Philosophy 148 — Announcements & Such

• Administrative Stuff
  - HW #1 is due on Thursday (by the end of class).
  - HW #2 will be assigned on Thursday as well.
  - Today's Agenda
    * Back to filling-out our “guiding analogy”:
      \[
      \frac{\text{truth-on-} \mathcal{L}}{\text{probability}} \quad ; \\
      \frac{\text{probability-in-} \mathcal{M}}{\text{truth-on-} \mathcal{M}}
      \]
    * Today, this will involve the following:
      - Brief review of theories of truth
      - Brief overview of some theories of objective probability
      - Then, Subjective Probability: A doxastic framework, and two arguments for pragmatic and epistemic probabilism

Brief Overview of Theories of Truth

• According to objective theories of truth, \( p \) is true if it corresponds to “the way the world really is”. In other words, there are mind-independent truthmakers, and these determine which statements are true.

• Subjective theories of truth tend to talk about beliefs being true if they are justified, coherent with one’s beliefs, and/or useful for one to believe.

• I mentioned last time that subjective theories face a regress problem (although, this may not be a fatal problem in the end). I also mentioned that, intuitively, subjective theories seem to yield incorrect verdicts.

• One thing about truth that seems clear is that it is redundant. When I assert “\( p \) is true”, this is just like asserting \( p \) itself. For instance, if I say “it is true that it is raining”, this is equivalent to just saying “it is raining”.

• Subjective theories seems to violate redundancy. Intuitively, when I say “\( p \) is justified” (useful, coherent, etc.), this is not equivalent to just saying \( p \). Intuitively, evidence can be misleading, and wishful thinking may be useful, etc.

Probability-in- \( \mathcal{M} \) as analogous to truth-on- \( \mathcal{L} \)

• Just as we can talk about \( p \) being true-on- \( \mathcal{L} \), which is synonymous with \( s_1 = p \), we can also talk about \( p \) having probability-on- \( \mathcal{M} \).

• And, like true-on- \( \mathcal{L} \), probability-on- \( \mathcal{M} \) is a logical/formal concept.

• That is, once we have specified a probability model \( \mathcal{M} \), this logically determines the probability-on- \( \mathcal{M} \) values of all sentences in \( \mathcal{L} \).

• Moreover, just as the true-on- \( \mathcal{L} \) of sentence \( p \) does not imply anything about \( p \)’s truth (simplex), neither does the probability-on- \( \mathcal{M} \) of \( p \) imply anything about \( p \)’s probability (simplectic) — if there be such a thing.

• Just as we have different philosophical “theories” of truth, we will also have different (and analogous) philosophical “theories” of probability.

• And, as in the case of truth, there will be objective theories and subjective theories of probability. However, there will be more compelling reasons for “going subjective” in the probability case than in the truth case.

• Let’s begin by looking at some objective theories of probability.

Brief Digression on Basic Set-Theoretic Notation

• The statement “\( a \in S \)” means that the object \( a \) is a member of the set \( S \):
  - \( 1 \in \{1, 2, 3\} \), but \( 4 \notin \{1, 2, 3\} \).

• The statement “\( X \subseteq Y \)” means that the set \( X \) is a subset of the set \( Y \) (in other words, all members of the set \( X \) are members of the set \( Y \)):
  - \( \{1, 2\} \subseteq \{1, 2, 3\} \), but \( \{1, 4\} \notin \{1, 2, 3\} \).

• We use “\( X \subset Y \)” to say that \( X \) is a proper subset of \( Y \).
  - \( \{1, 2, 3\} \subseteq \{1, 2, 3\} \), but \( \{1, 2, 3\} \subset \{1, 2, 3\} \).

• We can characterize sets using the \( \{ \cdot | \cdot \} \) notation, for instance:
  - \( \{a \mid a > 0 \& a \in \mathbb{Z}\} \) denotes the set of positive integers.

• \( X \cap Y \) denotes the intersection of the sets \( X \) and \( Y \).
  - \( \{1, 2, 3\} \cap \{2, 4, 6\} = \{2\} \), and \( \{2, 4, 8\} \cap \{8, 2, 1\} = \{2\} \).

