- *"All three could easily replicate themselves creating a cascade effect that could sweep through the physical world. . . . It is no exaggeration to say we are on the cusp of the further perfection of extreme evil."*

"The cusp of the further perfection of extreme evil"? Jill Kiljoy has no idea what *that* means, but it sounds like very, very bad news. So she tries to construct an argument from dominance to show that we should stop work on robotics, genetic engineering, and nanotechnology, now.

Present Jill K.'s argument briefly and precisely, with utility assignments and a simple partition of possibilities. Include a decision table. Then present general criticisms of the argument.

KEY WORDS FOR REVIEW

Decision under uncertainty
Pascal's wager
Partition
Live possibilities
Dominance
Causal influence
Dominance rule
Dominant expected value rule

11 What Do You Mean?

The idea of probability leads in two different directions: **belief** and **frequency**. Probability makes us think of the degree to which we can be confident of something uncertain, given what we know or can find out. Probability also makes us think of the relative frequency with which some outcome occurs on repeated trials on a chance setup.

Thus far we have used both ideas almost interchangeably, because the basic rules for calculating with each are virtually identical. But now we have to distinguish between them, because the philosophical and practical uses of these two ideas are very different. The distinction is essential for the rest of this book (and for all clear thinking about probability).

We have been doing all these calculations about probabilities, and have not said a word about what we mean by "probability" itself. Now we are going to set things right. Up to now it has not mattered a great deal what we mean by the word. From now on it will make all the difference.

This chapter is an example of one kind of philosophy, often called analytic philosophy. We will try to come to grips with different concepts associated with the idea of probability. Many students find this chapter the hardest one of all. Not surprising! The distinctions that we have to make have bedeviled probability theorists—including some of the very best—for more than 200 years. Debates between experts who take the frequency approach, and those who take the belief approach, continue to be acrimonious. *In this chapter and the next one, you will have to make up your own mind about where you stand on these issues.*

A BIASED COIN

In ordinary language we use the word "probability" in different ways. We say things like this:

(1) This coin is biased toward heads. The probability of getting heads is about 0.6.

We take for granted that (1) is a statement about the coin. It implicitly refers to some method of tossing in a definite chance setup. Here are five remarks about (1):

a Statement (1) is either true or false, regardless of what we know about the coin. If (1) is true, it is because of how the world is, especially how the coin and tossing device are.
b If (1) is true, we suppose that the coin is asymmetrical. (Or maybe there is something unusual about the tossing device.)
c We imagine that someone could explain (1) by facts drawn from the geometry of the coin, or the laws of physics.
d We could do experiments to test (1). We conduct many trials on the chance setup, and observe the relative frequency of heads. If we get 63 heads irregularly distributed in 100 tosses, we are ready to accept (1) as true. But if we get only 37 heads, we are very dubious about (1).
e In short: (1) states a fact about how the world is, and we can collect evidence to see whether (1) is true or false.

Notice that we could say very much the same thing about radioactive decay. If you put a Geiger counter next to a small piece of radium, you hear clicks every time a product of radioactive decay passes through the detector. Holding the counter four feet away from the source, you will notice a random sequence of clicks. This is a chance setup, and you might find out that:

The probability of getting a click in any given three-second interval is 0.6.

a This statement is true or false, regardless of what we know about the radium and the detector.
c We imagine that someone could explain it by facts drawn from the way the counter is made, facts about the sample of radium, and the laws of physics.
d We could do experiments to test the statement. We observe the relative frequency of 3-second periods in which there are clicks.
e In short: the statement states a fact about how the world is, and we can collect evidence to see whether it is true or false.

More sophisticated statements about the half-life of radium, for example, are to be understood in the same way.

THE EXTINCT DINOSAURS

We also say things like this:

(2) It is probable that the dinosaurs were made extinct by a giant asteroid hitting the Earth.

