfrak{s} = 0$, which avoids *this* Book. I'll discuss the *converse* DBT further, below.
- **If Mr. B violates Axiom 1, then his q is incoherent**. Proof:
  - If $q(p) = a < 0$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is $\mathfrak{s} - a\mathfrak{s} < 0$ if p, and $-a\mathfrak{s} < 0$ if $\sim p$. [If $q(p) \geq 0$, then Mr. B's payoff is $\mathfrak{s} - q\mathfrak{s} \geq 0$ if $\mathfrak{s} > 0$ and p is true, and $-q\mathfrak{s} \geq 0$ if $\mathfrak{s} < 0$ and $\sim p$, avoiding *this* Book.]

The Dutch Book Argument for (Pragmatic) Probabilism IV

- Recall, our Axiom 3 requires that

$$\Pr(p \vee r) = \Pr(p) + \Pr(r)$$

  if p and r cannot both be true (*i.e.*, if they are mutually exclusive).

- The argument for this *additivity* axiom is more controversial. The main source of controversy is the "package principle". We'll just *assume* it for the proof of the *Theorem*. But, for the *Claim*, we will re-think it.
- I will now go through the additivity case of the *Theorem*, and then I will discuss Maher's (and Schick's) objection to the "package principle".
- **Setup**: Let p and r be some pair of mutually exclusive propositions in the agent's doxastic state at t. And, suppose Mr. B announces these b's:

$$q(p) = a,\ q(r) = b,\ \text{and } q(p \vee r) = c,\ \text{where } c \neq a + b.$$

- This will leave Mr. B susceptible to a Dutch Book. Here's why ...

The Dutch Book Argument for (Pragmatic) Probabilism V

- **Case 1**: $c < a + b$. Ms. A asks Mr. B to make *all 3* of these bets ($\mathfrak{s} = +\$1$):

  1. Bet $\$a$ on p to win $\$(1 - a)$ if p, and to lose $\$a$ if $\sim p$.
  2. Bet $\$b$ on r to win $\$(1 - b)$ if r, and to lose $\$b$ if $\sim r$.
  3. Bet $\$(1 - c)$ *against* $p \vee r$ to win $\$c$ if $\sim(p \vee r)$, and lose $\$(1 - c)$ o.w.

- Since p and r are mutually exclusive (by assumption of the additivity axiom), the conjunction $p \,\&\, r$ cannot be true. ∴ There are 3 cases:

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, \sim r$	$1 - a$	$-b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, r$	$-a$	$1 - b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, \sim r$	$-a$	$-b$	c	$c - (a + b)$

- Since $c < a + b$, $c - (a + b)$ is negative. So, Mr. B loses $\$[c - (a + b)]$.
- **Case 2**: $c > a + b$. Ms. A simply reverses the bets ($\mathfrak{s} = -\$1$), and a parallel argument shows that the total payoff for Mr. B is $\$-[c - (a + b)] < 0$.
- Note: he can avoid *this* Book, by setting $c = a + b$. More on CDBT, below.

The Dutch Book Argument for (Pragmatic) Probabilism VI

- We also need to show that an agent's conditional betting quotients are coherent only if they satisfy our ratio definition of conditional probability. There's a DBT for this too (and it also assumes the "package principle").
- Suppose Mr. B announces: $q(p \,\&\, r) = b$, $q(r) = c > 0$, and $q(p \mid r) = a$. Ms. A asks Mr. B to make *all 3* of these bets (stakes depend on quotients!):

1. Bet $\$(b \cdot c)$ on $p \,\&\, r$ to win $\$[(1-b) \cdot c]$ if $p \,\&\, r$, and lose $\$(b \cdot c)$ o.w. $[\mathfrak{s} = c]$
2. Bet $\$[(1-c) \cdot b]$ against r to win $\$(b \cdot c)$ if r, and lose $\$[(1-b) \cdot c]$ o.w. $[\mathfrak{s} = b]$
3. Bet $\$[(1-a) \cdot c]$ against p, conditional on r, to win $\$(a \cdot c)$ if $r \,\&\, p$, and lose $\$[(1-a) \cdot c]$ if $r \,\&\, {\sim}p$. If ${\sim}r$, then the bet is *called off*, and payoff is \$0. $[\mathfrak{s} = c]$

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, r$	$(1-b) \cdot c$	$-[(1-c) \cdot b]$	$-[(1-a) \cdot c]$	$(a \cdot c) - b$
${\sim}p \,\&\, r$	$-(b \cdot c)$	$-[(1-c) \cdot b]$	$a \cdot c$	$(a \cdot c) - b$
${\sim}r$	$-(b \cdot c)$	$b \cdot c$	0	0

- If $a < \frac{b}{c}$, then Mr. B loses *come what may*. If $a > \frac{b}{c}$, then Ms. A just asks Mr. B to take the other side on all three bets. So, coherence requires: $q(p \mid r) = \frac{q(p \& r)}{q(r)}$.