• \( X \cup Y \) denotes the union of the sets \( X \) and \( Y \).
  - \( \{1, 2, 3\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 6\} \), and \( \{2, 4, 8\} \cup \{8, 2, 1\} = \{4, 2, 8, 1\} \).
Objective Theories of Probability (I)

- The simplest objective theory is the actual (finite) frequency theory.
- First, we must verify that actual frequencies in finite populations satisfy the probability axioms (otherwise, they aren’t probabilities at all).
- Let P be an actual (non-empty, finite) population, let χ be a property, and let X denote the set of (all) objects that actually have property χ.
- Let #(S) denote the number of objects in a set S. Using #(·), we can define the actual frequency of χ in such a population P in the following way:
  \[ f_P(\chi) \equiv \frac{\#(\chi \cap P)}{\#(P)} \]

Next, let X be the proposition that an (arbitrary) object a ∈ P has property χ. Using \( f_P(\chi) \), we can define \( P_P(X) \), as follows:

- \( P_P(X) \equiv f_P(\chi) \).

- We need to show that \( P_P(X) \) is in fact a probability function. There are various ways to do this. I will show that \( P_P(X) \) satisfies our three axioms.

Objective Theories of Probability (II)

- Axiom 1. We need to show that \( P_P(X) \geq 0 \), for any property χ. This is easy, since the ratio \( \frac{\#(\chi \cap P)}{\#(P)} \) must be non-negative, for any property χ. This is because P is non-empty (\#(P) > 0), and \#(χ ∩ P) must be non-negative.

- Axiom 2. We need to show that, if \( X =\models \top \), then \( P_P(X) = 1 \). In this context, we’re taking about properties χ that — by logic alone — must be satisfied by all objects in the universe (e.g., \( \chi X = FX \lor \neg FX \)). In this case, we have \( \chi ∩ P = P \), since every object is in χ. Therefore, \( P_P(X) = \frac{\#(P)}{\#(P)} = 1 \).

- Axiom 3. To be shown: If \( X \land Y =\models \bot \), then \( P_P(X \land Y) = P_P(X) + P_P(Y) \). In this context, \( X \land Y =\models \bot \) means we are talking about properties χ and ψ such that — by logic alone — no object can satisfy both properties at once (e.g., \( \chi a \land \psi a =\models \bot \)). In such a case, we will have the following:

\[
P_P(X \land Y) = \frac{\#(\chi \cup \psi \cap P)}{\#(P)} = \frac{\#(\chi \cap P) \cup (\psi \cap P)}{\#(P)} = \frac{\#(\chi \cap P) + \#(\psi \cap P)}{\#(P)} = P_P(X) + P_P(Y)
\]

Objective Theories of Probability (III)

- OK, so actual frequencies in populations determine probabilities. But, they are rather peculiar probabilities, in several respects.

- First, they are population-relative. If an object a is a member of multiple populations \( P_1, \ldots, P_n \), then this may yield different values for \( P_{P_1}(X), \ldots, P_{P_n}(X) \). This is related to the reference class problem from last time.

- Another peculiarity of finite actual frequencies is that they sometimes seem to be misleading about intuitive objective probabilities.

- For instance, imagine tossing a coin n times. This gives a population P of size n, and we can compute the P-frequency-probability of heads \( P_P(H) \).

- As n gets larger, the value of this frequency tends to “settle down” to some small range of values (see Mathematica notebook). Intuitively, none of these finite actual frequencies is exactly equal to the bias of the coin.

- So, finite frequencies seem, at best, to provide “estimates” of probabilities in some deeper objective sense. What might such a “deeper sense” be?

Objective Theories of Probability (IV)

- The law of large numbers ensures that (given certain underlying assumptions about the coin) the “settling down” we observe in many actual frequency cases (coin-tossing) will converge in the limit (n → ∞).

- If we do have convergence to some value (say \( \frac{1}{2} \) for a fair coin), then this value seems a better candidate for the “intuitive” objective probability. This leads to the hypothetical limiting frequency theory of probability.

- According to the hypothetical limiting frequency theory, probabilities are frequencies we would observe in a population — if that population were extended indefinitely (e.g., if we were to toss the coin \( \infty \) times).

- There are various problems with this theory. First, convergence is not always guaranteed. In fact, there are many hypothetical infinite extensions of any P for which the frequencies do not converge as \( n \rightarrow \infty \).

