Philosophy 148 — Announcements & Such

- Administrative Stuff
  - HW #1 is due on Thursday (by the end of class).
  - HW #2 will be assigned on Thursday as well.
    * This will cover arguments for pragmatic and epistemic probabilism — "Dutch Books" and "Epistemic Accuracy" Arguments.
  - Today's Agenda
    * Back to filling-out our "guiding analogy":

$$\frac{\text{truth-on-}\mathcal{I}}{\text{truth}} :: \frac{\text{probability-in-}\mathcal{M}}{\text{probability}}$$

    * Today, this will involve the following:
      · Brief review of theories of truth
      · Brief overview of some theories of objective probability
      · Then, Subjective Probability: A doxastic framework, and two arguments for pragmatic and epistemic probabilism

Brief Overview of Theories of Truth

- According to objective theories of truth, p is true if it *corresponds* to "the way the world really is". In other words, there are *mind-independent truthmakers*, and these determine which statements are true.
- Subjective theories of truth tend to talk about *beliefs* being true if they are *justified*, *coherent* with one's beliefs, and/or *useful* for one to believe.
- I mentioned last time that subjective theories face a regress problem (although, this may not be a fatal problem in the end). I also mentioned that, intuitively, subjective theories seem to yield *incorrect* verdicts.
- One thing about truth that seems clear is that it is *redundant*. When I assert "p is true", this is just like asserting p itself. For instance, if I say "*it is true that* it is raining", this is equivalent to just saying "*it is raining*".
- Subjective theories seems to *violate* redundancy. Intuitively, when I say "p is justified" (useful, coherent, *etc*.), this is *not* equivalent to just saying p. [Intuitively, evidence can be *misleading*, and *wishful thinking* may be useful, *etc*.]

Probability-in-$\mathcal{M}$ as analogous to *truth-on-$\mathcal{I}$*

- Just as we can talk about p being *true-on-$\mathcal{I}_i$*, which is synonymous with $s_i \vDash p$, we can also talk about p having *probability-r-on-$\mathcal{M}$*.
- And, like *truth-on-$\mathcal{I}_i$*, *probability-on-$\mathcal{M}$* is a *logical/formal* concept.
- That is, once we have *specified* a probability model $\mathcal{M}$, this *logically determines* the *probability-on-$\mathcal{M}$* values of all sentences in $\mathcal{L}$.
- Moreover, just as the *truth-on-$\mathcal{I}_i$* of sentence p does not imply anything about p's *truth* (*simpliciter*), neither does the *probability-on-$\mathcal{M}$* of p imply anything about p's *probability* (*simpliciter*) — *if there be such a thing*.
- Just as we have different philosophical "theories" of truth, we will also have different (and analogous) philosophical "theories" of probability.
- And, as in the case of truth, there will be objective theories and subjective theories of probability. However, there will be more compelling reasons for "going subjective" in the probability case than in the truth case.
- Let's begin by looking at some objective theories of probability.

Brief Digression on Basic Set-Theoretic Notation

- The statement "$a \in \mathbf{S}$" means that the object a is a member of the set $\mathbf{S}$:
  - $1 \in \{1, 2, 3\}$, but $4 \notin \{1, 2, 3\}$.
- The statement "$\mathbf{X} \subseteq \mathbf{Y}$" means that the set $\mathbf{X}$ is a subset of the set $\mathbf{Y}$ (in other words, all members of the set $\mathbf{X}$ are members of the set $\mathbf{Y}$):
  - $\{1, 2\} \subseteq \{1, 2, 3\}$, but $\{1, 4\} \nsubseteq \{1, 2, 3\}$.
  - We use "$\mathbf{X} \subset \mathbf{Y}$" to say that $\mathbf{X}$ is a *proper* subset of $\mathbf{Y}$.
    * $\{1, 2, 3\} \subseteq \{1, 2, 3\}$, but $\{1, 2, 3\} \not\subset \{1, 2, 3\}$.
- We can characterize sets using the $\{\cdot \mid \cdot\}$ notation, for instance:
  - $\{a \mid a > 0 \,\&\, a \in \mathbb{Z}\}$ denotes the set of positive integers.
- $\mathbf{X} \cap \mathbf{Y}$ denotes the *intersection* of the sets $\mathbf{X}$ and $\mathbf{Y}$.
  - $\{1, 2, 3\} \cap \{2, 4, 6\} = \{2\}$, and $\{2, 4, 8\} \cap \{8, 2, 1\} = \{2, 8\}$.
- $\mathbf{X} \cup \mathbf{Y}$ denotes the *union* of the sets $\mathbf{X}$ and $\mathbf{Y}$.
  - $\{1, 2, 3\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 6\}$, and $\{2, 4, 8\} \cup \{8, 2, 1\} = \{4, 2, 8, 1\}$.

