Dutch Book Arguments

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1. Introduction

Beliefs come in varying degrees. I am more confident that this coin will land Heads when tossed than I am that it will rain in Canberra tomorrow, and I am more confident still that 2 + 2 = 4. It is natural to represent my *degrees of belief*, or *credences*, with numerical values. *Dutch Book arguments* purport to show that there are rational constraints on such values. They provide the most famous justification for the Bayesian thesis that degrees of belief should obey the probability calculus. They are also offered in support of various further principles that putatively govern rational subjective probabilities.

Dutch Book arguments assume that your credences match your betting prices: you assign probability *p* to *X* if and only if you regard *pS* as the value of a bet that pays *S* if *X*, and nothing otherwise (where *S* is a positive stake). Here we assume that your highest buying price equals your lowest selling price, with your being indifferent between buying and selling at that price; we will later relax this assumption. For example, my credence in Heads is ½, corresponding to my valuing a $1 bet on Heads at 50 cents. A *Dutch Book* is a set of bets bought or sold at such prices as to guarantee a net loss. An agent is susceptible to a Dutch Book, and her credences said to be 'incoherent', if there exists such a set of bets, bought or sold at prices that she deems acceptable (by the lights of her credences).

There is little agreement on the origins of the term. Some say that Dutch merchants and actuaries in the 17th century had a reputation for being canny businessmen, but this provides a rather speculative etymology. By the time Keynes wrote in 1920, the proprietary sense of the term 'book' was apparently familiar to his readership: 'In fact underwriters themselves distinguish between risks which are properly insurable, either because their probability can be estimated between narrow numerical limits or because it is possible to make a 'book' which covers all possibilities…' (21). Ramsey's ground-breaking paper 'Truth and Probability' (1926), which inaugurates the Dutch Book argument[1], speaks of 'a book being made against you' (1980, 44; 1990, 79). Lehman (1955) writes:

> If a bettor is quite foolish in his choice of the rates at which he will bet, an opponent can win money from him no matter what happens… Such a losing book is called by [bookmakers] a "dutch book" (251).

So certainly 'Dutch Books' appear in the literature under that name by 1955. Note that Dutch Book arguments typically take the 'bookie' to be the clever person who is assured of winning money off some irrational agent who has posted vulnerable odds, whereas at the racetrack it is the 'bookie' who posts the odds in the first place.

This article will concentrate on the many forms of Dutch Book argument, as found especially in the philosophical literature, canvassing their interpretation, their cogency, and their prospects for unification.

2. Classic Dutch Book arguments for probabilism

2.1 Probabilism

Philosophers use the term 'probabilism' for the traditional Bayesian thesis that agents have degrees of belief that are rationally required to conform to the laws of probability. (This is silent on other issues that divide Bayesians, such as how such

[1] Earman (1992) finds some anticipation of the argument in the work of Bayes (1764).

degrees of belief should be updated.) These laws are taken to be codified by Kolmogorov's (1933) axiomatization, and the best-known Dutch Book arguments aim to support probabilism, so understood. However, several aspects of that axiomatization are presupposed, rather than shown, by Dutch Book arguments. Kolmogorov begins with a finite set Ω, and an algebra $\mathcal{F}$ of subsets of Ω (closed under complementation and finite union); alternatively, we may begin with a finite set *S* of sentences in some language, closed under negation and disjunction. We then define a real-valued, bounded (unconditional) probability function *P* on $\mathcal{F}$, or on *S*. Dutch Book arguments cannot establish any of these basic framework assumptions, but rather take them as given.

The heart of probabilism, and of the Dutch Book arguments, are the numerical axioms governing *P* (here presented sententially):

1. *Non-negativity*: $P(X) \geq 0$ for all *X* in *S*.
2. *Normalization:* $P(T) = 1$ for any tautology *T* in *S*.
3. *Finite additivity:* $P(X \vee Y) = P(X) + P(Y)$ for all *X*, *Y* in *S* such that *X* is incompatible with *Y*.

2.2 Classic Dutch Book arguments for the numerical axioms

We now have a mathematical characterization of the probability calculus. Probabilism involves the normative claim that if your degrees of belief violate it, you are irrational. The Dutch Book argument begins with a mathematical theorem:

Dutch Book Theorem: if a set of betting prices violate the probability calculus, then there is a Dutch Book consisting of bets at those prices.

The argument for probabilism involves the normative claim that if you are susceptible to a Dutch Book, then you are irrational. The sense of 'rationality' at issue here is an ideal, suitable for logically omniscient agents rather than for humans; 'you' are understood to be such an agent.

The gist of the proof of the theorem is as follows (all bets are assumed to have a stake of \$1):

Non-negativity: Suppose that your betting price for some proposition N is negative—that is, you value a bet that pays \$1 if N, 0 otherwise at some negative amount \$–n, where $n > 0$. Then you are prepared to sell a bet on N for \$–n—that is, you are prepared to pay someone \$$n$ to take the bet (which must pay at least \$0). You are thus guaranteed to lose at least \$$n$.

Normalization: Suppose that your betting price \$$t$ for some tautology T is less than \$1. Then you are prepared to sell a bet on T for \$$t$. Since this bet must win, you face a guaranteed net loss of \$$(1–t) > 0$. If \$$t$ is greater than \$1, you are prepared to buy a bet on T for \$$t$, guaranteeing a net loss of \$$(t–1) > 0$.

Finite additivity: Suppose that your betting prices on some incompatible P and Q are \$$p$ and \$$q$ respectively, and that your betting price on $P \vee Q$ is \$$r$, where \$$r$ > \$$(p + q)$. Then you are prepared to sell separate bets on P (for \$$p$) and on Q (for \$$q$), and to buy a bet on $P \vee Q$ for \$$r$, assuring an initial loss of \$$(r – (p + q)) > 0$. But however the bets turn out, there will be no subsequent change in your fortune, as is easily checked.

Now suppose that \$$r$ < \$$(p + q)$. Reversing 'sell' and 'buy' in the previous paragraph, you are guaranteed a net loss of \$$((p + q) – r) > 0$.

So much for the Dutch Book theorem; now, a first pass at the argument.

P1. Your credences match your betting prices.
P2. Dutch Book theorem: if a set of betting prices violate the probability calculus, then there is a Dutch Book consisting of bets at those prices.
P3. If there is a Dutch Book consisting of bets at your betting prices, then you are susceptible to losses, come what may, at the hands of a bookie.
P4. If you are so susceptible, then you are irrational.
∴C. If your credences violate the probability calculus, then you are irrational.
∴C'. If your credences violate the probability calculus, then you are *epistemically* irrational.

