Philosophy 148 — Day 1: INTRODUCTION & ADMINISTRATION

- Administrative Stuff (i.e., Syllabus)
  - Me & Raul (intros., personal data, office hours, etc.)
  - Prerequisites (Boolean logic, some simple algebra, no math phobia!)
  - Texts & Supplementary Readings (all online via website)
  - Requirements [Quiz (10), Assignments (30), Mid-Term (30), Final (30)]
  - Sections (determined this week, via index cards — meet next week)
    ∗ Index Cards: Name, email, section time ranking. The 8 possible times are: Tu or Th: 9–10, 10–11, 1–2, or 2–3. Give a ranking of those among the 8 that you can do. Indicate those you cannot do.
  - Website (main source of course information — stay tuned!)
  - Tentative Schedule (somewhat loose, time-wise, but all readings set)

- Next: Brief Overview/Outline of the Course
Philosophy 148 — Day 1: Fundamental Underlying Questions

• I am writing a book on inductive logic (a.k.a., confirmation theory).
• My main focus is on “quantitative generalizations” of deductive logic.
• The notion of validity is the deductive ideal for “logical goodness”.
• But, some invalid arguments seem “better”/“stronger” than others:

  \[ P_1. \] Someone is wise. \[ P_2. \] Someone is either wise or unwise.
  \[ \therefore C_1. \] Plato is wise. \[ \therefore C_2. \] Socrates is wise.

• The argument from \( P_1 \) to \( C_1 \) seems “better” than the one from \( P_2 \) to \( C_2 \).
• Is there a satisfying explication of this “better than” concept?
• And, if so, is this best understood a logical concept or an epistemic one or a pragmatic one, etc.? Moreover, if there is a logical “better than”, how is it related to epistemology? For that matter, how is validity related to epistemology? These are the sorts of questions in the air.
Philosophy 148 — Day 1: Course Overview/Outline

• The precise timing of the course is not fixed. But all readings are up.

• The *order* of topics in the course is also (more or less) set:
  - Review of Boolean Logic and Boolean Algebra [12A review + FBAs]
    * Propositional Logic
    * Monadic Predicate Logic
    * Finite Boolean Algebras [general logical framework for course]
  - Introduction of the (formal) Probability Calculus
    * Axiomatic Treatments
    * Algebraic Treatments
  - “Personalistic” Interpretations/Kinds of Probability
    * Pragmatic: betting odds / betting quotients / *rational* dob’s
    * Epistemic: degrees of *credence* / *justified* degrees of belief
- Confirmation Theory and Inductive Logic
  - Deductive Approaches to Confirmation
    - Hempelian
    - Hypothetico-Deductive
  - Probabilistic Approaches to Confirmation
    - Logical (Carnapian)
    - Subjective/Personalistic ("Bayesian")
- The Paradoxes of Confirmation
  - The Raven Paradox
  - The Grue Paradox
- Other Problems for Confirmation Theory (mainly, for "Bayesian" CT)
  - Old Evidence/Logical Omniscience/maybe others
- Three Psychological Puzzles Involving Probability & Confirmation
  - The Base Rate Fallacy
  - The Conjunction Fallacy
  - The Wason Selection Task
Syntax of Sentential Logic (SL)

- The syntax of SL is simple. Its lexicon contains the following symbols:
  - Upper-case letters ‘A’, ‘B’, … which stand for *basic sentences*.
  - Five *sentential connectives* (or *sentential operators*):

<table>
<thead>
<tr>
<th>Operator</th>
<th>Name</th>
<th>Logical Function</th>
<th>Used to translate</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘∼’</td>
<td>tilde</td>
<td>negation</td>
<td>not, it is not the case that</td>
</tr>
<tr>
<td>‘&amp;’</td>
<td>ampersand</td>
<td>conjunction</td>
<td>and, also, moreover, but</td>
</tr>
<tr>
<td>‘∨’</td>
<td>vee</td>
<td>disjunction</td>
<td>or, either … or …</td>
</tr>
<tr>
<td>‘→’ (⊃)</td>
<td>arrow</td>
<td>conditional</td>
<td>if … then …, only if</td>
</tr>
<tr>
<td>‘↔’ (≡)</td>
<td>double arrow</td>
<td>biconditional</td>
<td>if and only if</td>
</tr>
</tbody>
</table>

- Parentheses ‘(’, ‘)’, brackets ‘[’, ‘]’, and braces ‘{’, ‘}’ for grouping.