Objective Theories of Probability (I)

- The simplest objective theory is the *actual (finite) frequency* theory.
- First, we must verify that actual frequencies in finite populations satisfy the probability axioms (otherwise, they aren't *probabilities* at all).
- Let $\mathbf{P}$ be an actual (non-empty, finite) population, let χ be a property, and let $\boldsymbol{\chi}$ denote the set of (all) objects that actually have property χ.
- Let $\#(S) \overset{\text{def}}{=}$ the number of objects in a set S. Using $\#(\cdot)$, we can define the actual frequency of χ in such a population $\mathbf{P}$ in the following way:
  - $f_{\mathbf{P}}(\chi) \overset{\text{def}}{=} \dfrac{\#(\boldsymbol{\chi} \cap \mathbf{P})}{\#(\mathbf{P})}$
- Next, let X be the proposition that an (arbitrary) object $a \in \mathbf{P}$ has property χ. Using $f_{\mathbf{P}}(\chi)$, we can define $\Pr_{\mathbf{P}}(X)$, as follows:
  - $\Pr_{\mathbf{P}}(X) \overset{\text{def}}{=} f_{\mathbf{P}}(\chi)$.
- We need to show that $\Pr_{\mathbf{P}}(X)$ is in fact a *probability* function. There are various ways to do this. I will show that $\Pr_{\mathbf{P}}(X)$ satisfies our three axioms.

Objective Theories of Probability (II)

- **Axiom 1**. We need to show that $\Pr_{\mathbf{P}}(X) \geq 0$, for any property χ. This is easy, since the ratio $\frac{\#(\boldsymbol{\chi} \cap \mathbf{P})}{\#(\mathbf{P})}$ must be non-negative, for any property χ. This is because $\mathbf{P}$ is non-empty [$\#(\mathbf{P}) > 0$], and $\#(\boldsymbol{\chi} \cap \mathbf{P})$ must be non-negative.
- **Axiom 2**. We need to show that, if $X \dashv\vDash \top$, then $\Pr_{\mathbf{P}}(X) = 1$. In this context, we're taking about properties χ that — by logic alone — must be satisfied by all objects in the universe (*e.g.*, $\chi x = Fx \vee {\sim}Fx$). In this case, we have $\boldsymbol{\chi} \cap \mathbf{P} = \mathbf{P}$, since *every* object is in $\boldsymbol{\chi}$. Therefore, $\Pr_{\mathbf{P}}(X) = \frac{\#(\mathbf{P})}{\#(\mathbf{P})} = 1$.
- **Axiom 3**. To be shown: If $X \mathbin{\&} Y \dashv\vDash \bot$, then $\Pr_{\mathbf{P}}(X \vee Y) = \Pr_{\mathbf{P}}(X) + \Pr_{\mathbf{P}}(Y)$. In this context, $X \mathbin{\&} Y \dashv\vDash \bot$ means we are talking about properties χ and ψ such that — by logic alone — no object can satisfy both properties at once (*e.g.*, $\chi a \mathbin{\&} \psi a \vDash \bot$). In such a case, we will have the following:

$$\Pr_{\mathbf{P}}(X \vee Y) = \frac{\#[(\boldsymbol{\chi} \cup \boldsymbol{\psi}) \cap \mathbf{P}]}{\#(\mathbf{P})} = \frac{\#[(\boldsymbol{\chi} \cap \mathbf{P}) \cup (\boldsymbol{\psi} \cap \mathbf{P})]}{\#(\mathbf{P})} = \frac{\#(\boldsymbol{\chi} \cap \mathbf{P}) + \#(\boldsymbol{\psi} \cap \mathbf{P})}{\#(\mathbf{P})}$$
$$= \Pr_{\mathbf{P}}(X) + \Pr_{\mathbf{P}}(Y)$$

Objective Theories of Probability (III)