The bookie is usually assumed to seek cunningly to win your money, to know your betting prices, but to know no more than you do about contingent matters. None of these assumptions is necessary. Even if he is a bumbling idiot or a kindly benefactor, and even if he knows nothing about your betting prices, he *could* sell/buy you bets that ensure your loss, perhaps by accident; you are still *susceptible* to such loss. And even if he knows everything about the outcomes of the relevant bets, he cannot thereby expose you to losses *come what may*; rather, he can fleece you in the actual circumstances that he knows to obtain, but not in various possible circumstances in which things turn out differently.

The irrationality that is brought out by the Dutch Book argument is meant to be one *internal* to your degrees of belief, and in principle detectable by you by *a priori* reasoning alone. Much of our discussion will concern the exact nature of such ‘irrationality’. Offhand, it appears to be *practical* irrationality—your openness to financial exploitation. Let us start with this interpretation; in §4 we will consider other interpretations.

2.3 Converse Dutch Book theorem

There is a gaping loophole in this argument as it stands. For all it says, it may be the case that *everyone* is susceptible to such sure losses, and that obeying the probability calculus provides no inoculation. In that case, we have seen no reason so far to obey that calculus. This loophole is closed by the equally important, but often neglected

> *Converse Dutch Book Theorem*: If a set of betting prices obey the probability calculus, then there does not exist a Dutch Book consisting of bets at those prices.

This theorem was proved independently by Kemeny (1955) and Lehman (1955). Ramsey seems to have been well aware of it (although we have no record of him proving it): 'Having degrees of belief obeying the laws of probability implies a further measure of consistency, namely such a consistency between the odds acceptable on different propositions as shall prevent a book being made against you.' (1980, 41; 1990, 79). A proper presentation of the Dutch Book argument should include this theorem as a further premise.

A word of caution: As we will see, there are many Dutch Book arguments of the form:

'If you violate Φ, then you are susceptible to a Dutch Book;

∴ You should obey Φ.'

None of these arguments has any force without a converse premise. (If you violate Φ, then you will eventually die. A sobering thought, to be sure, but hardly a reason to join the ranks of the equally mortal Φ–ers!) Ideally, the converse premise will have the form:

‘If you *obey* Φ, then you are *not* susceptible to a Dutch Book’.

But a weaker premise may suffice:

‘If you *obey* Φ, then *possibly* you are *not* susceptible to a Dutch Book’.[2]

If *all* those who violate Φ are susceptible, and at least *some* who obey Φ are not, you apparently have an incentive to obey Φ. If you don’t, we know you are susceptible; if you do, at least there is some hope that you are not.

2.4 Extensions

Kolmogorov goes on to extend his set-theoretic underpinnings to infinite sets, closed further under *countable* union; we may similarly extend our set of sentences *S* so that it is also closed under infinitary disjunction. There is a Dutch Book argument for the corresponding infinitary generalization of the finite additivity axiom:

3′. Countable additivity:
If $A_1, A_2, \ldots$ is a sequence of pairwise incompatible sentences in *S,* then

$$P(\mathrm{V}A_n) = \sum_{n=1}^{\infty} P(A_n)$$

[NOTE TO COPY EDITOR: THE 'V' SHOULD BE NOT HAVE SERIPHS (IT'S THE LARGE 'VEL' SYMBOL OF LOGIC), AND IT SHOULD HAVE 'n=1' BELOW IT, AND '∞' ABOVE IT, JUST AS THE SIGMA DOES.]

Adams (1962) proves a Dutch Book theorem for countable additivity; Skyrms (1984) and Williamson (1999) give simplified versions of the corresponding argument.

Kolmogorov then analyzes the *conditional probability* of *A* given *B* by the ratio formula:

(Conditional Probability) $$P(A|B) = \frac{P(A \quad B)}{P(B)} \qquad (P(B) > 0)$$

2

This too has a Dutch Book justification. Following de Finetti (1937), we may introduce the notion of a *conditional bet* on *A*, given *B*, which

- pays $1 if A & B
- pays 0 if ¬A & B
- is called off if ¬B (that is, the price you pay for the bet is refunded).

Identifying an agent's value for $P(A|B)$ with the value she attaches to this conditional bet, if she violates (Conditional Probability) she is susceptible to a Dutch Book consisting of bets involving A & B, ¬B, and a conditional bet on A given B.

3. Objections

We will not question here the Dutch Book theorem or its converse. But there are numerous objections to premises P1, P3, and P4.

There are various circumstances in which an agent's credence in *X* can come apart from her betting price for *X*: when *X* is unverifiable or unfalsifiable; when betting on *X* has other collateral benefits or costs; when the agent sees a correlation between *X* and any aspect of a bet on *X* (its price, its stake, or even its placement); and so on. More generally, the betting interpretation shares a number of problems with operational definitions of theoretical terms, and in particular behaviorism about mental states. (See Eriksson and Hájek 2007.) The interpretation also assumes that an agent values money linearly—implausible for someone who needs $1 to catch a bus home, and who is prepared to gamble at otherwise unreasonable odds for a chance of getting it. Since in cases like this it seems reasonable for prices of bets with monetary prizes to be non-additive, if we identify credences with those prices, non-additivity of credences in turn seems reasonable. On the other hand, if we weaken the connection between credences and betting prices posited by P1, then we cannot infer probabilism from any results about rational betting prices—the latter may be required to obey the probability calculus, but what about *credences*? We could instead appeal to bets with prizes of *utilities* rather than monetary amounts. But the usual way of defining utilities is via a 'representation theorem', again dating back to Ramsey's 'Truth and Probability' (1926). Its upshot is that an agent whose preferences obey certain constraints (transitivity, and so on) is representable as an expected utility maximizer according to some utility and probability function. This threatens to render the Dutch Book argument otiose—the representation theorem has already provided an argument for probabilism. Perhaps some independent, probability-neutral account of 'utility' can be given, but in any case, a proponent of any Dutch Book argument should modify P1 appropriately.

All these problems carry over immediately to de Finetti's Dutch Book argument for (*Conditional Probability*), and further ones apparently arise for his identification of *conditional* credences with *conditional* betting odds. Here is an example adapted from one given by Howson (1995) (who in turn was inspired by a well-known counterexample, attributed to Richmond Thomason, to the so-called 'Ramsey test' for the acceptability of a conditional): You may assign low conditional probability to your ever knowing that you are being spied on by the CIA, given that in fact you are—they are clever about hiding such surveillance. But you presumably place a high value on the corresponding conditional bet—once you find out that the condition of the bet has been met, you will be very confident that you know it!

