

and sociological commentators have sometimes generalized this mistaken reaction into a full-scale attack on the rationality of men of science, and as a result have mistakenly looked for purely sociological explanations for many changes in scientists' beliefs, or the absence of such changes, which were in fact, as we now see, rationally de rigueur.' (Dorling, sec. 5)

2.5 Old News Explained

In sec. 2.2 we analyzed the case Huyghens identified as the principal one in his *Treatise on Light*: A prediction C long known to follow from a hypothesis H is now found to be true. Here, if the rigidity condition is satisfied, $new(H) = old(H|C)$, so that the probability factor is $\pi(H) = 1/old(C)$.

But what if some long-familiar phenomenon C , a phenomenon for which $old(C) = 1$, is newly found to follow from H in conjunction with familiar background information B about the solar system, and thus to be explained by $H \wedge B$? Here, if we were to update by conditioning on C , the probability factor would be 1 and $new(H)$ would be the same as $old(H)$. No confirmation here.¹⁵

Wrong, says Daniel Garber:¹⁶ The information prompting the update is not that C is true, but that $H \wedge B$ implies C . To condition on that news Garber proposes to enlarge the domain of the probability function old by adding to it the hypothesis that C follows from $H \wedge B$ together with all further truth-functional compounds of that new hypothesis with the old domain. Using some extension old^* of old to the enlarged domain, we might then have $new(H) = old^*(H|H \wedge B \text{ implies } C)$. That is an attractive approach to the problem, if it can be made to work.¹⁷

The alternative approach that we now illustrate defines the new assignment on the same domain that old had. It analyzes the $old \mapsto new$ transition by embedding it in a larger process, $ur \xrightarrow{obs} old \xrightarrow{expl} new$, in which ur represents an original state of ignorance of C 's truth and its logical relationship to H :

¹⁵This is what Clark Glymour has dubbed 'the paradox of old evidence': see his *Theory and Evidence*, Princeton University Press, 1980.

¹⁶See his 'Old evidence and logical omniscience in Bayesian confirmation theory' in *Testing Scientific Theories*, ed. John Earman, University of Minnesota Press, 1983. For further discussion of this proposal see 'Bayesianism with a human face' in my *Probability and the Art of Judgment*, Cambridge University Press, 1992.

¹⁷For a critique, see chapter 5 of John Earman's *Bayes or Bust?* (MIT Press, 1992).

EXAMPLE 1, The perihelion of Mercury.

Notation.

H: GTR applied to the Sun-Mercury system

C: Advance of 43 seconds of arc per century¹⁸

In 1915 Einstein presented a report to the Prussian Academy of Sciences explaining the then-known advance of approximately 43'' per century in the perihelion of Mercury in terms of his (“GTR”) field equations for gravitation. An advance of 38'' per century had been reported by Leverrier in 1859, due to ‘some unknown action on which no light has been thrown’.¹⁹

“The only way to explain the effect would be ([Leverrier] noted) to increase the mass of Venus by at least 10 percent, an inadmissible modification. He strongly doubted that an intramercurial planet [“Vulcan”], as yet unobserved, might be the cause. A swarm of intramercurial asteroids was not ruled out, he believed.”

Leverrier’s figure of 38'' was soon corrected to its present value of 43'', but the difficulty for Newtonian explanations of planetary astronomy was still

¹⁸Applied to various Sun-planet systems, the GTR says that all planets’ perihelions advance, but that Venus is the only one for which that advance should be observable. This figure for Mercury has remained good since its publication in 1882 by Simon Newcomb.

¹⁹‘*Subtle is the Lord...*’, *the Science and Life of Albert Einstein*, Abraham Pais, Oxford University Press, 1982, p. 254.

in place 65 years later, when Einstein finally managed to provide a general relativistic explanation ‘without special assumptions’ (such as Vulcan)—an explanation which was soon accepted as strong confirmation for the GTR.

In the diagram above, the left-hand path, $ur \mapsto prn \mapsto new$, represents an expansion of the account in sec. 2.2 of Huyghens’s “principal” case (‘*prn*’ for ‘principal’), in which the confirming phenomenon is verified *after* having been predicted via the hypothesis which its truth confirms. The “*ur*” probability distribution, indicated schematically at the top, represents a time before C was known to follow from H . To accommodate that discovery the ‘*b*’ in the *ur*-distribution is moved left and added to the ‘*a*’ to obtain the top row of the *prn* distribution. The reasoning has two steps. First: Since we now know that H is true, C must be true as well. Therefore $prn(H \wedge \neg C)$ is set equal to 0. Second: Since implying a prediction that may well be false neither confirms nor infirms H , the *prn* probability of H is to remain at its *ur*-value even though the probability of $H \wedge \neg C$ has been nullified. Therefore the probability of $H \wedge C$ is increased to $prn(H \wedge C) = a + b$. And this *prn* distribution is where the Huyghens on light example in sec. 2.2 *begins*, leading us to the bottom distribution, which assigns the following odds to H :

$$\frac{new(H)}{new(\neg H)} = \frac{a + b}{c}.$$

In terms of the present example this *new* distribution is what Einstein arrived at from the distribution labelled *old*, in which

$$\frac{old(H)}{old(\neg H)} = \frac{a}{c}.$$

The rationale is

COMMUTATIVITY:

The *new* distribution is the same, no matter if the observation or the explanation comes first.