- OK, so actual frequencies in populations determine *probabilities*. But, they are rather peculiar probabilities, in several respects.
- First, they are *population-relative*. If an object a is a member of multiple populations $\mathbf{P}_1, \ldots, \mathbf{P}_n$, then this may yield different values for $\Pr_{\mathbf{P}_1}(X)$, $\ldots$, $\Pr_{\mathbf{P}_n}(X)$. This is related to the *reference class problem* from last time.
- Another peculiarity of finite actual frequencies is that they sometimes seem to be misleading about intuitive objective probabilities.
- For instance, imagine tossing a coin n times. This gives a population $\mathbf{P}$ of size n, and we can compute the $\mathbf{P}$-frequency-probability of heads $\Pr_{\mathbf{P}}(H)$.
- As n gets larger, the value of this frequency tends to "settle down" to some small range of values (see *Mathematica* notebook). Intuitively, none of these finite actual frequencies is exactly equal to the bias of the coin.
- So, finite frequencies seem, at best, to provide "estimates" of probabilities in some deeper objective sense. What might such a "deeper sense" be?

Objective Theories of Probability (IV)

- The *law of large numbers* ensures that (given certain underlying assumptions about the coin) the "settling down" we observe in many actual frequency cases (coin-tossing) will converge *in the limit* ($n \to \infty$).
- If we do have convergence to some value (say $\frac{1}{2}$ for a fair coin), then this value seems a better candidate for the "intuitive" objective probability. This leads to the *hypothetical limiting frequency theory* of probability.
- According to the hypothetical limiting frequency theory, probabilities are frequencies we *would* observe in a population — *if* that population were extended indefinitely (*e.g.*, if we were to toss the coin ∞ times).
- There are various problems with this theory. First, convergence is not always guaranteed. In fact, there are *many* hypothetical infinite extensions of any **P** for which the frequencies do *not* converge as $n \to \infty$.
- Second, even among those extensions that *do* converge, there can be *many different* possible convergent values. Which is "the" probability?

Objective Theories of Probability (V)

- *Propensity* or *chance* theories of probability posit the existence of a deeper kind of physical probability, which manifests itself empirically in finite frequencies, and which constrains limiting frequencies.
- Having a theory that makes sense of quantum mechanical probabilities was one of the original inspirations of propensity theorists (Popper).
- In quantum mechanics, probability seems to be a fundamental physical property of certain systems. The theory entails exact *probabilities* of certain token events in certain experimental set-ups/contexts.
- These probabilities seem to transcend both finite and infinite frequencies. They seem to be basic *dispositional properties* of certain physical systems.
- In classical (deterministic) physics, all token events are *determined* by the physical laws + initial conditions of the universe. In quantum mechanics, only *probabilities* of token events are determined by the laws + i.c.'s.
- This leaves room for (non-extreme) *objective chances* of token events.

Objective Theories of Probability (VI)

- We saw that finite frequencies satisfy the (classical) probability axioms.
- Infinite frequencies don't satisfy the (classical) axioms of (infinite) probability calculus, for two reasons (beyond the scope of our course).
  - The underlying (infinite) logical space is non-Boolean.
  - Infinite frequencies do not satisfy the (infinite) additivity axiom.
- Some have claimed that QM-probabilities are also non-classical, owing to the fact that the underlying "quantum *logic*" is non-Boolean. But, there are also interpretations of QM in terms of classical probabilities.
- It is often *assumed* that objective chances satisfy the probability axioms, but it is not quite clear *why* (especially, in light of the above remarks).
- Since I won't be dwelling on these sorts of objective (physical) theories of probability in this course, I won't fuss about these technical puzzles.
- Next, we'll discuss subjective probability, and we *will* dwell on that.

Subjective Theories of Probability (I)

- I'll begin by motivating subjective probability with an example/context:

  I'm holding a coin behind my back. It is either 2-headed or 2-tailed. You do not know which kind of coin it is (and you have no reason to favor one of these possibilities over the other). I'm about to toss it. What probability (or odds) would you assign to the proposition that it will land heads?

- Many people have the intuition that $\frac{1}{2}$ (or even 50:50 odds) would be a reasonable answer to this question. However, it seems clear that $\frac{1}{2}$ *cannot* be the *objective* probability/chance of heads in this example.
- After all, we know that the coin is either 2-headed or 2-tailed. As such, the objective probability of heads is either 1 or 0 in this example.
- One might describe this as *epistemic* probability, because it seems *epistemically reasonable* to be *50% confident* that the coin will land heads.
- Also, taking a bet at even odds on heads seems *pragmatically* reasonable. This suggests a *pragmatic* theory of probability is also plausible here.