It may seem curious how the Dutch Book argument—still understood literally—moves from a mathematical theorem concerning the existence of abstract bets with certain properties, to a normative conclusion about rational credences, *via* a premise about some bookie. Presumably the agent had better assign positive credence to the bookie's existence, his nefarious motives, and his readiness to take either side of the relevant bets as required to ensnare the agent in a Dutch Book—otherwise, the bare possibility of such a scenario ought to play no role in her deliberations. (Compare: if you go to Venice, you face the possibility of painful death in Venice; if you do not go to Venice, you do not face this possibility. That is hardly a reason for you to avoid Venice; your appropriate course of action has to be more sensitive to your credences and utilities.) But probabilism should not legislate on what credences the agent has about such contingent matters. Still less should probabilism require this kind of paranoia when it is in fact unjustified—when she rightly takes her neighborhood to be free of such mercenary characters, as most of us do. And even if such characters abound, she can simply turn down all offers of bets when she sees them coming. So violating the probability calculus may not be a practical liability after all. Objections of this kind cast doubt on an overly literal interpretation of the Dutch Book argument. (See Kennedy and Chihara 1979, Kyburg 1978, Christensen 1991, Hájek 2005.)

But even granting the ill effects, practically speaking, of violating the probability calculus, it is a further step to show that there is some *epistemic* irrationality in such violation—yet it is this conclusion (C') that presumably the probabilist really seeks. After all, as Christensen (1991) argues, if those who violated probability theory were tortured by the Bayesian Thought Police, that might show violating probability theory is irrational in some sense—but surely not in the sense that matters to the probabilist.

P3 presupposes a so-called *package principle*—the value that you attach to a collection of bets is the sum of the values that you attach to the bets individually. Various authors have objected to this principle (e.g., Schick 1986, Maher 1993). Let us look at two kinds of concern. Firstly, there may be interference effects between the *prizes* of the bets. Valuing money non-linearly is a clear instance. Suppose that the pay-off of each of two bets is not sufficient for your bus ticket, so taken individually they are of little value to you; but their combined pay-off *is* sufficient, so the package of the two of them is worth a lot to you. (Here we are still interpreting Dutch Book arguments as taking literally all this talk of bets and monetary gains and losses.) Secondly, you may regard the *placement* of one bet in a package as correlated with the *outcome* of another bet in the package. I may be confident that Labour will win the next election, and that my wife is in a good mood; but knowing that she hates my betting on politics, my placing a bet on Labour's winning changes my confidence in her being in a good mood. This interference effect could not show up in the bets taken individually. We cannot salvage the argument merely by restricting 'Dutch Books' to cases in which such interference effects are absent, for that would render false the Dutch Book theorem (so understood): your sole violations of the probability calculus might be over propositions for which such effects are present. Nor should the probabilist rest content with weakening the argument's conclusion accordingly; after all, *any* violation of the probability calculus is supposed to be irrational, even if it occurs solely in such problematic cases. The dilemma, then, is to make plausible the package principle without compromising the rest of the argument. This should be kept in mind when assessing any Dutch Book argument that involves multiple bets, as most do.

The package principle is especially problematic when the package is *infinite*, as it needs to be in the Dutch Book argument for countable additivity. Arntzenius, Elga, and Hawthorne (2004) offer a number of cases of infinite sets of transactions, each of which is favourable, but which are unfavourable in combination. Suppose, for example, that Satan has cut an apple into infinitely many pieces, labeled by the natural numbers, and that Eve can take as many pieces as she likes. If she takes only finitely many, she suffers no penalty; if she takes infinitely many she is expelled from the Garden. Her first priority is to stay in the Garden; her second priority is to eat as many pieces as she can. For each *n* (= 1, 2, 3, …), she is strictly better off choosing to eat piece #*n*. But the combination of all such choices is strictly worse than the status quo. Arntzenius, Elga and Hawthorne consider similar problems with the agglomeration of infinitely many bets, concluding: 'There simply need not be any tension between judging each of an infinite package of bets as favourable, and judging the whole package as unfavourable. So one can be perfectly rational even if one is vulnerable to an infinite Dutch Book.' (279)

P4 is also suspect unless more is said about the 'sure' losses involved. For there is a good sense in which you may be susceptible to sure losses without any irrationality on your part. For example, it may be rational of you, and even rationally *required* of you, to be less than certain of various necessary *a posteriori* truths—that Hesperus is Phosphorus, that water is H_20, and so on—and yet bets on the falsehood of these propositions are (metaphysically) guaranteed to lose. Some sure losses are not at all irrational; in §4 we will look more closely at which are putatively the irrational ones.

Moreover, for all we have seen, those who obey the probability calculus, while protecting themselves from sure monetary losses, may be guilty of *worse* lapses in rationality. After all, there are worse financial choices than sure monetary losses—for example, even greater *expected* monetary losses. (You would do better to choose the sure loss of a penny over a 0.999 chance of losing a million dollars.) And there are other ways to be irrational besides exposing yourself to monetary losses.

4. Intepretations and variations

4.1 A game-theoretic interpretation

A game-theoretic interpretation of the Dutch Book argument can be given. It is based on de Finetti's proposal of a game-theoretic basis for subjective expected utility theory. A simplified presentation is given in Seidenfeld (2001), although it is still far more general than we will need here. Inspired by this presentation, we will simplify again, as follows. Imagine a 2-person, zero-sum game, between players whom for mnemonic purposes we will call the Agent and the Dutchman. The Agent is required to play first, revealing a set of real-valued numbers assigned to a finite partition of states—think of this as her probability assignment. The Dutchman sees this assignment, and chooses a finite set of weights over the partition—think of these as the stakes of corresponding bets, with the sign of each stake indicating whether the agent buys or sells that bet. The Agent wins the maximal total amount that she can, given this system of bets—think of the actual outcome being the most favorable it could be, by her lights. The Dutchman wins the negative of that amount—that is, whatever the Agent wins, the Dutchman loses, and vice versa. Since the Dutchman may choose all the weights to be 0, he can ensure that the value of the game to the Agent is bounded above by 0. The upshot is that the Agent will suffer a sure loss from a clever choice of weights by the Dutchman if and only if her probability assignments violate the probability calculus.

This interpretation of the Dutch Book argument takes rather literally the story of a two-player interaction between an agent and a bookie that is usually associated with it. However, in light of some of the objections we saw in the last section, there are reasons for looking for an interpretation of the Dutch Book argument that moves beyond considerations of strategic conflict and maximizing one's gains.