We can add details and precision to (2), as in this conversation from the TV program "The Science of Yesterday":

Science journalist *Betty Glossop*: There is a lot of new evidence about a layer of iridium deposits in many parts of the Earth. Geologists have identified them as contemporary with the extinction of the dinosaurs.
The interviewer *Joe Penchant*: What's that got to do with it?
Betty: Iridium is an uncommon element that is the most corrosion-resistant substance found in nature. We have identified an asteroid crater rich in iridium, presumably from the asteroid. We think that it produced a gigantic cloud of dust, including iridium, that covered the earth. Plants needed for vegetarian dinosaurs, and for the prey of carnivorous ones, simply didn't grow well and the dinosaurs starved to death.
Joe: So how probable is it, in the light of all this new information, that it was an asteroid that killed off the dinosaurs?
(3) *Betty*: Taking all the evidence into consideration, the probability is about 90%.

The word "probability" seems to be used differently in (1) and (3). In (3), *Betty Glossop* is talking about the probability of a ***proposition***, namely:

(4) The dinosaurs were made extinct by a giant asteroid hitting the Earth.

Let us first of all look just at (3), where *Betty* refers to her evidence about iridium and so on. Let's make some contrasts, item by item, between (3) and points a–e about (1), the biased coin. Read the following statements, and the previous one, very carefully. See if you agree with every single statement.

First, review exactly what (1), (2), (3), and (4) are. Notice that (4) is not a probability-statement at all. Notice that only (3) mentions evidence.

DINOSAURS AND PROBABILITY

a Statements (1) and (4) [but not (3)] are similar in one respect. Statement (4), like (1), is either true or false, regardless of what we know about the dinosaurs. If (4) is true, it is because of how the world is, especially what happened at the end of the dinosaur era. If (3) is true, it is not true because of "how the world is," but because of how well the evidence supports statement (4).
b If (3) is true, it is because of inductive logic, not because of how the world is.
c The evidence ***mentioned*** in (3) will go back to laws of physics (iridium), geology (the asteroid), geophysics, climatology, and biology. But these special sciences do not explain why (3) is true. Statement (3) states a relation between the evidence provided by these special sciences, and statement (4), about dinosaurs.
d We can do experiments to test the claims about iridium and so on. But we cannot do experiments to test (3). Notice that the tests of (1) may involve

repeated tosses of the coin. But *it makes no sense at all* to talk about repeatedly testing (3).

e In short: statement (3) makes a claim about how the evidence supports statement (4).

ON NOT MENTIONING EVIDENCE

We have bypassed (2), which is like (3), but does not mention the evidence. To make (2) look more like (1) and (3), we will put a number on that "probable":

(2.1) The probability that the dinosaurs were made extinct by a giant asteroid hitting the Earth is very high—about 0.9.

Statement (2.1) is different from (3), because it does not mention evidence. But in some ways it is like (3). Point d is one way they are alike:

d We can do experiments to test the claims about iridium and so on, which might lead someone to assert (2.1). But we cannot do experiments to test (2.1). Notice that the tests of (1) may involve repeated tosses of the coin. But *it makes no sense at all* to talk about repeatedly testing (2.1).

Unfortunately, there are at least two ways to understand (2.1). We may think that (2.1) is "really" just short for (3). When people say that so and so is probable, they mean that relative to the available evidence, so and so is probable. We will call this the *interpersonal/evidential* way to understand (2.1). We will call the other way to understand (2.1) the *personal* way.

INTERPERSONAL/EVIDENTIAL

On this way of understanding (2.1), it is short for:

(2.2) Relative to the available evidence, the probability that the dinosaurs were made extinct by a giant asteroid hitting the Earth is very high—about 0.9.

We could say of someone who long-windedly states (2.2):

- She is taking for granted that any *reasonable* person who thought about the evidence, would find it very *reasonable to think that* an asteroid wiped out the dinosaurs.
- She thinks that it is *rational* to be pretty *confident* that the asteroids caused the extinction of the dinosaurs.
- She thinks that to say (2.1) is to mean something like (2.2)—or *Betty Glossop*'s less long-winded (3).
- She thinks that (2.2) is *interpersonal*—because it is about what it is reasonable for any reasonable person to believe. And since the degree of belief should depend on the available *evidence*, we call this *interpersonal/evidential*.

