Philosophy 148 — Day 1: Introduction & Administration

- Administrative Stuff (*i.e.*, Syllabus)
  - Me & Raul (intros., personal data, office hours, etc.)
  - Prerequisites (Boolean logic, some simple algebra, no math phobia!)
  - Texts & Supplementary Readings (all online *via* website)
  - Requirements [Quiz (10), Assignments (30), Mid-Term (30), Final (30)]
  - Sections (determined this week, *via* index cards — meet next week)
    - Index Cards: Name, email, section time ranking. The 8 possible times are: Tu or Th: 9–10, 10–11, 1–2, or 2–3. Give a *ranking* of those among the 8 that you *can* do. Indicate those you *cannot* do.
  - Website (main source of course information — stay tuned!)
  - Tentative Schedule (somewhat loose, time-wise, but all readings set)
- Next: Brief Overview/Outline of the Course

Philosophy 148 — Day 1: Fundamental Underlying Questions

- I am writing a book on inductive logic (*a.k.a.*, confirmation theory).
- My main focus is on "quantitative generalizations" of deductive logic.
- The notion of *validity* is the deductive ideal for "logical goodness".
- But, some invalid arguments seem "better"/"stronger" than others:

  P_1. Someone is wise. P_2. Someone is either wise or unwise.

  $\therefore C_1$. Plato is wise. $\therefore C_2$. Socrates is wise.

- The argument from P_1 to C_1 seems "better" than the one from P_2 to C_2.
- Is there a satisfying *explication* of this "better than" concept?
- And, if so, is this best understood a *logical* concept or an *epistemic* one or a *pragmatic* one, *etc.*? Moreover, if there is a *logical* "better than", how is it related to *epistemology*? For that matter, how is *validity* related to epistemology? These are the sorts of questions in the air.

Philosophy 148 — Day 1: Course Overview/Outline

- The precise timing of the course is not fixed. But all readings are up.
- The *order* of topics in the course is also (more or less) set:
  - Review of Boolean Logic and Boolean Algebra [12A review + FBAs]
    - Propositional Logic
    - Monadic Predicate Logic
    - Finite Boolean Algebras [general logical framework for course]
  - Introduction of the (formal) Probability Calculus
    - Axiomatic Treatments
    - Algebraic Treatments
  - "Personalistic" Interpretations/Kinds of Probability
    - Pragmatic: betting odds / betting quotients / *rational* dob's
    - Epistemic: degrees of *credence* / *justified* degrees of belief

- Confirmation Theory and Inductive Logic
  * Deductive Approaches to Confirmation
    - Hempelian
    - Hypothetico-Deductive
  * Probabilistic Approaches to Confirmation
    - Logical (Carnapian)
    - Subjective/Personalistic ("Bayesian")
- The Paradoxes of Confirmation
  * The Raven Paradox
  * The Grue Paradox
- Other Problems for Confirmation Theory (mainly, for "Bayesian" CT)
  * Old Evidence/Logical Omniscience/maybe others
- Three *Psychological* Puzzles Involving Probability & Confirmation
  * The Base Rate Fallacy
  * The Conjunction Fallacy
  * The Wason Selection Task

Syntax of Sentential Logic (SL)

- The syntax of SL is simple. Its lexicon contains the following symbols:
  - Upper-case letters ‘A’, ‘B’, … which stand for *basic sentences*.
  - Five *sentential connectives* (or *sentential operators*):

Operator	**Name**	**Logical Function**	**Used to translate**
‘$\sim$’	tilde	negation	not, it is not the case that
‘&’	ampersand	conjunction	and, also, moreover, but
‘$\vee$’	vee	disjunction	or, either … or …
‘$\rightarrow$’ (‘$\supset$’)	arrow	conditional	if … then …, only if
‘$\leftrightarrow$’ (‘$\equiv$’)	double arrow	biconditional	if and only if

  - Parentheses ‘(’, ‘)’, brackets ‘[’. ‘]’, and braces ‘{’, ‘}’ for grouping.
- If a string of symbols contains anything other than these, it is not an SL sentence. And, only certain strings of these symbols are SL sentences.
- I assume you all know which SL strings are *sentences* and which are not…