4.2 The 'dramatizing inconsistency' interpretation

Ramsey's original paper offers such an interpretation. Here is the seminal passage:

> These are the laws of probability, which we have proved to be necessarily true of any consistent set of degrees of belief. Any definite set of degrees of belief which broke them would be inconsistent in the sense that it violated the laws of preference between options, such as that preferability is a transitive asymmetrical relation, and that if α is preferable to β, β for certain cannot be preferable to α if *p*, β if not-*p*. If anyone's mental condition violated these laws, his choice would depend on the precise form in which the options were offered him, which would be absurd. He could have a book made against him by a cunning better and would then stand to lose in any event.
>
> We find, therefore, that a precise account of the nature of partial belief reveals that the laws of probability are laws of consistency, an extension to partial beliefs of formal logic, the logic of consistency…
>
> Having any definite degree of belief implies a certain measure of consistency, namely willingness to bet on a given proposition at the same odds for any stake, the stakes being measured in terms of ultimate values. Having degrees of belief obeying the laws of probability implies a further measure of consistency, namely such a consistency between the odds acceptable on different propositions as shall prevent a book being made against you. (1980, 41-2; 1990, 78-9)

This interpretation has been forcefully defended by Skyrms in a number of works (1980, 1984, 1987a); e.g.,

> Ramsey and de Finetti have provided a way in which the fundamental laws of probability can be viewed as pragmatic consistency conditions: conditions for the consistent evaluation of betting arrangements no matter how described. (1980, 120)

Similarly, Armendt (1993, 4) writes of someone who violates the laws of probability:

> I say it *is* a flaw of rationality to give, at the same time, two different choice-guiding evaluations to the same thing. Call this *divided-mind* inconsistency.

Notice an interesting difference between the quote by Ramsey and those of Skyrms and Armendt. Ramsey is apparently also making the considerably more controversial point that a violation of the *laws of preference*—not merely the laws of probability—is tantamount to inconsistency. This is more plausible for some of his laws of preference (e.g. transitivity, which he highlights) than for others (e.g. the Archimedean axiom or continuity, which are imposed more for the mathematical convenience of ensuring that utilities are real-valued).

This version of the argument begins with P1 and P2 as before. But Skyrms and Armendt insist that the considerations of sure losses at the hands of a bookie are merely a *dramatization* of the real defect inherent in an agent's violating probability theory: an underlying inconsistency in the agent's evaluations. So their version of the argument focuses on that inconsistency instead. We may summarize it as follows:

P1. Your credences match your betting prices.
P2. Dutch Book theorem: if a set of betting prices violate the probability calculus, then there is a Dutch Book consisting of bets at those prices.
P3'. If there is a Dutch Book consisting of bets at your betting prices, then you give inconsistent evaluations of the same state of affairs (depending on how it is presented).
P4'. If you give inconsistent evaluations of the same state of affairs, then you are irrational.

∴C. If your credences violate the probability calculus, then you are irrational.
∴C'. If your credences violate the probability calculus, then you are *epistemically* irrational.

This talk of credences being 'irrational' is implicit in Skyrms's presentation—he focuses more on the notion of inconsistency per se—but it is explicit in Armendt's.

This version of the argument raises new objections. 'Inconsistency' is not a straightforward notion, even in logic. For starters, it is controversial just what counts *as* logic in this context. It would be glib to say that classical logic is automatically assumed. Apparently it is *not* assumed when we formulate the countable additivity axiom sententially—the logic had better be infinitary. In that case, is omega-inconsistency, the kind that might arise in countable Dutch Books, inconsistency of the troubling kind? (Consider an infinite set of sentences that has as members 'Fn' for every natural number n, but also '$\neg(\forall x)Fn$'.) Once we countenance non-classical logics, which should guide our judgments of inconsistency? Weatherson (2003) argues that the outcomes of the bets appealed to in Dutch Book arguments must be *verified*, and thus that the appropriate logic is *intuitionistic*. Note that nothing in the Dutch Book arguments resolves these questions, and yet the notion of *sure* losses looks rather different depending on what we take to be logically 'sure'.

However we resolve such questions, the 'inconsistency' at issue here is apparently something different again: a property of conflicting *evaluations*, and it is thus essentially preference-based. Offhand, giving 'two different choice-guiding evaluations', as Armendt puts it, seems to be a matter of not giving *identical* evaluations, a problem regarding the *number* of evaluations—two, rather than one. Understood this way, the alleged defect prima facie seems to be one of *inconstancy,* rather than *inconsistency*. To be sure, being 'consistent' in ordinary English sometimes means repeating a particular task without noticeable variation, as when we say that Tiger Woods is a consistent golfer, or when we complain that the chef at a particular restaurant is inconsistent. But this is trading on a pun on the word, and it need not have anything to do with logic. Notice that it is surely this *non*-logical sense of the word that Ramsey has in mind when he speaks of 'a certain measure of consistency, namely willingness to bet on a given proposition at the same odds for any stake', for it is hard to see how *logic* could legislate on that.

But arguably, the kind of inconstancy evinced by Dutch Book susceptibility *is* a kind of inconsistency. Crucial is Armendt's further rider, that of giving 'two different choice-guiding evaluations *to the same thing*'. The issue becomes one of how we individuate the 'things', the objects of preference. Skyrms writes that the incoherent agent 'will consider two different sets of odds as fair for an option depending on how that option is described; the equivalence of the descriptions following from the underlying Boolean logic.' (1987b, 2) But even two logically equivalent sentences are not the same thing—they are two, rather than one. To be sure, they may correspond to a single profile of payoffs across all logically possible worlds (keeping in mind our previous concerns about rival logics). But is a failure to recognize this the sin of *inconsistency*, a sin of *commission*, or is it rather a failure of logical omniscience, a sin of *omission*? (In the end, it might not matter much either way if both are failures to meet the demands of epistemic rationality, at least in an ideal sense.) See Vineberg (2001) for skepticism of the viability of the 'inconsistency' interpretation of the Dutch Book argument for the normalization axiom. This remains an area of lively debate.

That interpretation for the additivity axiom is controversial in a different way. Again, it may be irrational to give two different choice-guiding evaluations to the same thing. But those who reject the package principle deny that they are guilty of this kind of double-think. They insist that being willing to take bets individually does not rationally require being willing to take them in combination; recall the possibility of interference effects between the bets taken in combination. The interpretation is strained further for the *countable* additivity axiom; recall the problems that arose with the agglomeration of infinitely many transactions. In §6 we will canvass other Dutch Book arguments for which the interpretation seems quite implausible (not that Ramsey, Skyrms or Armendt ever offered it for them).

Christensen (1996) is dubious of the inference from C to C': while Dutch Books, so understood, may reveal an irrationality in one's *preferences*, that falls short of revealing some *epistemic* irrationality. Indeed, we may imagine an agent in whom the connection between preferences and epistemic states is sundered altogether. (Cf. Eriksson and Hájek 2007.) As Christensen rhetorically asks, 'How plausible is it, after all, that the intellectual defect exemplified by an agent's being more confident in P than in $(P \vee Q)$ is, at bottom, a defect in the agent's *preferences?*' (453).