- Since she thinks that (2.2) is interpersonal, and about rational degrees of belief, she thinks that (2.2) is "objective."

SUBJECTIVE/OBJECTIVE—NOT

If you read a lot about probability, you will often read about "objective" and "subjective" probabilities. These are terrible terms, loaded with ideology. "I'm objective, you're subjective, he is prejudiced." How often have you heard this sort of conversation?

JAMES: That's just your subjective opinion.
MARY: Nonsense, it is an objective fact.

How often have *you* talked just like that?

Don't get into that rut, in probability or in the rest of your life. *James* and *Mary* are not arguing, they are just slinging mud at each other.

But do notice that both (1), about the bias and the die, and (2.2) or (3), can be called "objective"—although for quite different reasons. Statement (1) is called "objective" because it is a statement about how the world is. Statement (2.2) is called "objective" because of a supposed logical relation between the evidence and a proposition [namely, (4)].

PERSONAL DEGREE OF BELIEF

There is another way to understand (2). When someone says (2), they may mean only something about themselves, something like:

(2.3) I personally am very confident that the dinosaurs were made extinct by a giant asteroid hitting the Earth.

Or even this way to understand (2.1):

(2.4) If I had to make a bet on it, I would bet 9 to 1 that the dinosaurs were made extinct by a giant asteroid hitting the Earth.

TIME TO THINK ABOUT YOURSELF

What do *you* mean, when you say things like (2), "It is probable that the dinosaurs were made extinct by a giant asteroid hitting the Earth"?

What do you mean when you say things like:

It will probably rain today.
It is probable that I will flunk my geology test; I just can't tell those stupid rock samples one from the other.
The probability that there was a second gunman involved in the John F. Kennedy assassination is negligible.

In all probability, we are going through a period of extreme global warming due to burning fossil fuels, aerosols, methane produced by manure from beef cattle grown so we can kill and eat them, and so on.

Despite all the propaganda, the global warming hypothesis is not at all probable; we are just going through a routine climatic cycle.

It is very probable that the company my father works for will be downsized after the takeover, and he will be out of a job.

BELIEF-TYPE

Statement (4) was a proposition about dinosaur extinction; (2)–(2.2) and (3) are about how *credible* (believable) (4) is. They are about the degree to which someone *believes*, or *should believe*, (4). They are about how *confident* one can or should be, in the light of that evidence.

The use of the words "probable" and "probability" in (2)–(2.2) and (3) is related to ideas such as:

belief	confidence
credibility	evidence

We need a name for this family of uses of probability words. Philosophers have used a lot of different names. The easiest to remember is:

BELIEF-TYPE PROBABILITY

This is not the end of the matter. We just saw that belief-type probabilities can be thought of in at least two ways: *interpersonal/evidential* and *personal*.

FREQUENCY-TYPE

Now look at (1) again:

(1) The probability of getting heads with this coin is 0.6.

The truth of this statement seems to have nothing to do with what we believe. We seem to be making a completely factual statement about a material object, namely the coin (and the device for tossing it). We could be simply wrong, whether we know it or not. This might be a fair coin, and we may simply have been misled by the small number of times we tossed it. We are talking about a physical property of the coin, which can be investigated by experiment.

What is this physical property? We may be saying something like:

- In repeated tossing, the *relative frequency* of heads settles down to a stable proportion, 6/10.
- The coin has a *tendency* to come down heads far more often than tails.
- It has a *propensity* or *disposition* to favor heads.
- Or we are saying something more basic about the asymmetry of the coin and tossing device. We may be referring to the *geometry* and *physics* of the coin, which cause it to come down more often heads than tails.

The use of the word "probability" in (1) is related to ideas such as

frequency	disposition
tendency	symmetry
propensity	

We need a name for this family of uses of probability words. Philosophers have used a lot of different names. The easiest to remember is:

FREQUENCY-TYPE PROBABILITY

OTHER NAMES

You won't believe the number of names that philosophers have given to these two groups of uses of the word "probability." In case you look at other books, here is a little dictionary.