The Dutch Book Argument for (Pragmatic) Probabilism VII

- Here are some problems with/limitations of the Dutch Book *Argument*:
  - It assumes that bets (or gambles) which are severally acceptable are jointly acceptable. This "package principle" is not obvious (see below).
  - It is couched in terms of *money*. It tacitly assumes that *utility* is *linear* in *money*. But, money seems to have *diminishing marginal utility*.
    * More generally, DBAs involve *betting behavior*. One might wonder whether one's betting behavior (when *forced* to post odds) is representative of *rational* behavior *generally* (gambling aversion?).
  - The DBA requires the *converse* DBT to be persuasive. And, *even with* the CDBT, it's still not clear whether pragmatic probabilism follows from the DBA. Does it show that Pr is *better than* non-Pr?
  - It does not address *process* requirements, only state requirements, *i.e.*, it does not constrain *transitions* from one doxastic state to another.
  - It presupposes P-rational agents are *logically omniscient* – that they can *recognize all tautologies* (in $\mathcal{B}^t$). Does P-rationality require this?

Postscripts to DBA I: The Package Principle

- The Dutch Book argument for Additivity may appear as airtight as the Dutch Book arguments for Normality and Non-Negativity. But, Schick (and others) have spotted a possible flaw [see the Schick paper on website].
- In our Dutch Book argument for Additivity, we (implicitly) assumed:
  - Bets that are severally acceptable are jointly acceptable.
  - The value of a set of bets is the sum of values of its elements.
- Might a rational agent be willing to accept *each* of the bets (1)–(3) without being willing to accept *all three at once*? And, if not, why not?
- Might they not value (1) at $\$a$, (2) at $\$b$, and (3) at $\$-c$, *without* valuing the *collection* at $\$[a+b-c]$? After all, they may *see* that, *taken jointly*, bets (1)–(3) lead to a sure loss, whereas *no individual bet does*.
- It is important to note that the DBA for the ratio definition of conditional probability also presupposes this sort of "package principle".
- Are there counterexamples to the "package principle"? Maher thinks so:

Suppose that after a night on the town, you want to catch the bus home. Alas, you find that you only have 60 cents in your pocket, but the bust costs \$1. A bookie, learning of your plight, offers you the following deal: If you give him your 60 cents, he will toss a coin, and if the coin lands heads then he will give you \$1; otherwise, you have lost your 60 cents. If you accept the bookie's proposal, then you stand a 50-50 chance of being able to take the bus home, while rejecting it means you will certainly have to walk. Under these conditions, you may well feel that the bookie's offer is acceptable; let us suppose you do. Presumably, the offer would have been equally well acceptable if you were betting on tails rather than heads...

- As betting quotients were defined for the simple DBA, your quotient for heads in the above example is 0.6, and so is your quotient for tails.
- By additivity, $\Pr(\text{Heads} \vee \text{Tails}) = \Pr(\text{Heads}) + \Pr(\text{Tails})$, since Heads and Tails are mutually exclusive. But, then, $q(\text{Heads} \vee \text{Tails}) = 1.2 > 1$, which violates the axioms (recall, we have a theorem saying $\Pr(p) \leq 1$, for all p).
- Assumption: if you find a bet on Heads acceptable, and find a bet on Tails acceptable, then you should also find both bets acceptable, *taken jointly*.
- I have made a handout which shows exactly when the PP is required.

Postscripts to the DBA II: The Value of Money

- We have the caveat about $\$|\mathfrak{s}|$ being "small in comparison to the agent's total wealth" for two reasons. First, if the agent could lose *everything* on a bet, this would undermine the probative value of the argument.
- Also, real agents (and, arguably, also rational agents!) marginally value money in a way that is *non-linear*, especially for larger sums of money.
- The difference in value between \$1 and \$1000 is pretty substantial. But, the difference in value between \$1M + \$1 and \$1M + \$1000 is not.
- This is called the *diminishing marginal utility of money*. Empirical studies show that actual agents have marginal utilities that are close to being linear only for \$ amounts that are "small relative to their total wealth."
- It's not implausible to suppose that rational agents are like this too. So, if we want something with *linear* marginal value (for *additivity* purposes – think "package principle"!), money is probably not the best thing to use.

Postscripts to the DBA III: The Need for the *Converse* DBT

- The DBT *by itself* cannot secure pragmatic probabilism. All the DBT establishes is that q's coherence entails that q is a probability function. What if the *converse* of this were *false*? Would the DBA still be persuasive?
- If there were some probability functions that were *also* susceptible to Dutch Book, then pragmatic probabilism would *not follow* from the DBA.
- Remember, pragmatic probabilism says *all* probabilistic doxastic states are (pragmatically) better than *all* non-probabilistic doxastic states.
- The mere fact that (DBT) *all non*-probabilistic states are *susceptible* to Dutch Book is *not* sufficient to establish this. One *also* needs to show that (CDBT) *all probabilistic* doxastic states are *immune* from Dutch Book.
- Luckily, the (CDBT) is *true*. We have *not* proven this (and we will not). All we have shown is that *particular* Dutch Books for each of the axioms is *blocked* by satisfying *that* axiom. This does *not* establish (CDBT).
- See the Kemeny paper (on website), if you want to see the proof of (CDBT).