4.3 'Depragmatized' Dutch Book arguments

Such considerations lead Christensen to offer an alternative interpretation of Dutch Books (1996, 2001). Firstly, he insists that the relationship of credences to preferences is normative: degrees of belief *sanction as fair* certain corresponding bets. Secondly, he restricts attention to what he calls 'simple agents', ones who value only money, and do so linearly. He argues that if a simple agent's beliefs sanction as fair each of a set of betting odds, and that set allows construction of a set of bets whose payoffs are logically guaranteed to leave him monetarily worse off, then the agent's beliefs are rationally defective. He then generalizes this lesson to all rational agents.

Vineberg (1997) criticizes the notion of 'sanctioning as fair' as vague and argues that various ways of precisifying it render the argument preference-based after all. Howson and Urbach (1993) present a somewhat similar argument to Christensen's—although without its notion of 'simple agents'—cast in terms of a Dutch Bookable agent's inconsistent beliefs about subjectively fair odds. Vineberg levels similar criticisms against their argument. See also Maher (1997) for further objections to Christensen's argument, Christensen's (2004) revised version of it, and Maher's (2006) critique of that version.

5. Diachronic Dutch Book arguments

The Dutch Book arguments that we have discussed are *synchronic*—all the bets are placed at the same time. *Diachronic* Dutch Book, or *Dutch strategy,* arguments are an important class in which the bets are spread across at least two times.

5.1 Conditionalization

Suppose that initially you have credences given by a probability function $P_{initial}$, and that you become certain of E (where E is the strongest such proposition). What should be your new probability function P_{new}? The favored updating rule among Bayesians is conditionalization; P_{new} is related to $P_{initial}$ as follows:

(Conditionalization) $P_{new}(X) = P_{initial}(X|E)$ (provided $P_{initial}(E) > 0$).

The Dutch Book argument for conditionalization begins by assuming that you are committed to following a *policy* for updating—a function that takes as inputs your initial credence function, and the member of some partition of possible evidence propositions that you learn, and that outputs a new probability function. It is further assumed that this rule is known by the bookie (although even if it isn't, the bookie could presumably place the necessary bets in any case, perhaps by luck). The diachronic Dutch Book *theorem*, due to Lewis (1999), states that if your updating rule is anything other than conditionalization, you are susceptible to a diachronic Dutch Book. (Your updating policy is codified in the conditional bets that you take.) The *argument* continues that such susceptibility is irrational; thus, rationality requires you to update by conditionalizing. As usual, a converse theorem is needed to complete the argument; Skyrms (1987b) provides it.

5.2 Objections

Many of the objections to synchronic Dutch Book arguments reappear, some with extra force; and some new objections arise. Indeed, the conclusion of this argument is not as widely endorsed as is the conclusion of the classic synchronic Dutch Book argument (namely, probabilism). There are thus authors who think that the diachronic argument, unlike the synchronic argument, proves too much, citing various cases in which one is putatively *not* required to conditionalize. Arntzenius (2003), Bacchus, Kyburg and Thalos (1990), and Bradley (2005) offer some.

Christensen (1996) argues that much as degrees of belief should be distinguished from corresponding betting prices (as we saw in §3), having a particular updating rule must be distinguished from corresponding conditional betting prices. The objection that 'the agent will see the Dutch Book coming' has also been pursued with renewed vigor in the diachronic setting. Developing an argument by Levi (1987), Maher (1992) offers an analysis of the game tree that unfolds between the bettor and the bookie. Skyrms (1993) gives a rebuttal, showing how the bookie can ensure that the bettor loses nevertheless. Maher (1993, section 5.1.3) replies by distinguishing between accepting a sure loss and choosing a dominated act, and he argues that only the latter is irrational.

The package principle faces further pressure. Since there must be a time lag between a pair of the diachronic Dutch Book bets, the later one is placed in the context of a changed world and must be evaluated in that context. It is clearly permissible to revise your betting prices when you know that the world has changed since you initially posted those prices. The subsequent debate centres on just how much is built into the *commitments* you incur in virtue of having the belief revision policy that you do.

Then there are objections that have no analogue in the synchronic setting. Unlike the synchronic arguments, the diachronic argument for conditionalization makes a specific assumption about how the agent interacts with the world, and that learning takes place by acquiring new *certainties.* But need evidence be so authoritative? Jeffrey (1965) generalizes conditionalizing to allow for less decisive learning experiences in which your probabilities across a partition $\{E_1, E_2,...\}$ change to $\{P_{new}(E_1), P_{new}(E_2), ..., \}$, where none of these values need be 0 or 1:

(Jeffrey conditionalization) $P_{new}(X) = \sum_i P_{initial}(X|E_i)P_{new}(E_i)$

Jeffrey conditionalization is again supported by a Dutch Book and converse Dutch Book theorem (although some further assumptions are involved; see Armendt 1980, Skyrms 1987b). Lewis insists that the ideally rational agent's learning episodes *do* come in the form of new certainties; he regards Jeffrey conditionalization as a fall-back rule for less-than-ideal agents. Rationality for Lewis thus involves more than just appropriately responding to evidence in the formation of one's beliefs; more tendentiously, it also involves the nature of that evidence itself. And it requires a commitment to some rule for belief revision. van Fraassen (1989) disputes this. There is even controversy over what it is to follow a rule in the first place (Kripke 1982), which had no analogue in the synchronic argument. Note, however, that an agent who fails to conditionalize is surely susceptible to a Dutch Book *whether or not she follows some rival rule.* A bookie could diachronically Dutch Book her by accident, rather than by strategically exploiting her use of such a rule—even if the bookie merely stumbles upon the appropriate bets, they do still guarantee her loss.

How does the interpretation that Dutch Books dramatize evaluational inconsistencies fare in the diachronic setting? Christensen (1991) contends that there need be no irrationality in an agent's evaluations at different times being inconsistent with each other, much as there is no irrationality in a husband and wife having evaluations inconsistent with each other (thereby exposing them jointly to a Dutch Book). He offers a synchronic Dutch Book argument for conditionalization, appealing again to the idea that credences *sanction as fair* the relevant betting prices. See Vineberg (1997) for criticisms.

van Fraassen (1984) gives a diachronic Dutch Book argument for the *Reflection Principle,* the constraint that an ideally rational agent's credences mesh with her expected future credences according to:

$P_t(X \mid P_{t'}(X) = x) = x$, for all X and for all x such that $P_t(P_{t'}(X) = x) > 0$,

where P_t is the agent's probability function at time t, and $P_{t'}$ is her function at later time t'. Various authors (e.g. Christensen 1991, Howson and Urbach 1993) find conditionalization plausible but the Reflection Principle implausible; and various authors find all the more that the argument for Reflection Principle proves too much.