- *Subjective/objective.* The oldest pair of words for the belief-type and frequency-type distinction is "subjective" (belief-type) and "objective" (frequency-type). Objection: why say that the long statement (3), referring to the available evidence about the iridium layer and so forth, is "subjective"? Many scientists would claim that it is an "objective" assessment of the evidence.
- *It's all Greek to me.* Belief-type probabilities have been called "epistemic"—from *episteme*, a Greek word for knowledge. Frequency-type probabilities have been called "aleatory," from *alea*, a Latin word for games of chance, which provide clear examples of frequency-type probabilities. Objection: these words have never caught on. And it is much easier for most of us to remember plain English words rather than fancy Greek and Latin ones.
- *Number 1 and number 2.* The philosopher and logician Rudolf Carnap (1891–1970) called belief-type probability "probability$_1$," and frequency-type probability "probability$_2$." Objection: Carnap's proposal never caught on. And it is hard to remember which is number 1 and which is number 2.

Many other labels for the two groups of probability ideas have been used, but that is enough for now. Now let us look at some of our own examples earlier in

the book. Where did we implicitly think of frequency-type probability? Where did we implicitly think of belief-type probability?

SHOCKS

Page 52 gave some data about two suppliers of shock absorbers, Bolt & Co. and Acme Inc. We began with the information that:

> Bolt supplies 40% of the shock absorbers for a manufacturer, and Acme 60%.
> We took the probability of a randomly selected shock absorber being made by Bolt to be 0.4.
> Of Acme's shocks, 96% test reliable. But Bolt has been having some problems on the line, and recently only 72% of Bolt's shock absorbers have tested reliable.

We asked:

> What is the probability that a randomly chosen shock absorber will test reliable? The probability worked out to be 0.864.
> What is the conditional probability that a randomly chosen shock absorber, which is tested and found to be reliable, is made by Bolt? The answer was ⅓.

Here it is natural to take a frequency perspective.

We are talking about a mass-produced product from an assembly line. We observe the relative frequency of defective and reliable products from the two lines, one at Bolt, one at Acme. These frequencies must reflect a difference between the two companies.

STREP THROAT

Page 75 discussed Odd Question 6, about strep throat. You were asked to imagine that you were a physician examining a patient, and sending swabs to the lab for testing. We said,

> You think it likely that the patient has strep throat. Let us, to get a sense of the problem, put a number to this, the probability is 90% that the patient has strep throat. Pr(S) = 0.9.

You, as physician, were asked to reach a conclusion on the basis of seemingly inconsistent reports from the lab: 3 positive tests and 2 negative ones. You concluded,

> It is very much more likely than not, that the patient does have strep throat.
> The probability that the patient has strep throat, given the data, is 343/344, or 0.997.

Then we moved to another case, called "sheer ignorance." We said that the ignorant person might say,

> It is 50–50 whether this patient has strep throat.
> The probability that the patient has strep throat is 0.5.
> That is, Pr(S) = 0.5.
> You computed your probability of strep throat, in the light of the news from the lab, as the rather surprising 343/352, or 0.974.

In this example we do have some frequency data, namely the probability of false positives in the lab test. Nevertheless, we are plainly talking about the beliefs of the physician or the ignorant amateur, both before and after getting the lab results.

In this case it is natural to take a belief perspective.

Notice that the problem about shock absorbers and the problem about strep throat can be solved in exactly the same way, using Bayes' Rule. The formal, logical, arithmetical problem is the same, but the meaning is somewhat different.

STATING THAT, AND REASONS FOR

Frequency-type probability statements state *how the world is*. They state, for example, a physical property about a coin and tossing device, or the production practices of Acme and Bolt.

Belief-type probability statements express a person's confidence in a belief, or state the credibility of a conjecture or proposition in the light of evidence.

Beware of a confusion that troubles a lot of students.