Subjective Theories of Probability (II)

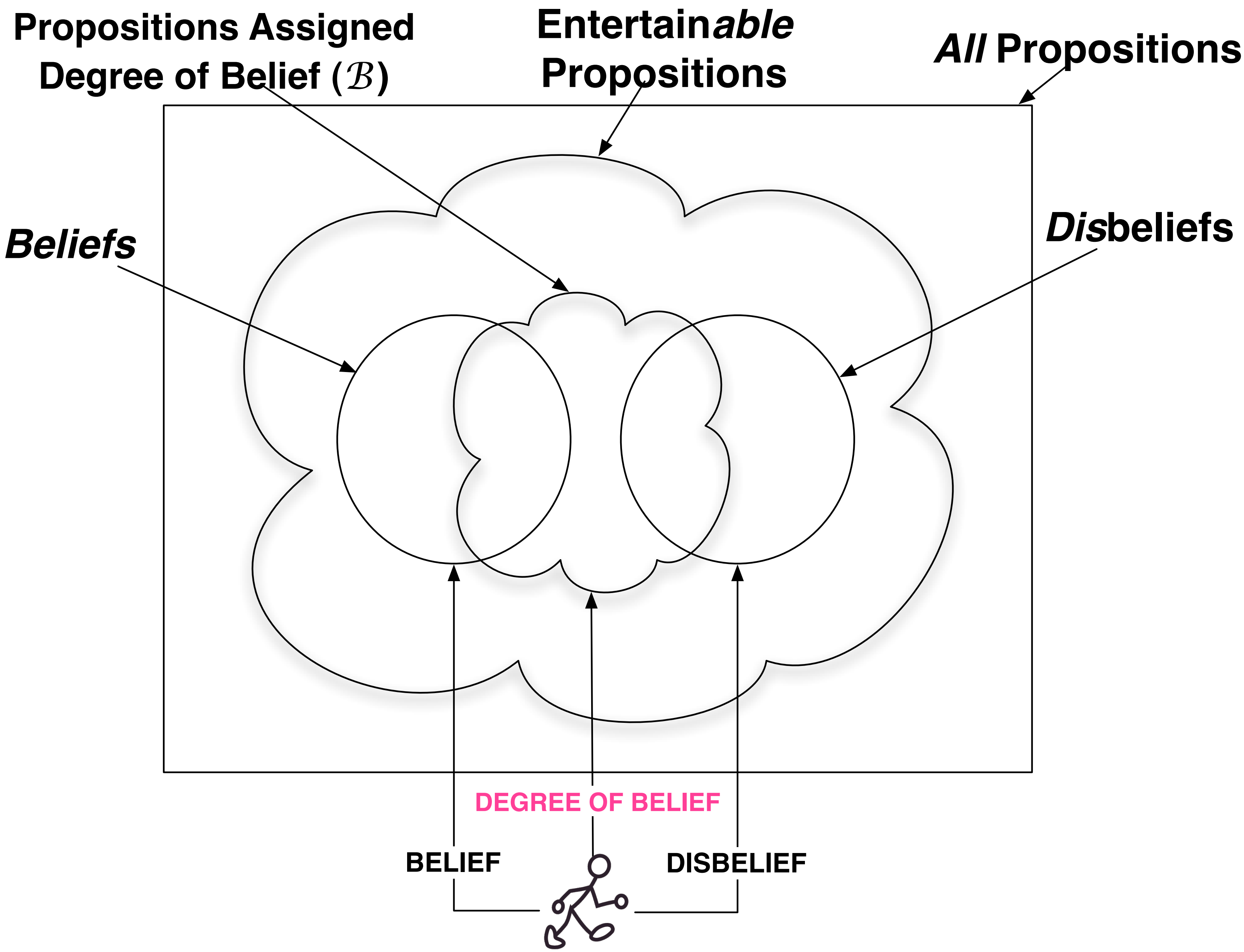
- It seems clear that there is such a thing as "degree of belief". And, it also seems clear that there are *some* sorts of constraints on such degrees.
- But, why should degrees of belief obey the *probability axioms*?
- There are arguments that epistemic and pragmatic probabilities should each be *probabilities*. We will examine examples of each type of argument.
- We will begin with *pragmatic* subjective probability.
- There are various arguments for *pragmatic probabilism*: that *pragmatically rational* degrees of belief obey the probability axioms.
- All such arguments must do two things:
  - Identify a *necessary condition* (N) for pragmatic rationality.
  - Show that having non-probabilistic degrees of belief *violates* N.
- The first argument we will examine is the Dutch Book Argument (DBA).
- But, first, a framework for *evaluating* agents' doxastic states.