- Second, even among those extensions that do converge, there can be many different possible convergent values. Which is “the” probability?
**Objective Theories of Probability (V)**

- *Propensity or chance* theories of probability posit the existence of a deeper kind of physical probability, which manifests itself empirically in finite frequencies, and which constrains limiting frequencies.
- Having a theory that makes sense of quantum mechanical probabilities was one of the original inspirations of propensity theorists (Popper).
- In quantum mechanics, probability seems to be a fundamental physical property of certain systems. The theory entails exact probabilities of certain token events in certain experimental set-ups/contexts.
- These probabilities seem to transcend both finite and infinite frequencies. They seem to be basic *dispositional properties* of certain physical systems.
- In classical (deterministic) physics, all token events are *determined* by the physical laws + initial conditions of the universe. In quantum mechanics, only probabilities of token events are determined by the laws + i.c.’s.
- This leaves room for (non-extreme) *objective chances* of token events.

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**Objective Theories of Probability (VI)**

- We saw that finite frequencies satisfy the (classical) probability axioms.
- Infinite frequencies don’t satisfy the (classical) axioms of (infinite) probability calculus, for two reasons (beyond the scope of our course).
  - The underlying (infinite) logical space is non-Boolean.
  - Infinite frequencies do not satisfy the (infinite) additivity axiom.
- Some have claimed that QM-probabilities are also non-classical, owing to the fact that the underlying “quantum logic” is non-Boolean. But, there are also interpretations of QM in terms of classical probabilities.
- It is often assumed that objective chances satisfy the probability axioms, but it is not quite clear why (especially, in light of the above remarks).
- Since I won’t be dwelling on these sorts of objective (physical) theories of probability in this course, I won’t fuss about these technical puzzles.
- Next, we’ll discuss subjective probability, and we will dwell on that.

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**Subjective Theories of Probability (I)**

- I’ll begin by motivating subjective probability with an example/context:
  I’m holding a coin behind my back. It is either 2-headed or 2-tailed. You do not know which kind of coin it is (and you have no reason to favor one of these possibilities over the other). I’m about to toss it. What probability (or odds) would you assign to the proposition that it will land heads?
- Many people have the intuition that \( \frac{1}{2} \) (or even 50:50 odds) would be a reasonable answer to this question. However, it seems clear that \( \frac{1}{2} \) cannot be the *objective* probability/chance of heads in this example.
- After all, we know that the coin is either 2-headed or 2-tailed. As such, the objective probability of heads is either 1 or 0 in this example.
- One might describe this as *epistemic* probability, because it seems *epistemically reasonable* to be 50% confident that the coin will land heads.
- Also, taking a bet at even odds on heads seems *pragmatically* reasonable. This suggests a *pragmatic* theory of probability is also plausible here.

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**Subjective Theories of Probability (II)**

- It seems clear that there is such a thing as “degree of belief”. And, it also seems clear that there are *some* sorts of constraints on such degrees.
- But, why should degrees of belief obey the *probability axioms*?
- There are arguments that epistemic and pragmatic probabilities should each be probabilities. We will examine examples of each type of argument.
- We will begin with *pragmatic* subjective probability.
- There are various arguments for *pragmatic probabilism*: that *pragmatically rational* degrees of belief obey the probability axioms.
- All such arguments must do two things:
  - Identify a *necessary condition* (N) for pragmatic rationality.
  - Show that having non-probabilistic degrees of belief violates N.
- The first argument we will examine is the Dutch Book Argument (DBA).
- But, first, a framework for evaluating agents’ doxastic states.
An Evaluative Doxastic Framework (I)

- We will assume that rational agents have attitudes toward propositions. One of these attitudes is belief. What is belief? This is not entirely clear.
- We will say that belief is a relation between an agent $S$ and a proposition $p$. We needn't worry too much about the precise conditions under which $S$ believes that $p$. Intuitively, belief is dispositional property.
- When $S$ believes $p$, this will be accompanied by various dispositions to behave in certain ways: to provide arguments in favor of $p$ should it be challenged, to act in accordance with the assumption that $p$ is true, etc.
- We will not assume that an agent must actively (or consciously) attend to $p$ in order to believe it. Thus, we can have implicit beliefs. But, all propositions believed by an agent must be entertainable for that agent.
- We'll also talk about degrees of belief (degrees of confidence, credences). This is a quantitative or comparative generalization of belief, which will have its own sorts of dispositional manifestations (e.g., betting behavior).