Suppose that you violate one of the axioms of probability—say, additivity. Then by the Dutch Book theorem, you are Dutch Bookable. Suppose further that you obey conditionalization. Then by the converse Dutch Book theorem for conditionalization, you are not Dutch Bookable. So you both are and are not Dutch Bookable—contradiction? Something has gone wrong. Presumably, these theorems need to have certain 'ceteris paribus' clauses built in, although it is not obvious how they should be spelled out exactly.

More generally, the problem is that there are Dutch Book arguments for various norms—we have considered the norms of obeying the probability calculus, the Reflection Principle, updating by conditionalization, and updating by Jeffrey conditionalization. For a given norm N, the argument requires both a Dutch Book theorem:

> if you violate N, then you are susceptible to a Dutch Book

and a converse Dutch Book theorem:

> if you obey N, then you are immune to a Dutch Book.

But the latter theorem must have a ceteris paribus clause to the effect that you obey all the other norms. For if you violate, say, norm N', then by *its* Dutch Book theorem you are susceptible to a Dutch Book. So the converse Dutch Book theorem for N as it stands must be false: if you obey N *and violate N'* then you are susceptible to a Dutch Book after all. One might wonder how a theorem could ever render precise the required ceteris paribus clause in all its detail.

This problem only becomes more acute when we pile on still more Dutch Book arguments for still more norms. As we now will.

6. Some more exotic Dutch Book arguments, and recent developments

We have discussed several of the most important Dutch Book arguments, but they are just the tip of the iceberg. In this section we will survey briefly a series of such arguments for more specific or esoteric theses.

6.1 Semi-Dutch Book argument for strict coherence

The first, due to Shimony (1955), is not strictly speaking a Dutch Book argument, but it is related closely enough to merit attention here. Call a *semi-Dutch Book* a set of bets that can at best break even, and that in at least one possible outcome has a net loss. Call an agent *strictly coherent* if she obeys the probability calculus, and assigns $P(H|E)$ = 1 only if *E* entails *H*. (These pieces of terminology are not Shimony's, but they have become standard more recently.) Simplifying his presentation, Shimony essentially shows that if you violate strict coherence, you are susceptible to a semi-Dutch Book. Such susceptibility, moreover, is thought to be irrational, since you risk a loss with no compensating prospect of a gain. Where Dutch Books militate against *strictly* dominated actions (betting according to Dutch Bookable credences), semi-Dutch Books militate against *weakly* dominated actions.

Semi-Dutch Book arguments raise new problems. Strict coherence cannot be straightforwardly added to the package of constraints supported by the previous Dutch Book arguments, since it is incompatible with updating by conditionalization. After all, an agent who conditionalizes on *E* becomes certain of *E* (given any possible condition), despite its not being a tautology. Earman (1992) takes this to reveal a serious internal problem with Bayesianism: a tension between its fondness for Dutch Book arguments on the one hand, and conditionalization on the other. But there is *no* sense, not even analogical, in which semi-Dutch Books dramatize inconsistencies. An agent who violates strict coherence can grant that the outcomes in which she would face a loss are *logically* possible, but she can *consistently* retort that this does not trouble her—after all, she is 100% confident that they will not obtain! Indeed, an omniscient God would be semi-Dutch Bookable, and none the worse for it.

6.2 Imprecise probabilities

Few of our actual probability assignments are precise to infinitely many decimal places; and arguably even ideally rational agents can have *imprecise* probability assignments. Such agents are sometimes modeled with *sets* of precise probability functions (Levi 1974, Jeffrey 1992), or with lower and upper probability functions (Walley 1991). There are natural extensions of the betting interpretation to accommodate imprecise probabilities. For example, we may say that your probability for *X* lies in the interval [p, q] if and only if $*p* is the highest price at which you will buy, and $*q* is the lowest price at which you sell, a bet that pays $1 if *X*, 0 otherwise. (Note that on this interpretation, maximal imprecision over the entire [0, 1] interval regarding everything would immunize you from all Dutch Books—you would never buy a bet with a stake of $1 for more than $0, and never sell it for less than $1, so nobody could ever profit from your betting prices.) C. A. B. Smith (1961) shows that an agent can make lower and upper probability assignments that avoid sure loss but that nevertheless violate probability theory. Thus, the distinctive connection between probability incoherence and Dutch Bookability is cleaved for imprecise probabilities; probabilistic coherence is demoted to a sufficient but not necessary condition for the avoidance of sure loss. Walley (1991) provides Dutch Book arguments for various constraints on upper and lower probabilities.

6.3 'Incompatibilism' about chance and determinism

Call the thesis that determinism is compatible with intermediate objective chances *compatibilism*, and call someone who holds this thesis a *compatibilist.* Schaffer (2007) argues that a compatibilist who knows that some event *E* is determined to occur, and yet who regards the chance of *E* at some time to be less than 1, is susceptible to a Dutch Book.

6.4 Popper's axioms on conditional probability functions

Unlike Kolmogorov, who axiomatized unconditional probability and then defined conditional probability thereafter, Popper (1959) axiomatized *conditional* probability directly. Stalnaker (1970) gives what can be understood as a Dutch Book argument for this axiomatization.

6.5 More infinite books

Suppose that your probability function is not concentrated at finitely many points—this implies that the range of that function is infinite (assuming an infinite state space). It is surely rational for you to have such a probability function; indeed, given the evidence at our disposal, it would surely be *irrational* for us to think that we can rule out, with probability 1, all but finitely many possible ways the world might be. Suppose further that your utility function is unbounded (although your utility for each possible outcome is finite). This too seems to be rationally permissible. McGee (1999) shows that you are susceptible to an infinite Dutch Book (involving a sequence of unconditional and conditional bets). He concludes: 'in situations in which there can be infinitely many bets over an unbounded utility scale, no rational plan of action is available' (257). McGee's argument is different from other Dutch Book arguments in two striking ways. Firstly, it makes a rather strong and even controversial assumption about the agent's *utility* function. Secondly, McGee does not argue for some rationality constraint on a credence function; on the contrary, since the relevant constraint in this case (being concentrated on finitely many points) is implausible, he drives the argument in the opposite direction. The upshot is supposed to be that irrationality is unavoidable. One might argue, on the other hand, that this just shows that Dutch Bookability is not always a sign of irrationality.

The theme of seemingly being punished for one's rationality in situations involving infinitely many choices is pursued further in Barrett and Arntzenius (1999). They imagine a rational agent repeatedly paying \$1 in order to make a more profitable transaction; but after infinitely many such transactions, he has made no total profit on those transactions and has paid an infinite amount. He is better off at every stage acting in an apparently irrational way. For more on this theme, see Arntzenius, Elga and Hawthorne (2004).