- If a string of symbols contains anything other than these, it is not an SL sentence. And, only certain strings of these symbols are SL sentences.

- I assume you all know which SL strings are *sentences* and which are not…
Semantics of Sentential Logic: Truth Tables I

- Sentential Logic is *truth-functional* because the truth value of a compound $S$ is a function of the truth values of $S$’s *atomic parts*.

- All statement forms $p$ are defined by *truth tables*, which tell us how to determine the truth value of $p$’s from the truth values of $p$’s parts.

- Truth-tables provide a precise way of thinking about *logical possibility*. Each row of a truth-table can be thought of as a *logical possibility*. And, the actual world falls into *exactly one* of these rows/logical possibilities.

- In this sense, truth-tables provide a way to “see” logical space.

- Once we have an understanding of all the logically possible truth-values that and SL sentence can have (which truth-tables provide for us), testing the validity of SL arguments is easy — *inspection* of truth-tables!

- We just look for possible worlds (rows of the salient truth-table) in which all the premises are true and the conclusion is false.
Semantics of Sentential Logic: Truth Tables II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use T and F for true and false):

<table>
<thead>
<tr>
<th>p</th>
<th>∼p</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

- In words, this says that if \( p \) is true than \( ∼p \) is false, and if \( p \) is false, then \( ∼p \) is true. This is quite intuitive, and corresponds well to the English meaning of ‘not’. So, SL negation is like English negation.

- Examples:
  - It is not the case that Wagner wrote operas. (\( ∼W \))
  - It is not the case that Picasso wrote operas. (\( ∼P \))

- ‘\( ∼W \)’ is false, since ‘\( W \)’ is true, and ‘\( ∼P \)’ is true, since ‘\( P \)’ is false (like English).
**Chapter 3 — Semantics of SL: Truth Tables III**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p &amp; q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

- Notice how we have four (4) rows in our truth table this time (not 2). There are four possible ways of assigning truth values to $p$ and $q$.

- The truth-functional definition of $\&$ is very close to the English ‘and’. A SL conjunction is true if *both* conjuncts are true; it’s false otherwise.
  - Monet and van Gogh were painters. ($M \& V$)
  - Monet and Beethoven were painters. ($M \& B$)
  - Beethoven and Einstein were painters. ($B \& E$)

- ‘$M \& V$’ is true, since both ‘$M$’ and ‘$V$’ are true. ‘$M \& B$’ is false, since ‘$B$’ is false. And, ‘$B \& E$’ is false, since ‘$B$’ and ‘$E$’ are both false (like English).
Semantics of Sentential Logic: Truth Tables IV

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p ∨ q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

- The truth-functional definition of ∨ is not as close to the English ‘or’. A SL disjunction is true if *at least one* disjunct is true; it’s false otherwise.

- In English, ‘A or B’ often implies that ‘A’ and ‘B’ are *not both true*. That is called *exclusive* or. In SL, ‘A ∨ B’ is *not* exclusive; it is *inclusive* (it is true if both disjuncts are true). We can express exclusive or in SL. How?
  - Either Jane austen or René Descartes was novelist. (J ∨ R)
  - Either Jane Austen or Charlotte Bronte was a novelist. (J ∨ C)
  - Either René Descartes or David Hume was a novelist. (R ∨ D)

- The first two disjunctions are true since at least one their disjuncts is true. The third disjunction is false, since both of its disjuncts are false.
Semantics of Sentential Logic: Truth Tables V

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
<td>$p \rightarrow q$</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

- The SL conditional ($\rightarrow$) is farther from the English ‘only if’. An SL conditional is false iff its antecedent is true and its consequent is false.

- Consider the following English conditionals. [$M =$ the moon is made of green cheese, $O =$ life exists on other planets, and $E =$ life exists on Earth]
  - If the moon is made of green cheese, then life exists on other planets.
  - If life exists on other planets, then life exists on earth.