We'll bracket questions about whether beliefs (d.o.b.’s) are the kinds of things you can choose to have. We'll think of this in evaluative rather than normative terms, and (for now) as about states rather than processes.
An Evaluative Doxastic Framework (V)

- When we make judgments about rationality, we can take two “stances”:
  - **Evaluative Stance**: Here, we’re merely *evaluating* some state or process against some standard(s) of ideal rationality. We’re not making any claims about what anyone *ought to do* (not advising, blaming, etc).
  - **Normative Stance**: Here, we are talking about what some agent(s) *ought to do*, from the point of view of some standard(s) of ideal (normative) rationality. Here, we do advise, prescribe, blame, etc.

- And, we can be making judgments about states or processes:
  - **State Judgments**: These are judgments about the rationality (“goodness”) of (some aspect of) the doxastic state of S at t.
  - **Process Judgments**: Judgments of the rationality (“goodness”) of some process leading S from one doxastic state (at t) to another (at t’ > t).

- We will be involved mainly with in *evaluation* of doxastic states. [We’ll say a little about doxastic processes, but also from an evaluative stance.]

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The Dutch Book Argument for (Pragmatic) Probabilism I

- The key assumptions/set-up of the Dutch Book argument are as follows:
  - For each proposition p ∈ 2^t in our agent’s (Mr. B’s) doxastic state at t, Mr. B must announce a number q(p) – called his *betting quotient* on p, at t – and then Ms. A (the bookie) will choose the *stake* s of the bet.
  - |s| should be small in relation to Mr. B’s total wealth (more on this later). But, it can be positive or negative (so, she can “switch sides”).

Mr. B’s payoff (in S) for a bet about p = \[ s - q(p) \cdot s \text{ if } p \text{ is true.} \]
\[ -q(p) \cdot s \text{ if } p \text{ is false.} \]

- **NOTE**: If s > 0, then the bet is on p, if s < 0, then the bet is against p.
- q(p) is taken to be a measure of Mr. B’s *degree of belief* in p (at t).
- If there is a sequence of multiple bets on multiple propositions, then Mr. B’s total payoff is the *sum* of the payoffs for each bet on each proposition. This is called “the package principle”. More on it later!

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The Dutch Book Argument for (Pragmatic) Probabilism II

- Examples of some evaluative doxastic claims/principles (“norms”):
  1. Logically *consistent* belief states are better than inconsistent states.
  2. If a belief state includes ‘p’ and ‘p → q’, then it would be better if it contained ‘q’ and did not contain ‘~q’ (than it would otherwise be).
  3. Degree-of-belief states that can be accurately represented as *probability models*, are better than those which cannot be.
  4. If an ideally rational agent S satisfies the following two conditions:
     (i) S’s doxastic state at t can be represented as a Pr-model \( (B^t_\mathcal{S}, \Pr^t_\mathcal{S}) \),
     (ii) Between t and t’, S learns q and nothing else (where q is in \( B^t_\mathcal{S} \)),

     then, (iii) the ideal doxastic state for S at t’ is \( (B_{t'}^t, \Pr_{t'}^t) \), where \( \Pr_{t'}^t(\bullet) = \Pr_{t}^t(\bullet | q) \). [d.o.b.–updating goes by *conditionalization.*]

- (1) and (2) are norms for *belief* states. (3) and (4) are norms for *degree-of-belief* states (or sequences of them). We’ll focus mainly on (3).

- Next: our first argument for *probabilism* — The Dutch Book Argument.
The Dutch Book Argument for (Pragmatic) Probabilism III

- The DBT has four parts: one for each of the three axioms, and one for the definition of conditional probability. In each case, we prove that if \( q(\cdot) \) or \( q(\cdot | \cdot) \) violates the axioms (or defn.), then \( q(\cdot) \) is incoherent.

- **If Mr. B violates Axiom 2, then his \( q \) is incoherent.** Proof:
  - If Mr. B assigns \( q(\top) = a < 1 \), then Ms. A sets \( s < 0 \), and Mr. B’s payoff is always \( s - as < 0 \), since \( \top \) cannot be false.
  - Similarly, if Mr. B assigns \( q(\top) = a > 1 \), then Ms. A sets \( s > 0 \), and Mr. B’s payoff is always \( s - as < 0 \), since \( \top \) cannot be false.
  * NOTE: if \( q(\top) = 1 \), then Mr. B’s payoff is always \( s - s = 0 \), which avoids this Book. I’ll discuss the converse DBT further, below.