If I state a matter of fact, you expect me to believe what I state. You expect me to have some reasons for thinking my belief is true. You expect me to be able to give you my reasons.

So some students think: every statement about how the world is, states reasons and beliefs. NO!

A statement about how the world is, is a statement made (we hope) because a person has some beliefs and some reasons for them. But what it *says* is not, "I have reasons for my belief that *p*." What it says is, "This is how the world is: *p*."

Distinguish:

- What a person says (what a proposition states).
- The reasons that a person may have, for stating or believing some proposition.

Actually, people often do not have reasons for what they say. Sometimes they do not even believe what they say. A person may say "how the world is" in many different circumstances. The person may:

- Have excellent reasons.
- Merely hope the statement is true but have no reasons.

- Be lying—believe that the proposition is false, have reasons for thinking so, but want to misinform someone.

THE SINGLE CASE

Frequency-type probabilities usually are about items of some kind that occur in a sequence: spins of a wheel, shock absorbers produced by a manufacturer.

It does not make sense to speak of the "frequency" of a single event. A patient either has, or has not, got strep throat. In the taxicab problem, Odd Question 5, either a blue cab sideswiped another car, or a green cab did it. As stated, these problems involve a single event, a single car, a single patient, a single case.

Such probabilities cannot literally be understood as a frequency. They cannot be understood as tendencies or propensities either.

In the taxicab story, the *witness* has a tendency or propensity or disposition to make a correct identification of cab color on a misty night. She gets things right 80% of the time. That is a frequency. But there is not a tendency on the part of the *sideswiper* to be green or blue. It was green. Or it was blue.

> If someone speaks of the probability of a single event, then they must be taking the belief perspective.

Sounds simple, but beware. *Dean* tosses a coin, and it falls on the table in front of me. Before *Dean* or anyone else has a chance to observe the outcome, he slams a book down on top of the coin.

> DEAN: What is the probability that the coin under the book is heads up?
> BEANO: 60%
> DEAN: You mean that the probability is 0.6, that the coin under the book is heads up?
> BEANO: Yes.
> DEAN: Why do you say that?
> BEANO: I thought you were tossing that biased coin you discussed in (1) at the start of this chapter. I thought that this coin is biased towards heads, and the probability of getting heads is about 0.6.

In this case, *Beano* is making a belief-type statement about *this* toss of *this* coin, a single case. He has a *reason* for this belief-type statement, namely a frequency-type statement about the coin.

SWITCHING BACK AND FORTH

We use one word, "probability," from both a frequency and a belief perspective. That is no accident. We switch back and forth between the two perspectives. We computed that:

> The conditional probability is 1/3 that a shock absorber is made by Bolt, if it has been tested at random and found to be reliable.

That is a statement about the production characteristics of shock absorbers bought by the automobile manufacturer. Tomorrow, *Rosie the Riveter*, the quality control engineer, tests a shock absorber at random from a batch of shocks. She finds that it is reliable.

> What is the probability that *this* shock absorber was made by Bolt?

Rosie knows the answer: the probability is 1/3.

But this probability is about a single case. There is no frequency with which this shock, or this batch, is made by Bolt. It is, or it is not.

Rosie made a belief-type statement. Her reason for making it was her knowledge of the relative frequencies with which randomly selected shock absorbers, in this setup, are reliable.

So *Rosie* has switched from a frequency perspective to a belief perspective.

THE FREQUENCY PRINCIPLE

We switch perspectives by a rule of thumb, which has been called the *frequency principle*. It connects belief-type and frequency-type probabilities.

It is a rule about *knowledge* and *ignorance*. Suppose that:

- You *know* the frequency-type probability of an event on trials of some kind.
- You are *ignorant* of anything else about the outcome of a single trial of that kind.

Then you take the frequency-type probability as the belief-type probability of the single case.