6.6 Group Dutch Books

If Jack assigns probability 0.3 to rain tomorrow and Jill assigns 0.4, then you can Dutch Book the pair of them: you buy a dollar bet on rain tomorrow from Jack for 30 cents and sell one to Jill for 40 cents, pocketing 10 cents. Hacking (1975) reports that the idea of guaranteeing a profit by judicious transactions with two agents with different betting odds can be found around the end of the ninth century A.D., in the writings of the Indian mathematician Mahaviracarya. We have already mentioned Christensen's observation of the same point involving a husband and wife. And there are interesting Dutch Books involving a greater number of agents (in e.g. Bovens and Rabinowicz (MS)).

6.7 The Sleeping Beauty problem

Most Dutch Book arguments are intended to support some general constraint on rational agents—*structural* features of their credence (or utility) profiles. We will end with an example of a Dutch Book argument for a very specific constraint: in a particular scenario, a rational agent is putatively required to assign a *particular* credence. The scenario is that of the Sleeping Beauty problem (Elga 2000). Someone is put to sleep, and then woken up either once or twice depending on the outcome of a fair coin toss (Heads: once; Tails: twice). But if she is to be woken up twice, her memory of the first awakening is erased. What probability should she give to Heads at the first awakening? There are numerous arguments for answering ½, and for answering 1/3. Hitchcock (2004) gives a Dutch Book argument for the 1/3 answer. Bradley and Leitgeb (2006) dispute this argument, offering further constraints on what a "Dutch Book" requires in order to reveal any irrationality in an agent.

7. Conclusion

We have seen a striking diversity of Dutch Book arguments. A challenge that remains is to give a *unified* account of them. Is there a single kind of fault that they all illustrate, or is there rather a diversity of faults as well? And if there is a single fault, is it epistemic, or some other kind of fault? The interpretation according to which Dutch Books reveal an inconsistency in an agent's evaluations, for example, is more plausible for some of the Dutch Books than for others—it is surely implausible for McGee's Dutch Book, and for some of the other infinitary books that we have seen. But in those cases, do we really want to say that the irrationality at issue literally concerns monetary losses at the hands of cunning bookies (which in any case is hardly an epistemic fault)?

Or perhaps irrationality comes in many varieties, and it is enough that a Dutch Book exposes it in *some* form or other. But if there are many different ways to be irrational, the *validity* of a Dutch Book argument for any particular principle is threatened. At best, it establishes that an agent who violates that principle is irrational *in one respect.* This falls far short of establishing that the agent is irrational all-things-considered; indeed, it leaves open the possibility that along all the other axes of rationality the agent is doing as well as possible, and even that overall there is nothing better that she could do. Moreover, it is worth emphasizing again that without a corresponding *converse* theorem that one can avoid a Dutch Book by obeying the principle, even the irrationality in that one respect has not been established—unless it is coherent that necessarily *all* agents are irrational in that respect. Dutch Books may reveal a pragmatic vulnerability of some kind, but it is a further step to claim that the vulnerability stems from irrationality.[3] Indeed, as some of the infinitary Dutch Books seem to teach us, some Dutch Books apparently do not evince any irrationality whatsoever. Sometimes your circumstances can be unforgiving through no fault of your own: you are damned whatever you do.[4]

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[3] I thank an anonymous reviewer for putting the point this way.

[4] I thank especially Brad Armendt, Jens-Christian Bjerring, Darren Bradley, Rachael Briggs, Andy Egan, Branden Fitelson, Carrie Jenkins, Stephan Leuenberger, Isaac Levi, Aidan Lyon, Patrick Maher, John Matthewson, Peter Menzies, Daniel Nolan, Darrell Rowbottom, Wolfgang Schwarz, Teddy Seidenfeld, Michael Smithson, Katie Steele, Michael Titelbaum, Susan Vineberg, and Weng Hong Tang, whose helpful comments led to improvements in this article.

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REFERENCES

Adams, Ernest, 'On Rational Betting Systems', *Archive für Mathematische Logik und Grundlagenforschung*, 6, 7-29, (1962), 112-28.

Armendt, Brad, 'Is There a Dutch Book Argument for Probability Kinematics?', *Philosophy of Science*, 47, No. 4, (1980), December, 583-88.

Armendt, Brad, 'Dutch Books, Additivity and Utility Theory', *Philosophical Topics*, 21, No. 1, (1993), 1-20.

Arntzenius, Frank, 'Some Problems for Conditionalization and Reflection', *Journal of Philosophy* 100, 7 (2003), 356-71.

Arntzenius, Frank, Adam Elga, and John Hawthorne, 'Bayesianism, Infinite Decisions, and Binding', *Mind* 113, (2004), 251-83.

Bacchus, F, H. E. Kyburg, and M. Thalos, 'Against Conditionalization', *Synthese* 85, (1990), 475-506.

Barrett, Jeff and Frank Arntzenius, 'An Infinite Decision Puzzle', *Theory and Decision* 46, (1999), 101-3.

Bayes, Thomas, 'An Essay Towards Solving a Problem in the Doctrine of Chances', *Philosophical Transactions of the Royal Society of London* 53, (1764), 370-418.

Bovens, Luc and Wlodek Rabinowicz, 'Dutch Books, Group Decision-Making, the Tragedy of the Commons and Strategic Jury Voting' (MS).

Bradley, Darren and Hannes Leitgeb, 'When Betting Odds and Credences Come Apart:

More Worries for Dutch Book Arguments', *Analysis* Vol. 66, No. 2, (2006), 119-27.

Bradley, Richard, 'Radical Probabilism and Bayesian Conditioning', *Philosophy of Science* 72, No. 2, (2005), 334-64.

Christensen, David, 'Clever Bookies and Coherent Beliefs.' *The Philosophical Review* C, No. 2, (1991), 229-47.

Christensen, David, 'Dutch-Book Arguments De-pragmatized: Epistemic Consistency for Partial Believers', *The Journal of Philosophy*, (1996), 450-79.

Christensen, David, 'Preference-Based Arguments for Probabilism' *Philosophy of Science* 68 (3), (2001), 356-76.

Christensen, David, *Putting Logic In Its Place: Formal Constraints on Rational Belief* (Oxford: Oxford University Press, 2004).

de Finetti, B. 'La Prévision: Ses Lois Logiques, Ses Sources Subjectives', *Annales de l'Institut Henri Poincaré*, 7, (1937), 1-68; translated as 'Foresight. Its Logical Laws, Its Subjective Sources', in *Studies in Subjective Probability*, H. E. Kyburg, Jr. and H. E. Smokler (eds.), Robert E. Krieger Publishing Company (1980).

Earman, John, *Bayes or Bust?* (Cambridge, MA: MIT Press, 1992).

Elga, Adam, 'Self-Locating Belief and the Sleeping Beauty Problem', *Analysis* Vol. 60, No. 2, (2000), 143-47.