- **If Mr. B violates Axiom 1, then his \( q \) is incoherent.** Proof:
  - If \( q(p) = a < 0 \), then Ms. A sets \( s < 0 \), and Mr. B’s payoff is \( s - as < 0 \) if \( p \), and \( as < 0 \) if \( \neg p \). If \( q(p) \geq 0 \), then Mr. B’s payoff is always \( s - qs \geq 0 \) if \( s > 0 \) and \( p \) is true, and \( -qs \geq 0 \) if \( s < 0 \) and \( \neg p \), avoiding this Book.

The Dutch Book Argument for (Pragmatic) Probabilism IV

- Recall, our Axiom 3 requires that:
  \[
  \Pr(p \lor r) = \Pr(p) + \Pr(r)
  \]
  if \( p \) and \( r \) cannot both be true (i.e., if they are mutually exclusive).

- The argument for this additivity axiom is more controversial. The main source of controversy is the “package principle”. We’ll just assume it for the proof of the Theorem. But, for the Claim, we will re-think it.

- I will now go through the additivity case of the Theorem, and then I will discuss Maher’s (and Schick’s) objection to the “package principle”.

- **Setup:** Let \( p \) and \( r \) be some pair of mutually exclusive propositions in the agent’s doxastic state at \( t \). And, suppose Mr. B announces these \( b's\):
  \[
  q(p) = a, \quad q(r) = b, \quad \text{and} \quad q(p \lor r) = c, \quad \text{where} \quad c \neq a + b.
  \]

- This will leave Mr. B susceptible to a Dutch Book. Here’s why . . .

The Dutch Book Argument for (Pragmatic) Probabilism V

- **Case 1:** \( c < a + b \). Ms. A asks Mr. B to make all 3 of these bets (\( s = +$1 \)):
  1. Bet \( $a \) on \( p \) to win \( $1 - a \) if \( p \), and to lose \( $a \) if \( \neg p \).
  2. Bet \( $b \) on \( r \) to win \( $1 - b \) if \( r \), and to lose \( $b \) if \( \neg r \).
  3. Bet \( $1 - c \) against \( p \lor r \) to win \( $c \) if \( \neg(p \lor r) \), and lose \( $1 - c \) o.w.

- Since \( p \) and \( r \) are mutually exclusive (by assumption of the additivity axiom), the conjunction \( p \land r \) cannot be true. \( \therefore \) There are 3 cases:

<table>
<thead>
<tr>
<th>Case</th>
<th>Payoff on (1)</th>
<th>Payoff on (2)</th>
<th>Payoff on (3)</th>
<th>Total Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \land \neg r )</td>
<td>( 1 - a )</td>
<td>(-b)</td>
<td>(-1 - c)</td>
<td>( c - (a + b))</td>
</tr>
<tr>
<td>( \neg p \land r )</td>
<td>(-a)</td>
<td>(1 - b)</td>
<td>(-1 - c)</td>
<td>( c - (a + b))</td>
</tr>
<tr>
<td>( \neg p \land \neg r )</td>
<td>(-a)</td>
<td>(-b)</td>
<td>(c)</td>
<td>( c - (a + b))</td>
</tr>
</tbody>
</table>

- Since \( c < a + b, c - (a + b) \) is negative. So, Mr. B loses \( $c - (a + b) \).

- **Case 2:** \( c > a + b \). Ms. A simply reverses the bets (\( s = -$1 \)), and a parallel argument shows that the total payoff for Mr. B is \( -$[c - (a + b)] < 0 \).

- Note: he can avoid this Book, by setting \( c = a + b \). More on CDBT, below.

The Dutch Book Argument for (Pragmatic) Probabilism VI

- We also need to show that an agent's conditional betting quotients are coherent only if they satisfy our ratio definition of conditional probability. There’s a DBT for this too (and it also assumes the “package principle”).

- Suppose Mr. B announces: \( q(p \land r) = b, \quad q(r) = c > 0 \), and \( q(p \lor r) = a \).