- The SL translations of these sentences are both true.
  - ‘$M \rightarrow O$’ is true because its antecedent ‘$M$’ is false.
  - ‘$O \rightarrow E$’ is true because its consequent ‘$E$’ is true.

- This does not capture the English ‘if’. Remember: $p \rightarrow q \equiv \neg p \lor q$. 

UCB Philosophy

Introduction & Administration

01/22/08
Semantics of Sentential Logic: Truth Tables VI

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \leftrightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

- The SL biconditional $\leftrightarrow$ inherits similar problems. An SL biconditional is true iff both of its components have the same truth value.

- Consider these two biconditionals. [$M = \text{the moon is made of green cheese, } U = \text{there are unicorns, } E = \text{life exists on Earth, and } S = \text{the sky is blue}$]
  - The moon is made of green cheese if and only if there are unicorns.
  - Life exists on earth if and only if the sky is blue.

- The SL translations of these sentences are both true.
  - $M \leftrightarrow U$ is true because $M$ and $U$ are false.
  - $E \leftrightarrow S$ is true because $E$ and $S$ are true.

- This does not capture the English ‘iff’. [$p \leftrightarrow q \models (p \land q) \lor (\neg p \land \neg q)$]
With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound SL statements.

A non-trivial example:

\[
\begin{array}{ccc|ccccccc}
 p & q & r & (p & (q \lor r)) & \rightarrow & ((p & q) \lor (p & r)) \\
 T & T & T & T & T & T & T & T \\
 T & T & F & T & T & T & T & F \\
 T & F & T & T & T & T & F & T \\
 T & F & F & T & F & T & F & F \\
 F & T & T & F & T & F & F & F \\
 F & T & F & F & T & F & F & F \\
 F & F & T & F & T & F & F & F \\
 F & F & F & F & T & F & F & F \\
\end{array}
\]

Thus, “\((p & (q \lor r)) \rightarrow ((p & q) \lor (p & r))\)” is a tautology.
Logical Truth, Logical Falsity, and Contingency: Definitions

- A statement is **logically true** (or tautologous) if it is true regardless of the truth-values of its components. Example: $p \leftrightarrow p$ is a tautology.

  \[
  \begin{array}{c|c|c|c|c}
  p & p & \leftrightarrow & p \\
  \hline
  T & T & T & T \\
  F & F & T & T \\
  \end{array}
  \]

- A statement is **logically false** (or self-contradictory) if it is false regardless of the truth-values of its components. Example: $p \& \neg p$.

  \[
  \begin{array}{c|c|c|c|c}
  p & p & \& & \neg & p \\
  \hline
  T & T & F & F & T \\
  F & F & F & T & F \\
  \end{array}
  \]

- A statement is **contingent** if its truth-value varies depending on the truth-values of its components. Example: $A$ (or any atom) is contingent.

  \[
  \begin{array}{c|c|c}
  A & A \\
  \hline
  T & T \\
  F & F \\
  \end{array}
  \]
Interpretations and Logical Equivalence

• An interpretation of an SL formula \( p \) is an assignment of truth-values to all of the sentence letters in \( p \).

• Each row of the truth-table of \( p \) is an interpretation of \( p \). Sometimes, I will also refer to rows of SL truth-tables as (logically) possible situations, or possible worlds.

• A tautology (contradiction) is an SL sentence whose truth value is \( T \) (\( F \)) on all of its interpretations (\( i.e. \), an SL sentence which is true (false) in all (logically) possible worlds).

• Two SL sentences are said to be logically equivalent iff they have the same truth-value on all (joint) interpretations.