Now the Bayes factor gives the clearest view of the transition $old(H) \mapsto new(H)$:²⁰

²⁰The approach here is taken from Carl Wagner, “Old evidence and new explanation III”, *PSA 2000* (J. A. Barrett and J. McK. Alexander, eds.), part 1, pp. S165-S175 (Proceedings of the 2000 biennial meeting of the Philosophy of Science Association, supplement to *Philosophy of Science* 68 [2001]), which reviews and extends earlier work, “Old evidence and new explanation I” and “Old evidence and new explanation II” in *Philosophy of Science* 64, No. 3 (1997) 677-691 and 66 (1999) 283-288.

$$\beta(H : \neg H) = \frac{a+b}{c} / \frac{a}{c} = 1 + \frac{ur(\neg C|H)}{ur(C|H)}.$$

Thus the new explanation of old news C increases the odds on H by a factor of $(1 + \text{the } ur\text{-odds against } C, \text{ given } H)$. Arguably, this is very much greater than 1, since, in a notation in which $C = C_{43} = \text{an advance of } (43 \pm .5)''/c$ and similarly for other C_i 's, $\neg C$ is a disjunction of very many "almost" incompatible disjuncts: $\neg C = \dots C_{38} \vee C_{39} \vee C_{40} \vee C_{41} \vee C_{42} \vee C_{44} \vee \dots$.²¹ That's the good news.²²

And on the plausible assumption²³ of *ur*-independence of H from C , the formula for β becomes even simpler:

$$\beta(H : \neg H) = 1 + \frac{ur(\bar{C})}{ur(C)}$$

The bad news is that *ur* and *prn* are new constructs, projected into a mythical methodological past. But maybe that's not so bad. As we have just seen in a very sketchy way, there seem to be strong reasons for taking the ratio $ur(\neg C|H)/ur(C|H)$ to be very large. This is clearer in the case of *ur*-independence of H and C :

EXAMPLE 2, Ur-independence. If Wagner's independence assumption is generally persuasive, the physics community's *ur*-odds against C (" C_{43} ") will be very high, since, behind the veil of *ur*-ignorance, there is a large array of C_i 's, which, to a first approximation, are equiprobable. Perhaps, 999 of them? Then $\beta(H : \neg H) \geq 1000$, and the weight of evidence (2.1) is $w(H : \neg H) \geq 3$.

2.6 Supplements

1 "Someone is trying decide whether or not T is true. He notes that T is a consequence of H . Later he succeeds in proving that H is false. How does this refutation affect the probability of T ?"²⁴

2 "We are trying to decide whether or not T is true. We derive a sequence of consequences from T , say C_1, C_2, C_3, \dots . We succeed in verifying C_1 , then

²¹"Almost": $ur(C_i \wedge C_j) = 0$ for any distinct integers i and j .

²²"More [*ur*]danger, more honor." See Pólya, quoted in sec. 2.2 here.

²³Carl Wagner's, again.

²⁴This problem and the next are from George Pólya, 'Heuristic reasoning and the theory of probability', *American Mathematical Monthly* **48** (1941) 450-465.

C_2 , then C_3 , and so on. What will be the effect of these successive verifications on the probability of T ?”

3 FALLACIES OF “YES/NO” CONFIRMATION.²⁵ Each of the following plausible rules is unreliable. Find counterexamples—preferably, simple ones.

- (a) If D confirms T , and T implies H , then D confirms H .
- (b) If D confirms H and T separately, it must confirm their conjunction, TH .
- (c) If D and E each confirm H , then their conjunction, DE , must also confirm H .
- (d) If D confirms a conjunction, TH , then it cannot infirm each conjunct separately.