Postscripts to the DBA IV: Is the *Converse* DBT *Enough*?

- *Even if* we use both DBT and CDBT (which *are* both theorems), does the Dutch Book *Argument* establish pragmatic probabilism? Maybe not.
- Alan Hájek thinks there is still a gap in the DBA, *even with* both directions of the DBT, *and even if* we grant the truth of the "package principle".
- As Hájek explains in his paper "Scotching Dutch Books" (now on website), the structure of the DBA is more-or-less something like the following:
  1. S's q is coherent $\iff$ S's q is a probability function. [DBT + CDBT]
  2. S is susceptible to Dutch Book $\iff$ S's q is incoherent. [definition]
  3. Susceptibility to DB is "bad" and immunity from DB is "good". [ass.]
  4. $\therefore$ S's q is *better* if it is probabilistic than if it is *non*-Pr. [P-probabilism]
- Does (4) follow from (1)–(3)? Clearly, (1)–(3) entail that there is *some sense* in which q is *guaranteed* to be "better" *in virtue of* being probabilistic.
- What if there is *also* some sense in which q is *guaranteed* to be "*worse*" *in virtue of* being probabilistic. Is there a symmetric "Good Book" argument?

Postscripts to the DBA V: Hájek's "Good Book" Argument

- A *Good Book* is a sequence of bets that *wins* money *come what may*. Betting quotients q are *good* if they are susceptible to a Good Book.
- It turns out that the following as *also* a theorem (we won't prove it, but it shouldn't be surprising, given the Dutch Book theorem and it Converse):

  Good Book Theorem (GBT). q is good $\iff$ q is *not* a probability function.

- So, why not the following, symmetric argument *against* P-probabilism?
  1. S's q is good $\iff$ S's q is not a probability function. [GBT]
  2. S is susceptible to Good Book $\iff$ S's q is good. [definition]
  3. Susceptibility to GB is "good" and immunity from GB is "bad". [ass.]
  4. $\therefore$ S's q is *worse* if it is probabilistic than if it is *non*-Pr. [$\sim$P-probabilism]
- The question now becomes: Is the DBA more compelling than the GBA? Which is pragmatically *better*: immunity from DB or susceptibility to GB?
- And, are there yet *other dimensions* of "goodness" we have overlooked?

Postscripts to the DBA VI: States *vs*. Processes Again 1

- The Dutch Book Argument we have seen only aims to establish that rationality requires an agent's doxastic state *at a particular time* to be representable as a probability model. This is a *state* requirement.
- These kinds of requirements are also sometimes called requirements of *synchronic* rationality. One might wonder: what about *processes* that lead us from one probability model to another — *diachronic* rationality?
- Traditional subjective probabilists (like Ramsey and de Finetti) didn't think there were any process requirements in this sense. So long as you are "coherent" at each time, that's all there is to pragmatic rationality.
- Contemporary subjective probabilists (often called "Bayesians") do offer *diachronic* Dutch Book arguments in support of what I called (4) above, which is sometimes called the "rule of conditionalization" (ROC).
- We won't look at diachronic Dutch Book arguments in this course. But, there are various arguments of this kind in the literature.

Postscripts to the DBA VI: States *vs*. Processes Again 2

- [The "accuracy arguments" we will look at next in our *epistemic* probability unit do not seem to have any diachronic analogues. This is an interesting asymmetry in the literature on subjective probability.]
- There are other kinds of process requirements in the literature on pragmatic subjective probability. One of these is (roughly) as follows:
  - When an agent goes from one doxastic state to another, they should do so in a way that constitutes a "minimal change"— the new state should be "closest" to their old one, subject to some "constraints".
- This rough idea can be made more precise, and it can lead to answers that diverge from the (ROC), depending on how one precisifies the terms "learning", "minimal change", "closest", and "constraints".
- *E.g.*, if "learning" does not require *assigning probability 1* to what is learned, then this sort of "minimal change" approach leads to a more general form of conditionalization known as *Jeffrey Conditionalization*. [We may touch on this later, when we discuss Bayesian confirmation.]

Postscripts to the DBA VII: Logical Omniscience

- If the DBA is sound, then *pragmatically* rational agents are *logically omniscient* — they can demarcate the logical truths from the non-logical-truths in their doxastic state $\mathcal{B}^t$. This is a strong requirement!
- Several authors (*e.g.*, Hacking, Harman) have objected that this is simply too strong a requirement to place on a pragmatically rational agent.
- One could try to weaken this requirement in some way. Hacking's "Slightly More Realistic Personal Probability" (website) is a good example.
- It is not so easy to weaken this requirement and still ensure that the resulting doxastic states are *probability models*. Consider two proposals:
  - S is required to have $\Pr(p) = 1$ *only* for p's S *knows* to be logical $\top$'s.
  - S is required to have $\Pr(p) = 1$ *only* for p's that are expressible (say, in some language S uses) with *at most* some degree k of *complexity*.
- Problem: how will we ensure that $\mathcal{B}^t$ is a *Boolean algebra* on these proposals? We'll return to this issue in the Bayesian confirmation unit.