- Ms. A asks Mr. B to make all 3 of these bets (stakes depend on quotients!):
  1. Bet \( $(b \cdot c) \) on \( p \land r \) to win \( $(1 - b) \cdot c \) if \( p \land r \), and lose \( $(b \cdot c) \) o.w. \( [s = c] \)
  2. Bet \( $(1 - c) \cdot b \) against \( r \) to win \( $(b \cdot c) \) if \( r \), and lose \( $(1 - c) \cdot b \) o.w. \( [s = b] \)
  3. Bet \( $(1 - a) \cdot c \) against \( p \), conditional on \( r \), to win \( $(a \cdot c) \) if \( r \land p \), and lose \( $(1 - a) \cdot c \) if \( r \land \neg p \). If \( \neg r \), then the bet is called off, and payoff is \( 0 \). \( [s = c] \)

<table>
<thead>
<tr>
<th>Case</th>
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<th>Payoff on (3)</th>
<th>Total Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \land r )</td>
<td>( (1 - b) \cdot c )</td>
<td>(-[(1 - c) \cdot b])</td>
<td>(-[(1 - a) \cdot c])</td>
<td>( (a \cdot c) - b )</td>
</tr>
<tr>
<td>( \neg p \land r )</td>
<td>(-[(b \cdot c))</td>
<td>(-[(1 - c) \cdot b])</td>
<td>(a \cdot c)</td>
<td>( (a \cdot c) - b )</td>
</tr>
<tr>
<td>( \neg r )</td>
<td>( (b \cdot c))</td>
<td>(b \cdot c)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

- If \( a > \frac{b}{c} \), then Mr. B loses come what may. If \( a < \frac{b}{c} \), then Ms. A just asks Mr. B to take the other side on all three bets. So, coherence requires: \( q(p \lor r) = \frac{q(b \lor r)}{q(r)} \).
The Dutch Book Argument for (Pragmatic) Probabilism VII

- Here are some problems with/limitations of the Dutch Book Argument:
  - It assumes that bets (or gambles) which are severally acceptable are jointly acceptable. This “package principle” is not obvious (see below).
  - It is couched in terms of money. It tacitly assumes that utility is linear in money. But, money seems to have diminishing marginal utility.
  - More generally, DBAs involve betting behavior. One might wonder whether one’s betting behavior (when forced to post odds) is representative of rational behavior generally (gambling aversion?).
  - The DBA requires the converse DBT to be persuasive. And, even with the CDBT, it’s still not clear whether pragmatic probabilism follows from the DBA. Does it show that Pr is better than non-Pr?
  - It does not address process requirements, only state requirements, i.e., it does not constrain transitions from one doxastic state to another.
  - It presupposes P-rational agents are logically omniscient — that they can recognize all tautologies (in $B^i$). Does P-rationality require this?

Postscripts to DBA I: The Package Principle

- The Dutch Book argument for Additivity may appear as airtight as the Dutch Book arguments for Normality and Non-Negativity. But, Schick (and others) have spotted a possible flaw [see the Schick paper on website].
- In our Dutch Book argument for Additivity, we (implicitly) assumed:
  - Bets that are severally acceptable are jointly acceptable.
  - The value of a set of bets is the sum of values of its elements.
- Might a rational agent be willing to accept each of the bets (1)–(3) without being willing to accept all three at once? And, if not, why not?
- Might they not value (1) at $\$a$, (2) at $\$b$, and (3) at $\$c$, without valuing the collection at $\$a + b − c$? After all, they may see that, taken jointly, bets (1)–(3) lead to a sure loss, whereas no individual bet does.
- It is important to note that the DBA for the ratio definition of conditional probability also presupposes this sort of “package principle”.
- Are there counterexamples to the “package principle”? Maher thinks so:

Suppose that after a night on the town, you want to catch the bus home. Alas, you find that you only have 60 cents in your pocket, but the bust costs $1. A bookie, learning of your plight, offers you the following deal: If you give him your 60 cents, he will toss a coin, and if the coin lands heads then he will give you $1; otherwise, you have lost your 60 cents. If you accept the bookie’s proposal, then you stand a 50-50 chance of being able to take the bus home, while rejecting it means you will certainly have to walk. Under these conditions, you may well feel that the bookie’s offer is acceptable; let us suppose you do. Presumably, the offer would have been equally well acceptable if you were betting on tails rather than heads...

- As betting quotients were defined for the simple DBA, your quotient for heads in the above example is 0.6, and so is your quotient for tails; thus your betting quotients violate the probability calculus in this example.
- By additivity, $\Pr(\text{Heads} \lor \text{Tails}) = \Pr(\text{Heads}) + \Pr(\text{Tails})$, since Heads and Tails are mutually exclusive. But, $q(\text{Heads} \lor \text{Tails}) = 1.2 > 1$, which violates the axioms (recall, we have a theorem saying $\Pr(p) \leq 1$, for all $p$).
- Assumption: if you find a bet on Heads acceptable, and find a bet on Tails acceptable, then you should also find both bets acceptable, taken jointly.