4 HEMPEL’S RAVENS. It seems evident that black ravens confirm (H) ‘All ravens are black’ and that nonblack nonravens do not. Yet H is logically equivalent to ‘All nonravens are nonblack’. Use probabilistic considerations to resolve this paradox of “yes/no” confirmation.²⁶

5 WAGNER III (sec 2.5).²⁷ Carl Wagner extends his treatment of old evidence newly explained to cases in which updating is by generalized conditioning on a countable sequence $\mathcal{C} = \langle C_1, C_2, \dots \rangle$ where none of the updated probabilities need be 1, and to cases in which each of the C ’s “follows only probabilistically” from H in the sense that the conditional probabilities of the C ’s, given H , are high but < 1 . Here we focus on the simplest case, in which \mathcal{C} has two members, $\mathcal{C} = \langle C, \neg C \rangle$. Where (as in the diagram in sec. 2.6) updates are not always from *old* to *new*, we indicate the prior and posterior probability measures by a subscript and superscript: β_{prior}^{post} . In particular, we shall write $\beta_{old}^{new}(H : \neg H)$ explicitly, instead of $\beta(H : \neg H)$ as in (3) and (4) of sec. 2.6. And we shall use ‘ β^* ’ as shorthand: $\beta^* = \beta_{ur}^{old}(C : \neg C)$.

(a) Show that in the framework of sec. 2.6 COMMUTATIVITY is equivalent to UNIFORMITY:

$$\beta_{old}^{new}(A : A') = \beta_{ur}^{prn}(A : A') \text{ if } A, A' \in \{H \wedge C, H \wedge \neg C, \neg H \wedge C, \neg H \wedge \neg C\}.$$

(b) Show that uniformity holds whenever the updates $ur \mapsto old$ and $prn \mapsto new$ are both by generalized conditioning.

²⁵These stem from Carl G. Hempel’s ‘Studies in the logic of confirmation’, *Mind* 54 (1945) 1-26 and 97-121, which is reprinted in his *Aspects of Scientific Explanation*, The Free Press, New York, 1965.

²⁶The paradox was first floated (1937) by Carl G. Hempel, in an abstract form: See pp. 50-51 of his *Selected Philosophical Essays*, Cambridge University Press, 2000. For the first probabilistic solution, see “On confirmation” by Janina Hosiasson-Lindenbaum, *The Journal of Symbolic Logic* 5 (1940) 133-148. For a critical survey of more recent treatments, see pp. 69-73 of John Earman’s *Bayes or Bust?*

²⁷This is an easily detachable bit of Wagner’s “Old evidence and new explanation III” (cited above in sec. 3.6).

Now verify that where uniformity holds, so do the following two formulas:

$$(c) \beta_{ur}^{new}(H : \neg H) = \frac{(\beta^* - 1)prn(C|H) + 1}{(\beta^* - 1)prn(C|\neg H) + 1}$$

$$(d) \beta_{ur}^{old}(H : \neg H) = \frac{(\beta^* - 1)ur(C|H) + 1}{(\beta^* - 1)ur(C|\neg H) + 1}$$

And show that, given uniformity,

(e) if H and C are *ur*-independent, then $old(H) = ur(H)$ and, therefore,

$$(f) \beta_{old}^{new}(H : \neg H) = \frac{(\beta^* - 1)prn(C|H) + 1}{(\beta^* - 1)prn(C|\neg H) + 1}, \text{ so that}$$

(g) Given uniformity, if H and C are *ur*-independent then

(1) $\beta_{old}^{new}(H : \neg H)$ depends only on β^* , $prn(C|H)$ and $prn(C|\neg H)$;

(2) if $\beta > 1$ and $prn(C|H) > prn(C|\neg H)$, then $\beta_{old}^{new}(H : \neg H) > 1$;

(3) $\beta_{old}^{new}(H : \neg H) \rightarrow \frac{prn(C|H)}{prn(C|\neg H)}$ as $\beta \rightarrow \infty$.

6 SHIMONY ON HOLT-CLAUSER.²⁸ “Suppose that the true theory is local hidden variables, but Clauser’s apparatus [which supported QM] was faulty. Then you have to accept that the combination of systematic and random errors yielded (within the rather narrow error bars) the quantum mechanical prediction, which is a definite number. The probability of such a coincidence is very small, since the part of experimental space that agrees with a numerical prediction is small. By contrast, if quantum mechanics is correct and Holt’s apparatus is faulty, it is not improbable that results in agreement with Bell’s inequality (hence with local hidden variables theories) would be obtained, because agreement occurs when the relevant number falls within a rather large interval. Also, the errors would usually have the effect of disguising correlations, and the quantum mechanical prediction is a strict correlation. Hence good Bayesian reasons can be given for voting for Clauser over Holt, even if one disregards later experiments devised in order to break the tie.”

Compare: Relative ease of the Rockies eventually wearing down to Adirondacks size, as against improbability of the Adirondacks eventually reaching the size of the Rockies.

²⁸Personal communication, 12 Sept. 2002.