Semantics of Sentential Logic: Truth Tables I

- Sentential Logic is *truth-functional* because the truth value of a compound S is a function of the truth values of S's *atomic parts*.
- All statement forms p are defined by *truth tables*, which tell us how to determine the truth value of p's from the truth values of p's parts.
- Truth-tables provide a precise way of thinking about *logical possibility*. Each row of a truth-table can be thought of as a *logical possibility*. And, the actual world falls into *exactly one* of these rows/logical possibilities.
- In this sense, truth-tables provide a way to "see" logical space.
- Once we have an understanding of all the logically possible truth-values that and SL sentence can have (which truth-tables provide for us), testing the validity of SL arguments is easy — *inspection* of truth-tables!
- We just look for possible worlds (rows of the salient truth-table) in which all the premises are true and the conclusion is false.

Semantics of Sentential Logic: Truth Tables II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use T and F for true and false):

p	$\sim p$
T	F
F	T

- In words, this says that if p is true than $\sim p$ is false, and if p is false, then $\sim p$ is true. This is quite intuitive, and corresponds well to the English meaning of 'not'. So, SL negation is like English negation.
- Examples:
  - It is not the case that Wagner wrote operas. ($\sim W$)
  - It is not the case that Picasso wrote operas. ($\sim P$)
- '$\sim W$' is false, since 'W' is true, and '$\sim P$' is true, since 'P' is false (like English).

Chapter 3 — Semantics of SL: Truth Tables III

p	q	$p \& q$
T	T	T
T	F	F
F	T	F
F	F	F

- Notice how we have four (4) rows in our truth table this time (not 2). There are four possible ways of assigning truth values to p and q.
- The truth-functional definition of & is very close to the English 'and'. A SL conjunction is true if *both* conjuncts are true; it's false otherwise.
  - Monet and van Gogh were painters. ($M \& V$)
  - Monet and Beethoven were painters. ($M \& B$)
  - Beethoven and Einstein were painters. ($B \& E$)
- '$M \& V$' is true, since both 'M' and 'V' are true. '$M \& B$' is false, since 'B' is false. And, '$B \& E$' is false, since 'B' and 'E' are both false (like English).

Semantics of Sentential Logic: Truth Tables IV

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The truth-functional definition of $\vee$ is not as close to the English ‘or’. A SL disjunction is true if *at least one* disjunct is true; it’s false otherwise.
- In English, ‘A or B’ often implies that ‘A’ and ‘B’ are *not both true*. That is called *exclusive* or. In SL, ‘$A \vee B$’ is *not* exclusive; it is *inclusive* (it is true if both disjuncts are true). We *can* express exclusive or in SL. How?
  - Either Jane austen or René Descartes was novelist. ($J \vee R$)
  - Either Jane Austen or Charlotte Bronte was a novelist. ($J \vee C$)
  - Either René Descartes or David Hume was a novelist. ($R \vee D$)
- The first two disjunctions are true since at least one their disjuncts is true. The third disjunction is false, since both of its disjuncts are false.

Semantics of Sentential Logic: Truth Tables V

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- The SL conditional ($\rightarrow$) is farther from the English 'only if'. An SL conditional is false iff its antecedent is true and its consequent is false.
- Consider the following English conditionals. [M = the moon is made of green cheese, O = life exists on other planets, and E = life exists on Earth]
  - If the moon is made of green cheese, then life exists on other planets.
  - If life exists on other planets, then life exists on earth.
- The SL translations of these sentences are both true.
  - '$M \rightarrow O$' is true because its antecedent 'M' is false.
  - '$O \rightarrow E$' is true because its consequent 'E' is true.
- This does *not* capture the English 'if'. Remember: $p \rightarrow q \mathrel{=\!\!|\!\!\models} \sim p \vee q$.