• I’ll abbreviate “\( p \) and \( q \) are logically equivalent” as “\( p \equiv q \)” \( [i.e., \ p \ follows \ from \ q \ (q \vdash p), \ and \ q \ follows \ from \ p \ (p \vdash q)]. \)
Equivalence, Contradictoriness, Consistency, and Inconsistency

- Two statements are said to be equivalent (written $p \equiv q$) if they have the same truth-value in all possible worlds (i.e., in all rows of a simultaneous truth-table of both statements). For instance, $A \rightarrow B \equiv \sim A \lor B$:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$A \rightarrow B$</th>
<th>$\sim A \lor B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

- Two statements are contradictory if they have opposite truth-values in all possible worlds (i.e., in all rows of a simultaneous truth-table of both statements). For instance, $A$ and $\sim A$:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\sim A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>T</td>
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<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
• Two statements are **inconsistent (mutually exclusive)** if they cannot both be true (i.e., no row of their simultaneous truth-table has them both being T). *E.g.*, \( A \leftrightarrow B \) and \( A \& \sim B \) are inconsistent (but not contradictory!):

\[
\begin{array}{ccc|c|c|cc}
A & B & A \leftrightarrow B & A \& \sim B \\
T & T & T & T & T & T \\
T & F & F & F & T & F \\
F & T & T & F & F & T \\
F & F & F & F & T & F \\
\end{array}
\]

• Two statements are **consistent** if they are both true in at least one possible world (i.e., in at least one row of a simultaneous truth-table of both statements). For instance, \( A \& B \) and \( A \lor B \) are consistent:

\[
\begin{array}{ccc|cc|ccc}
A & B & A \& B & A \lor B \\
T & T & T & T & T & T & T \\
T & F & F & F & T & T & F \\
F & T & F & F & T & T & T \\
F & F & F & F & F & F & F \\
\end{array}
\]
Logical Equivalence: Example #1

- I said that $p \rightarrow q$ is logically equivalent to $\neg p \lor q$.
- The following truth-table establishes this equivalence:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$\lor$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
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<td>T</td>
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<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

- The truth-tables of $\neg p \lor q$ and $p \rightarrow q$ are the same.
Logical Equivalence: Example #2

- \( p \leftrightarrow q \) is an abbreviation for \((p \rightarrow q) \& (q \rightarrow p)\).

- The following truth-table shows it is a legitimate abbreviation:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( (p \rightarrow q) )</th>
<th>&amp;</th>
<th>( (q \rightarrow p) )</th>
<th>( p \leftrightarrow q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

- \( p \leftrightarrow q \) and \((p \rightarrow q) \& (q \rightarrow p)\) have the same truth-table.
Some More Logical Equivalences

- Here is a simultaneous truth-table which establishes that

\[ A \leftrightarrow B \equiv (A \land B) \lor (\neg A \land \neg B) \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A ↔ B</th>
<th>(A &amp; B) \lor (\neg A &amp; \neg B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

- Can you prove the following equivalences with truth-tables?

- \( \neg (A \land B) \equiv \neg A \lor \neg B \)
- \( \neg (A \lor B) \equiv \neg A \land \neg B \)
- \( A \equiv (A \land B) \lor (A \land \neg B) \)
- \( A \equiv A \land (B \to B) \)
- \( A \equiv A \lor (B \land \neg B) \)
**Logical Equivalence, Contradictoriness, etc.: Relationships**

- What are the relationships between “p and q are equivalent”, “p and q are consistent”, “p and q are contradictory”, “p and q are inconsistent”?

<table>
<thead>
<tr>
<th>Equivalent</th>
<th>Contradictory</th>
</tr>
</thead>
<tbody>
<tr>
<td>▼</td>
<td>▼</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>↑</td>
<td>↑</td>
</tr>
</tbody>
</table>

- Answers:
  1. Equivalent $\not\equiv$ Consistent ($p \land \lnot p$ and $q \land \lnot q$)
  2. Consistent $\not\equiv$ Equivalent ($p \to q$ and $p \land q$)
  3. Contradictory $\Rightarrow$ Inconsistent (why?)
  4. Inconsistent $\not\Rightarrow$ Contradictory (example?)
Truth-Tables and Deductive Validity I

- Remember, an argument is valid if it is impossible for its premises to be true while its conclusion is false. Let \( p_1, \ldots, p_n \) be the premises of a SL argument, and let \( q \) be the conclusion of the argument. Then, we have:

\[
\begin{align*}
\quad & p_1 \\
\vdots \\
\quad & p_n \\
\therefore & \quad q
\end{align*}
\]

is valid if and only if there is no row in the simultaneous truth-table (interpretation) of \( p_1, \ldots, p_n \), and \( q \) which looks like:

<table>
<thead>
<tr>
<th>atoms</th>
<th>premises</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>\ldots</td>
<td>( p_1 )</td>
<td>\ldots</td>
</tr>
<tr>
<td>\ldots</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
Truth-Tables and Deductive Validity II

<table>
<thead>
<tr>
<th>atoms</th>
<th>premises</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$B$</td>
<td>$A \rightarrow B$</td>
</tr>
<tr>
<td>$A \rightarrow B$</td>
<td>is valid:</td>
<td>$A$ \rightarrow $B$</td>
</tr>
<tr>
<td>$\therefore B$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>atoms</th>
<th>premises</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$A \rightarrow B$</td>
<td>$A$</td>
</tr>
<tr>
<td>$A \rightarrow B$</td>
<td>is invalid:</td>
<td>$A \rightarrow B$</td>
</tr>
<tr>
<td>$\therefore A$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Finite Propositional Boolean Algebras I

- A *finite propositional Boolean algebra* is a finite set of *propositions* which is *closed* under the logical operations and satisfies the laws of SL.

- *Propositions* are the things expressed by sentences (abstract entities, distinct from sentences). If two sentences are logically equivalent, then they express the same proposition. *E.g.*, “$A \rightarrow B$” and “$\sim A \lor B$”.

- A set $S$ is *closed* under logical operations if applying a logical operation to a member (or pair of members) of $S$ always yields a member of $S$.

- Example: consider a sentential language with three atomic letters “$X$”, “$Y$”, and “$Z$”. The set of propositions expressible using the logical connectives and these three atomic letters forms a finite Boolean algebra.

- This Boolean algebra has $2^3 = 8$ *atomic propositions* or *states* (*i.e.*, rows of a 3-sentence truth-table!). Question: How many propositions does it contain *in total*? Answer: $2^8 = 256$ (255 plus the contradiction). *Why?*
Finite Propositional Boolean Algebras II

- A literal is either an atomic sentence or the negation of an atomic sentence (e.g., “A” and “¬A” are literals involving the atom “A”).

- A state of a Boolean algebra $B$ is a proposition expressed by a maximal conjunction of literals in a language $L_B$ describing $B$ (“maximal”: “containing exactly one literal for each atomic sentence in $B$”).

- Consider an algebra $B$ described by a 3-atom language $L_B$ (“X”, “Y”, “Z”). The states of $B$ are described by the $2^3 = 8$ state descriptions of $L_B$:

  $(s_1)\ X \& Y \& Z$
  $(s_2)\ X \& Y \& \sim Z$
  $(s_3)\ X \& \sim Y \& Z$
  $(s_4)\ X \& \sim Y \& \sim Z$
  $(s_5)\ \sim X \& Y \& Z$
  $(s_6)\ \sim X \& Y \& \sim Z$
  $(s_7)\ \sim X \& \sim Y \& Z$
  $(s_8)\ \sim X \& \sim Y \& \sim Z$
• We can “visualize” the states of $\mathcal{B}$ using a truth table or a Venn Diagram.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>$s_1$</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>$s_2$</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>$s_3$</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>$s_4$</td>
</tr>
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<td>F</td>
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<td>$s_5$</td>
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<td>F</td>
<td>$s_6$</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td>$s_7$</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>$s_8$</td>
</tr>
</tbody>
</table>

• Everything that can be expressed in the sentential language $\mathcal{L}_B$ can be expressed as a *disjunction of state descriptions* (think about why).

• Thus, every proposition expressible in $\mathcal{L}_B$ can be “visualized” simply by shading combinations of the 8 state-regions of the Venn Diagram of $\mathcal{B}$. It because of this that we can use Venn Diagrams to establish Boolean Laws.

• $p \vdash q$ (in $\mathcal{B}$) iff every shaded region in the Venn Diagram representation of $p$ (in $\mathcal{B}$) is also shaded in the Venn Diagram representation of $q$ (in $\mathcal{B}$).