An Evaluative Doxastic Framework (I)

- We will assume that rational agents have attitudes toward propositions. One of these attitudes is *belief*. What is belief? This is not entirely clear.
- We will say that belief is a relation between an agent S and a proposition p. We needn't worry too much about the precise conditions under which S believes that p. Intuitively, belief is *dispositional* property.
- When S believes p, this will be accompanied by various dispositions to behave in certain ways: to provide arguments in favor of p should it be challenged, to act in accordance with the assumption that p is true, *etc*.
- We will not assume that an agent must actively (or consciously) *attend to* p in order to believe it. Thus, we can have *implicit* beliefs. But, all propositions believed by an agent must be *entertainable* for that agent.
- We'll also talk about *degrees of belief* (*degrees of confidence*, *credences*). This is a quantitative or comparative *generalization* of belief, which will have its own sorts of dispositional manifestations (*e.g.*, betting behavior).

An Evaluative Doxastic Framework (II)

- We'll assume that, at each time t, an ideally rational agent S has a *doxastic state*, which includes a *Boolean algebra of propositions* $\mathcal{B}_S^t$ — a subset of the p's that are *entertainable* for S at t (those S has "access" to at t).
- $\mathcal{B}_S^t$ will contain those p's in which S has *some degree of confidence* at t. S will only *believe some subset* of this set of propositions at t. But, each member of $\mathcal{B}_S^t$ is, in some sense, a "live option" as a belief for S at t.
- The idea here is that there are three kinds of propositions for an agent S at a time t: (1) entertain*able* (but not entertain*ed*) p's, (2) entertain*ed* p's ($p \in \mathcal{B}_S^t$ receive some degree of confidence), and (3) *un*-entertain*able* p's.
- Restricting the set of candidate beliefs (at t) to some proper subset of *all* the propositions seems right. But, why assume $\mathcal{B}_S^t$ is a *Boolean algebra*? [This implies (among other things) *closure* under Boolean operations.]
- For now, we'll assume this form of *logical omniscience*. Note: we're *not* assuming that the agent's *beliefs* are closed under the logical operations, or even that the set of all entertain*able* propositions is closed. Picture:


Propositions Assigned Degree of Belief ($\mathcal{B}$)
Entertain*able* Propositions
All Propositions
Beliefs
*Dis*beliefs
DEGREE OF BELIEF
BELIEF
DISBELIEF

An Evaluative Doxastic Framework (IV)

- What's the difference between the *pragmatic* rationality of doxastic states (including beliefs, degrees of belief, *etc.*) and *epistemic* rationality thereof?
- Example: there is psychological evidence that (actual) agents S tend to perform better at certain activities ϕ if they *believe* that (p_S^ϕ) S is very good at ϕ-ing. This can remain true even when the belief is *unjustified*.
- A case could be made that it would (in some cases) be *pragmatically* rational for (some) S to believe that p_S^ϕ, even when such a belief is not supported by S's evidence. But, this seems *epistemically ir*rational.
- Simpler example: I offer you \$1M to believe that the number of pebbles on Pebble Beach is *exactly* 10^{12}. You have no evidence for this claim, and otherwise no reason to believe it. But, you *really* value money, *etc., etc.*
- We'll bracket questions about whether beliefs (d.o.b.'s) are the *kinds* of things you *can choose* to have. We'll think of this in *evaluative* rather than *normative* terms, and (for now) as about *states* rather than *processes*.

An Evaluative Doxastic Framework (V)

- When we make judgments about rationality, we can take two "stances":
  - **Evaluative Stance**: Here, we're merely *evaluating* some state or process against some standard(s) of ideal rationality. We're not making any claims about what anyone *ought to do* (not *advising*, *blaming*, *etc*).
  - **Normative Stance**: Here, we are talking about what some agent(s) *ought to do*, from the point of view of some standard(s) of ideal (*normative*!) rationality. Here, we *do advise*, *prescribe*, *blame*, *etc*.
- And, we can be making judgments about *states* or *processes*:
  - **State Judgments**: These are judgments about the rationality ("goodness") of (some aspect of) the *doxastic state* of S at t.
  - **Process Judgments**: Judgments of the rationality ("goodness") of some *process* leading S from one doxastic state (at t) to another (at $t' > t$).
- We will be involved mainly with in *evaluation* of doxastic *states*. [We'll say a little about doxastic processes, but also from an *evaluative stance*.]