Semantics of Sentential Logic: Truth Tables VI

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- The SL biconditional $\leftrightarrow$ inherits similar problems. An SL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [M = the moon is made of green cheese, U = there are unicorns, E = life exists on Earth, and S = the sky is blue]
  - The moon is made of green cheese if and only if there are unicorns.
  - Life exists on earth if and only if the sky is blue.
- The SL translations of these sentences are both true.
  - $M \leftrightarrow U$ is true because M and U are false.
  - $E \leftrightarrow S$ is true because E and S are true.
- This does *not* capture the English 'iff'. [$p \leftrightarrow q \dashv\vDash (p \,\&\, q) \vee ({\sim}p \,\&\, {\sim}q)$]

Semantics of Sentential Logic: Truth Tables VII

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound SL statements.
- A non-trivial example:

p	q	r	$(p$	$\&$	$(q \vee r))$	$\rightarrow$	$((p \& q)$	$\vee$	$(p \& r))$
T	T	T	T	T	T	**T**	T	T	T
T	T	F	T	T	T	**T**	T	T	F
T	F	T	T	T	T	**T**	F	T	T
T	F	F	T	F	F	**T**	F	F	F
F	T	T	F	F	T	**T**	F	F	F
F	T	F	F	F	T	**T**	F	F	F
F	F	T	F	F	T	**T**	F	F	F
F	F	F	F	F	F	**T**	F	F	F

- Thus, "$(p \& (q \vee r)) \rightarrow ((p \& q) \vee (p \& r))$" is a *tautology*.

Logical Truth, Logical Falsity, and Contingency: Definitions

- A statement is logically true (or tautologous) if it is true regardless of the truth-values of its components. Example: $p \leftrightarrow p$ is a tautology.

p	p	$\leftrightarrow$	p
T	T	T	T
F	F	T	F

- A statement is logically false (or self-contradictory) if it is false regardless of the truth-values of its components. Example: $p \,\&\, {\sim}p$.

p	p	$\&$	$\sim$	p
T	T	F	F	T
F	F	F	T	F

- A statement is contingent if its truth-value varies depending on the truth-values of its components. Example: A (or *any* atom) is contingent.

A	A
T	T
F	F

Interpretations and Logical Equivalence

- An *interpretation* of an SL formula p is an assignment of truth-values to all of the sentence letters in p.
- Each row of the truth-table of p is an *interpretation* of p. Sometimes, I will also refer to rows of SL truth-tables as (logically) *possible situations*, or *possible worlds*.
- A tautology (contradiction) is an SL sentence whose truth value is T (F) on *all* of its interpretations (*i.e.*, an SL sentence which is *true (false) in all (logically) possible worlds*).
- Two SL sentences are said to be *logically equivalent* iff they have the same truth-value on all (joint) interpretations.
- I'll abbreviate "p and q are logically equivalent" as "$p \mathrel{=\!|\!\models} q$" [*i.e.*, p follows from q ($q \models p$), and q follows from p ($p \models q$)].

Equivalence, Contradictoriness, Consistency, and Inconsistency

- Two statements are said to be equivalent (written $p \dashv\vDash q$) if they have the same truth-value in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, $A \to B \dashv\vDash {\sim}A \vee B$:

A	B	A	$\to$	B	$\sim$	A	$\vee$	B
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F

- Two statements are contradictory if they have opposite truth-values in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, A and ${\sim}A$:

A	A	$\sim$	A
T	T	F	T
F	F	T	F

- Two statements are inconsistent (mutually exclusive) if they cannot both be true (*i.e.*, no row of their simultaneous truth-table has them both being T). *E.g.*, $A \leftrightarrow B$ and $A \,\&\, \sim B$ are inconsistent (but *not* contradictory!):

A	B	A	$\leftrightarrow$	B	A	$\&$	$\sim$	B
T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	T	T	F
F	T	F	F	T	F	F	F	T
F	F	F	T	F	F	F	T	F

- Two statements are consistent if they are both true in at least one possible world (*i.e.*, in at least one row of a simultaneous truth-table of both statements). For instance, $A \,\&\, B$ and $A \vee B$ are consistent:

A	B	A	$\&$	B	A	$\vee$	B
T	T	T	T	T	T	T	T
T	F	T	F	F	T	T	F
F	T	F	F	T	F	T	T
F	F	F	F	F	F	F	F

Logical Equivalence: Example #1

- I said that $p \to q$ *is logically equivalent to* $\sim p \vee q$.
- The following truth-table establishes this equivalence:

p	q	$\sim p$	$\vee$	q	$p \to q$
T	T	F	**T**	T	**T**
T	F	F	**F**	F	**F**
F	T	T	**T**	T	**T**
F	F	T	**T**	F	**T**

- The truth-tables of $\sim p \vee q$ and $p \to q$ are the same.