Eriksson, Lina, and Alan Hájek, 'What Are Degrees of Belief?', *Studia Logica* 86, Special Issue Formal Epistemology I, Branden Fitelson (ed.), (2007), 183-213.

Hacking, Ian, *The Emergence of Probability: A Philosophical Study of Early Ideas About Probability, Induction and Statistical Inference* (Cambridge: Cambridge

University Press, 1975).

Hájek, Alan, 'Scotching Dutch Books?', *Philosophical Perspectives* 19 (issue on Epistemology), John Hawthorne (ed.), (2005), 139-51.

Hitchcock, Christopher 'Beauty and the Bets', *Synthese* Vol. 139, No. 3 (2004), April, 405-20.

Howson, C. and P. Urbach, *Scientific Reasoning: The Bayesian Approach*. La Salle, Illinois, Open Court, (1993).

Howson, Colin, 'Theories of Probability', *British Journal for the Philosophy of Science* 46 (1995), 1–32.

Jeffrey, Richard C., *The Logic of Decision* (Chicago: Chicago University Press, 1965).

Jeffrey, Richard C., 'Bayesianism With a Human Face', in *Probability and the Art of Judgment* (Cambridge: Cambridge University Press, 1992).

Kennedy, Ralph and Charles Chihara, 'The Dutch Book Argument: Its Logical Flaws, Its Subjective Sources', *Philosophical Studies* 36, (1979), 19-33.

Kemeny, J., 'Fair Bets and Inductive Probabilities', *Journal of Symbolic Logic,* 20: (1955), 263-73.

Keynes, John Maynard, *A Treatise on Probability* (London: Macmillan and Co., Limited, 1920).

Kolmogorov, Andrei. N., *Grundbegriffe der Wahrscheinlichkeitrechnung, Ergebnisse Der Mathematik*, (1933). Translated as *Foundations of Probability* (New York: Chelsea Publishing Company, 1956).

Kripke, Saul, *Wittgenstein on Rules and Private Language: An Elementary Exposition* (Cambridge, MA: Harvard University Press, 1982).

Kyburg, Henry, 'Subjective Probability: Criticisms, Reflections and Problems', *Journal* *of Philosophical Logic* 7, (1978), 157-80.

Lehman, R. Sherman, 'On Confirmation and Rational Betting', *The Journal of Symbolic Logic,* Vol. 20, No. 3, (1955), September, 251-62.

Levi, Isaac, 'On Indeterminate Probabilities', *Journal of Philosophy* 71, (1974), 391-418.

Levi, Isaac, 'The Demons of Decision', *The Monist* 70, (1987), 193-211.

Lewis, David, *Papers in Metaphysics and Epistemology* (Cambridge: Cambridge University Press, 1999).

Maher, Patrick, 'Diachronic Rationality', *Philosophy of Science* 59, (1992), 120-41.

Maher, Patrick, *Betting on Theories*, (Cambridge: Cambridge University Press, 1993).

Maher, Patrick, 'Depragmatized Dutch Book Arguments', *Philosophy of Science* 64, (1997), 291-305.

Maher, Patrick, Book Review of David Christensen's *Putting Logic In Its Place*, *Notre Dame Journal of Formal Logic* 47 (2006), 133-149.

McGee, Vann, 'An Airtight Dutch Book', *Analysis* Vol. 59, No. 264, (1999), October, 257-65.

Popper, Karl, *The Logic of Scientific Discovery* (Basic Books, 1959).

Ramsey, F. P., 'Truth and Probability', in *Foundations of Mathematics and other Essays,* R. B. Braithwaite (ed.), (Routledge & P. Kegan, 1931), 156-98; reprinted in *Studies in Subjective Probability*, H. E. Kyburg, Jr. and H. E. Smokler (eds.), 2nd ed., (R. E. Krieger Publishing Company, 1980), 23-52; reprinted in *F. P. Ramsey: Philosophical Papers*, D. H. Mellor (ed.) (Cambridge: University Press, Cambridge, 1990).

Schaffer, Jonathan, 'Deterministic Chance?', *British Journal of the Philosophy of Science* 58, (2007), 113-40.

Schick, Frederic, 'Dutch Bookies and Money Pumps', *Journal of Philosophy* 83, (1986), 112-19.

Seidenfeld, Teddy, *International Encyclopedia of the Social and Behavioral Sciences*, N. J. Smelser and P. B. Baltes (eds.), (Oxford: Elsevier, 2001).

Seidenfeld, Teddy and Mark J. Schervish, 'A Conflict Between Finite Additivity and Avoiding Dutch Book', *Philosophy of Science* 50, (1983), 398-412.

Shimony, A., 'Coherence and the Axioms of Confirmation', *Journal of Symbolic Logic* 20, (1955), 1-28.

Skyrms, Brian, 'Higher Order Degrees of Belief', in *Prospects for Pragmatism*, ed. D. H. Mellor (ed.) (Cambridge: Cambridge University Press, 1980), 109-37.

Skyrms, Brian, *Pragmatics and Empiricism* (New Haven: Yale University, 1984).

Skyrms, Brian, 'Coherence', in N. Rescher (ed.), *Scientific Inquiry in Philosophical Perspective*, Pittsburgh: University of Pittsburgh Press, (1987a), 225-42.

Skyrms, Brian, 'Dynamic Coherence and Probability Kinematics', *Philosophy of Science,* Vol. 54, No. 1 (March), (1987b), 1-20.

Skyrms, Brian, 'A Mistake in Dynamic Coherence Arguments?', *Philosophy of Science* 60, (1993), 320-28.

Smith, Cedric A. B., 'Consistency in Statistical Inference and Decision', *Journal of the Royal Statistical Society* B, 23, (1961), 1-25.

Stalnaker, Robert, 'Probability and Conditionals', *Philosophy of Science* 37, (1970), 64–80.

Stalnaker, Robert, *Inquiry* (Cambridge, MA: MIT Press, 1984).

van Fraassen, Bas, 'Belief and the Will', *Journal of Philosophy* 81, (1984), 235-56.

van Fraassen, B., *Laws and Symmetry* (Oxford: Clarendon Press, 1989).

Vineberg, Susan, 'Dutch Books, Dutch Strategies and What They Show About Rationality, *Philosophical Studies* 86, No. 2, (1997), 185-201.

Vineberg, Susan, 'The Notion of Consistency for Partial Belief', *Philosophical Studies* 102 (February), (2001), 281-96.

Walley, Peter, *Statistical Reasoning With Imprecise Probabilities* (Chapman and Hall, 1991).

Weatherson, Brian, 'Classical and Intuitionistic Probability', *Notre Dame Journal of Formal Logic* 44, (2003), 111-23.

Williamson, Jon, 'Countable Additivity and Subjective Probability', *The British Journal for the Philosophy of Science* 50, No. 3, (1999), 401-16.