An Evaluative Doxastic Framework (VI)

- Examples of some evaluative doxastic claims/principles ("norms"):
  1. Logically *consistent* belief states are better than inconsistent states.
  2. If a belief state includes $\ulcorner p \urcorner$ and $\ulcorner p \rightarrow q \urcorner$, then it would be better if it contained $\ulcorner q \urcorner$ and did not contain $\ulcorner \sim q \urcorner$ (than it would otherwise be).
  3. Degree-of-belief states that can be accurately represented as *probability models*, are better than those which cannot be.
  4. *If* an ideally rational agent S satisfies the following two conditions:
     (*i*) S's doxastic state at t can be represented as a Pr-model $\langle \mathcal{B}_S^t, \Pr_S^t \rangle$,
     (*ii*) Between t and t', S learns q *and nothing else* (where q is in $\mathcal{B}_S^t$),
     then, (*iii*) the *ideal doxastic state* for S at t' is $\langle \mathcal{B}_S^{t'}, \Pr_S^{t'} \rangle$, where $\Pr_S^{t'}(\bullet) = \Pr_S^t(\bullet \mid q)$. [d.o.b.–updating goes by *conditionalization*.]
- (1) and (2) are norms for *belief* states. (3) and (4) are norms for *degree-of-belief* states (or *sequences* of them). We ll focus mainly on (3).
- Next: our first argument for *probabilism* — The Dutch Book Argument.

The Dutch Book Argument for (Pragmatic) Probabilism I

- The key assumptions/set-up of the Dutch Book argument are as follows:
  - For each proposition $p \in \mathcal{B}^t$ in our agent's (Mr. B's) doxastic state at t, Mr. B must announce a number $q(p)$ – called his *betting quotient* on p, at t – and *then* Ms. A (the bookie) will choose the *stake* $\mathfrak{s}$ of the bet.
  - $|\mathfrak{s}|$ should be small in relation to Mr. B's total wealth (more on this later). But, it can be positive or negative (so, she can "switch sides").

$$\text{Mr. B's payoff (in \$) for a bet about } p = \begin{cases} \mathfrak{s} - q(p) \cdot \mathfrak{s} & \text{if } p \text{ is true.} \\ -q(p) \cdot \mathfrak{s} & \text{if } p \text{ is false.} \end{cases}$$

  - NOTE: If $\mathfrak{s} > 0$, then the bet is *on* p, if $\mathfrak{s} < 0$, then the bet is *against* p.
  - $q(p)$ is taken to be a measure of Mr. B's *degree of belief* in p (at t).
  - If there is a sequence of multiple bets on multiple propositions, then Mr. B's total payoff is the *sum* of the payoffs for each bet on each proposition. This is called "the package principle". More on *it* later!

The Dutch Book Argument for (Pragmatic) Probabilism II

- Mr. B's "degree of belief function" $q(\cdot)$ is *coherent* iff it is impossible for Ms. A to choose stakes $\mathfrak{s}$ such that she wins *no matter what happens*, *i.e.*, $q(\cdot)$ is *coherent* iff Ms. A cannot construct a "Dutch Book" against Mr. B.

  Theorem (DBT). $q(\cdot)$ is *coherent* **only if** $q(\cdot)$ is a *probability function*.

- Note: the "if" part is also a theorem. That is the *converse* DBT. We won't prove it, but I'll discuss it. Advocates of the DBA think DBT justifies this:

  Claim. The doxastic state $\langle \mathcal{B}_S^t, q(\cdot)\rangle$ of an agent S who is faced with such a scenario is (pragmatically) rational ***only if*** $q(\cdot)$ is coherent (hence a probability function). Does this imply P-probabilism? More, below.

- Does Theorem justify Claim? There are worries about the relationship between gambling and P-rationality, and the "package principle" (and various other assumptions) will also be called into question.

- We'll discuss these worries after we look at the Theorem itself.