Logical Equivalence: Example #2

- $p \leftrightarrow q$ is an *abbreviation* for $(p \to q)\,\&\,(q \to p)$.
- The following truth-table shows it is a *legitimate* abbreviation:

p	q	$(p \to q)$	**&**	$(q \to p)$	$p \leftrightarrow q$
T	T	T	**T**	T	**T**
T	F	F	**F**	T	**F**
F	T	T	**F**	F	**F**
F	F	T	**T**	T	**T**

- $p \leftrightarrow q$ and $(p \to q)\,\&\,(q \to p)$ have the same truth-table.

Some More Logical Equivalences

- Here is a simultaneous truth-table which establishes that

$$A \leftrightarrow B \dashv\vDash (A \,\&\, B) \vee (\sim A \,\&\, \sim B)$$

A	B	A	$\leftrightarrow$	B	$(A$	$\&$	$B)$	$\vee$	$(\sim$	A	$\&$	$\sim$	$B)$
T	T	T	T	T	T	T	T	T	F	T	F	F	T
T	F	T	F	F	T	F	F	F	F	T	F	T	F
F	T	F	F	T	F	F	T	F	T	F	F	F	T
F	F	F	T	F	F	F	F	T	T	F	T	T	F

- Can you prove the following equivalences with truth-tables?
  - $\sim(A \,\&\, B) \dashv\vDash \sim A \vee \sim B$
  - $\sim(A \vee B) \dashv\vDash \sim A \,\&\, \sim B$
  - $A \dashv\vDash (A \,\&\, B) \vee (A \,\&\, \sim B)$
  - $A \dashv\vDash A \,\&\, (B \rightarrow B)$
  - $A \dashv\vDash A \vee (B \,\&\, \sim B)$

Logical Equivalence, Contradictoriness, *etc.*: Relationships

- What are the relationships between "p and q are equivalent", "p and q are consistent", "p and q are contradictory", "p and q are inconsistent"?

Equivalent	Contradictory
$\Downarrow$? $\Uparrow$	$\Downarrow$? $\Uparrow$
Consistent	Inconsistent

- Answers:
  1. Equivalent $\not\Rightarrow$ Consistent ($p \,\&\, {\sim}p$ and $q \,\&\, {\sim}q$)
  2. Consistent $\not\Rightarrow$ Equivalent ($p \to q$ and $p \,\&\, q$)
  3. Contradictory $\Rightarrow$ Inconsistent (*why*?)
  4. Inconsistent $\not\Rightarrow$ Contradictory (example?)

Truth-Tables and Deductive Validity I

- Remember, an argument is valid if it is *impossible* for its premises to be true while its conclusion is false. Let $p_1, \ldots, p_n$ be the premises of a SL argument, and let q be the conclusion of the argument. Then, we have:

$$\begin{array}{l} p_1 \\ \vdots \\ p_n \\ \hline \therefore q \end{array}$$ is valid if and only if there is no row in the simultaneous truth-table (*interpretation*) of $p_1, \ldots, p_n$, and q which looks like:

atoms premises conclusion

$\cdots$	p_1	$\cdots$	p_n	q
$\cdots$	T	T	T	F

Truth-Tables and Deductive Validity II

A

$A \rightarrow B$ is *valid*:

$\therefore B$

atoms		premises				conclusion
A	B	A	A	$\rightarrow$	B	B
T	T	T	T	T	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	F	F	T	F	F

B

$A \rightarrow B$ is *in*valid:

$\therefore A$

	atoms		premises				conclusion
	A	B	B	A	$\rightarrow$	B	A
	T	T	T	T	T	T	T
	T	F	F	T	F	F	T
☞	F	T	T	F	T	T	F
	F	F	F	F	T	F	F

Finite Propositional Boolean Algebras I

- A *finite propositional Boolean algebra* is a finite set of *propositions* which is *closed* under the logical operations and satisfies the laws of SL.
- *Propositions* are the things expressed by sentences (abstract entities, distinct from sentences). If two sentences are logically equivalent, then they express the same proposition. *E.g.*, "$A \to B$" and "$\sim A \vee B$".
- A set S is *closed* under logical operations if applying a logical operation to a member (or pair of members) of S always yields a member of S.
- Example: consider a sentential language with three atomic letters "X", "Y", and "Z". The set of propositions expressible using the logical connectives and these three atomic letters forms a finite Boolean algebra.
- This Boolean algebra has $2^3 = 8$ *atomic propositions* or *states* (*i.e.*, rows of a 3-sentence truth-table!). Question: How many propositions does it contain *in total*? Answer: $2^8 = 256$ (255 plus the contradiction). *Why*?