The Dutch Book Argument for (Pragmatic) Probabilism III

- The DBT has four parts: one for each of the three axioms, and one for the definition of conditional probability. In each case, we prove that if $q(\cdot)$ [or $q(\cdot \mid \cdot)$] *violates* the axioms (or defn.), then $q(\cdot)$ is *in*coherent.
- **If Mr. B violates Axiom 2, then his q is incoherent**. Proof:
  - If Mr. B assigns $q(\top) = a < 1$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
  - Similarly, if Mr. B assigns $q(\top) = a > 1$, then Ms. A sets $\mathfrak{s} > 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
    - NOTE: if $q(\top) = 1$, then Mr. B's payoff is always $\mathfrak{s} - \mathfrak{s} = 0$, which avoids *this* Book. I'll discuss the *converse* DBT further, below.
- **If Mr. B violates Axiom 1, then his q is incoherent**. Proof:
  - If $q(p) = a < 0$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is $\mathfrak{s} - a\mathfrak{s} < 0$ if p, and $-a\mathfrak{s} < 0$ if $\sim p$. [If $q(p) \geq 0$, then Mr. B's payoff is $\mathfrak{s} - q\mathfrak{s} \geq 0$ if $\mathfrak{s} > 0$ and p is true, and $-q\mathfrak{s} \geq 0$ if $\mathfrak{s} < 0$ and $\sim p$, avoiding *this* Book.]

The Dutch Book Argument for (Pragmatic) Probabilism IV

- Recall, our Axiom 3 requires that

$$\Pr(p \vee r) = \Pr(p) + \Pr(r)$$

  if p and r cannot both be true (*i.e.*, if they are mutually exclusive).

- The argument for this *additivity* axiom is more controversial. The main source of controversy is the "package principle". We'll just *assume* it for the proof of the *Theorem*. But, for the *Claim*, we will re-think it.
- I will now go through the additivity case of the *Theorem*, and then I will discuss Maher's (and Schick's) objection to the "package principle".
- **Setup**: Let p and r be some pair of mutually exclusive propositions in the agent's doxastic state at t. And, suppose Mr. B announces these b's:

$$q(p) = a,\ q(r) = b, \text{ and } q(p \vee r) = c, \text{ where } c \neq a + b.$$

- This will leave Mr. B susceptible to a Dutch Book. Here's why …

The Dutch Book Argument for (Pragmatic) Probabilism V

- **Case 1**: $c < a + b$. Ms. A asks Mr. B to make *all 3* of these bets ($\mathfrak{s} = +\$1$):

  1. Bet $\$a$ on p to win $\$(1 - a)$ if p, and to lose $\$a$ if $\sim p$.
  2. Bet $\$b$ on r to win $\$(1 - b)$ if r, and to lose $\$b$ if $\sim r$.
  3. Bet $\$(1 - c)$ *against* $p \vee r$ to win $\$c$ if $\sim(p \vee r)$, and lose $\$(1 - c)$ o.w.

- Since p and r are mutually exclusive (by assumption of the additivity axiom), the conjunction $p \,\&\, r$ cannot be true. ∴ There are 3 cases:

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, \sim r$	$1 - a$	$-b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, r$	$-a$	$1 - b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, \sim r$	$-a$	$-b$	c	$c - (a + b)$

- Since $c < a + b$, $c - (a + b)$ is negative. So, Mr. B loses $\$[c - (a + b)]$.
- **Case 2**: $c > a + b$. Ms. A simply reverses the bets ($\mathfrak{s} = -\$1$), and a parallel argument shows that the total payoff for Mr. B is $\$-[c - (a + b)] < 0$.
- Note: he can avoid *this* Book, by setting $c = a + b$. More on CDBT, below.

The Dutch Book Argument for (Pragmatic) Probabilism VI

- We also need to show that an agent's conditional betting quotients are coherent only if they satisfy our ratio definition of conditional probability. There's a DBT for this too (and it also assumes the "package principle").
- Suppose Mr. B announces: $q(p \,\&\, r) = b$, $q(r) = c > 0$, and $q(p \mid r) = a$. Ms. A asks Mr. B to make *all 3* of these bets (stakes depend on quotients!):

1. Bet $\$(b \cdot c)$ on $p \,\&\, r$ to win $\$[(1-b) \cdot c]$ if $p \,\&\, r$, and lose $\$(b \cdot c)$ o.w. $[\mathfrak{s} = c]$
2. Bet $\$[(1-c) \cdot b]$ against r to win $\$(b \cdot c)$ if r, and lose $\$[(1-b) \cdot c]$ o.w. $[\mathfrak{s} = b]$
3. Bet $\$[(1-a) \cdot c]$ against p, conditional on r, to win $\$(a \cdot c)$ if $r \,\&\, p$, and lose $\$[(1-a) \cdot c]$ if $r \,\&\, {\sim}p$. If ${\sim}r$, then the bet is *called off*, and payoff is \$0. $[\mathfrak{s} = c]$

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, r$	$(1-b) \cdot c$	$-[(1-c) \cdot b]$	$-[(1-a) \cdot c]$	$(a \cdot c) - b$
${\sim}p \,\&\, r$	$-(b \cdot c)$	$-[(1-c) \cdot b]$	$a \cdot c$	$(a \cdot c) - b$
${\sim}r$	$-(b \cdot c)$	$b \cdot c$	0	0

- If $a < \frac{b}{c}$, then Mr. B loses *come what may*. If $a > \frac{b}{c}$, then Ms. A just asks Mr. B to take the other side on all three bets. So, coherence requires: $q(p \mid r) = \frac{q(p\&r)}{q(r)}$.

The Dutch Book Argument for (Pragmatic) Probabilism VII

- Here are some problems with/limitations of the Dutch Book *Argument*:
  - It assumes that bets (or gambles) which are severally acceptable are jointly acceptable. This "package principle" is not obvious (see below).
  - It is couched in terms of *money*. It tacitly assumes that *utility* is *linear* in *money*. But, money seems to have *diminishing marginal utility*.
    * More generally, DBAs involve *betting behavior*. One might wonder whether one's betting behavior (when *forced* to post odds) is representative of *rational* behavior *generally* (gambling aversion?).
  - The DBA requires the *converse* DBT to be persuasive. And, *even with* the CDBT, it's still not clear whether pragmatic probabilism follows from the DBA. Does it show that Pr is *better than* non-Pr?
  - It does not address *process* requirements, only state requirements, *i.e.*, it does not constrain *transitions* from one doxastic state to another.
  - It presupposes P-rational agents are *logically omniscient* – that they can *recognize all tautologies* (in $\mathcal{B}^t$). Does P-rationality require this?

Postscripts to DBA I: The Package Principle

- The Dutch Book argument for Additivity may appear as airtight as the Dutch Book arguments for Normality and Non-Negativity. But, Schick (and others) have spotted a possible flaw [see the Schick paper on website].
- In our Dutch Book argument for Additivity, we (implicitly) assumed:
  - Bets that are severally acceptable are jointly acceptable.
  - The value of a set of bets is the sum of values of its elements.
- Might a rational agent be willing to accept *each* of the bets (1)–(3) without being willing to accept *all three at once*? And, if not, why not?
- Might they not value (1) at $\$a$, (2) at $\$b$, and (3) at $\$-c$, *without* valuing the *collection* at $\$[a+b-c]$? After all, they may *see* that, *taken jointly*, bets (1)–(3) lead to a sure loss, whereas *no individual bet does*.
- It is important to note that the DBA for the ratio definition of conditional probability also presupposes this sort of "package principle".
- Are there counterexamples to the "package principle"? Maher thinks so:

Suppose that after a night on the town, you want to catch the bus home. Alas, you find that you only have 60 cents in your pocket, but the bust costs \$1. A bookie, learning of your plight, offers you the following deal: If you give him your 60 cents, he will toss a coin, and if the coin lands heads then he will give you \$1; otherwise, you have lost your 60 cents. If you accept the bookie's proposal, then you stand a 50-50 chance of being able to take the bus home, while rejecting it means you will certainly have to walk. Under these conditions, you may well feel that the bookie's offer is acceptable; let us suppose you do. Presumably, the offer would have been equally well acceptable if you were betting on tails rather than heads...

- As betting quotients were defined for the simple DBA, your quotient for heads in the above example is 0.6, and so is your quotient for tails; thus your betting quotients violate the probability calculus in this example.
- By additivity, $\Pr(\text{Heads} \vee \text{Tails}) = \Pr(\text{Heads}) + \Pr(\text{Tails})$, since Heads and Tails are mutually exclusive. But, $q(\text{Heads} \vee \text{Tails}) = 1.2 > 1$, which violates the axioms (recall, we have a theorem saying $\Pr(p) \leq 1$, for all p).
- Assumption: if you find a bet on Heads acceptable, and find a bet on Tails acceptable, then you should also find both bets acceptable, *taken jointly*.