Finite Propositional Boolean Algebras II

- A *literal* is either an atomic sentence or the negation of an atomic sentence (*e.g.*, "A" and "$\sim A$" are literals involving the atom "A").
- A *state* of a Boolean algebra $\mathcal{B}$ is a proposition expressed by a *maximal* conjunction of literals in a language $\mathcal{L}_{\mathcal{B}}$ describing $\mathcal{B}$ ("maximal": "containing exactly one literal for each atomic sentence in $\mathcal{B}$").
- Consider an algebra $\mathcal{B}$ described by a 3-atom language $\mathcal{L}_{\mathcal{B}}$ ("X", "Y", "Z"). The states of $\mathcal{B}$ are described by the $2^3 = 8$ *state descriptions* of $\mathcal{L}_{\mathcal{B}}$:

(s_1) $X \,\&\, Y \,\&\, Z$

(s_2) $X \,\&\, Y \,\&\, {\sim}Z$

(s_3) $X \,\&\, {\sim}Y \,\&\, Z$

(s_4) $X \,\&\, {\sim}Y \,\&\, {\sim}Z$

(s_5) ${\sim}X \,\&\, Y \,\&\, Z$

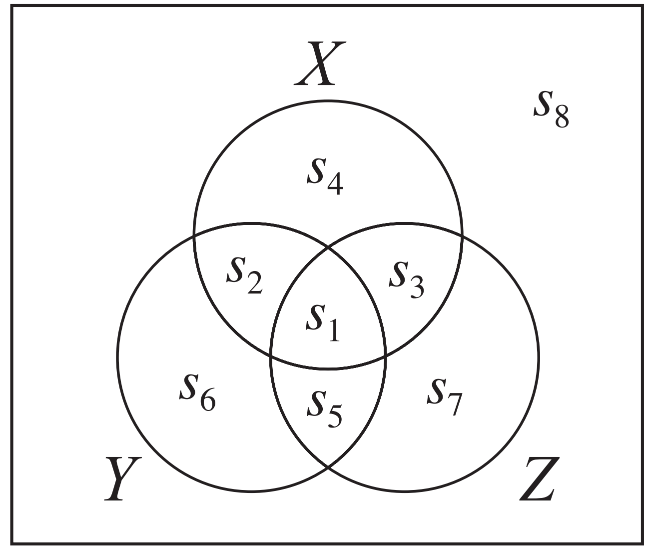
(s_6) ${\sim}X \,\&\, Y \,\&\, {\sim}Z$

(s_7) ${\sim}X \,\&\, {\sim}Y \,\&\, Z$

(s_8) ${\sim}X \,\&\, {\sim}Y \,\&\, {\sim}Z$

- We can "visualize" the states of $\mathcal{B}$ using a truth table or a Venn Diagram.

X	Y	Z	States
T	T	T	s_1
T	T	F	s_2
T	F	T	s_3
T	F	F	s_4
F	T	T	s_5
F	T	F	s_6
F	F	T	s_7
F	F	F	s_8



- Everything that can be expressed in the sentential language $\mathcal{L}_{\mathcal{B}}$ can be expressed as a *disjunction of state descriptions* (think about why).
- Thus, every proposition expressible in $\mathcal{L}_{\mathcal{B}}$ can be "visualized" simply by shading combinations of the 8 state-regions of the Venn Diagram of $\mathcal{B}$. It because of this that we can use Venn Diagrams to establish Boolean Laws.
- $p \vDash q$ (in $\mathcal{B}$) iff every shaded region in the Venn Diagram representation of p (in $\mathcal{B}$) is also shaded in the Venn Diagram representation of q (in $\mathcal{B}$).