We could call it a *know-almost-nothing-but-the-frequency* rule. Here is a very pedantic way to state the frequency principle:

> If S is an individual event of type E,
> and the *only* information about whether E occurred on a trial of a certain kind, on a certain chance setup, is that on trials of that kind on that setup the frequency-type probability Pr(E) = p,
> then the belief-type probability of S is also p.

RELEVANT SUBSETS

The frequency principle is a rule of thumb. When is a frequency-type probability absolutely "all" that we know about the occurrence of an event? Only in artificial situations, as when we toss a fair coin and hide the result. Nevertheless, in real life we quite often have "something like" this situation.

Usually we have a lot of not-very-tidy information. The weather office took the smallest subset of weeks like the week just ended, and worked out the tendency of such weeks to be followed by precipitation.

When we implicitly use the frequency principle, we often make a judgment

of relevance like that. Recall Tomer, the heavy smoker, from page 120. His friend Peggy was told by the statistician that:

> The probability of a male of about Tomer's age (and otherwise like Tomer) dying before age sixty-five, given that he smokes, is 0.36.

The statistician chose a *relevant subset* of males, chiefly by age, but also with a view to getting a trustworthy statement of frequency-type probabilities. Real-life application of the frequency principle requires a lot of judgment.

PROBABILITIES OF PROBABILITIES

Back to the gambler's fallacy. *Alert Learner* (page 31) noticed that the wheel had stopped at black on twelve spins in a row. She suspected bias. After doing some more experiments on the wheel, suppose she concludes that the wheel is heavily biased toward black:

$$0.91 \leqslant \Pr(B) \leqslant 0.93.$$

She intends this to be understood as a statement about the properties of the wheel. Because of the way in which the wheel is made and spun, it stops at black about 92% of the time.

This is a risky conclusion. *Alert Learner* may still feel confident enough to say,

> It is very probable that the probability of black is close to 0.92.

She might even try something more precise-sounding: "The probability that $0.91 \leqslant \Pr(B) \leqslant 0.93$ is at least 95%."

Apparently *Alert Learner* is expressing a belief-type probability of a frequency-type probability. How on earth can she do that? Easy. It is a matter of fact, whether the probability of getting black is between 0.91 and 0.93 or not. And we can discuss the probability of that being true. A probability (belief-type) of a probability (frequency-type).

EXERCISES

1 *Shock absorbers again.* (a) In the shock absorber example, Acme had a better record of reliability than Bolt. What might be the causes of this difference? (b) Why are causes relevant to the distinction between frequency-type and belief-type probabilities?

Classify each of the following statements in italics as frequency-type or belief-type. If you think that one or more could be understood as either frequency-type or belief-type, explain why.

2 *Influenza.* These are all quotations from a newspaper story.

(a) *When a flu epidemic strikes, the probability that a person who was exposed will get sick is between 10% and 15%.*

(b) *The disease is likely to run its course in 5 to 7 days in healthy young people.*

(c) But *it is far more probable that the flu will last for weeks in an old person than in a healthy young person.*

(d) Making a vaccine is always a guessing game, because there is no way to predict what flu strains will appear. But researchers can get a good idea by detecting what strains start cropping up at the end of a previous year. *If a strain appears at the end of the season, there is a good probability that it will make the rounds next year.*

(e) *The probability that the flu vaccine prevents flu in a young healthy person is 70 to 90 percent,* Dr. Gunn said. *The probability is only 50 percent in people over 65.* But, he added, *the probability that it prevents death in an older person is 0.85.*

3 (a) *January 31, 1996*: (from a news story that day): China is determined to abolish the local legislature . . . *It is also probable that it will weaken Hong Kong statutes that protect civil liberties.*

(b) *February 1, 2006*: (from a news story to appear that day): Contrary to recent reports, *it is not probable that the breakaway southern province will choose Hong Kong as its capital city.*

4 *The Fed.* The speaker was the chairman of the Federal Reserve Board (the "Fed") that determines the U.S. money supply and basic interest rates. According to a newspaper story, the chairman said:

(a) *The probability of a recession is less than 50%, in contrast with growing fears of an economic downturn a year ago.*

(b) Recent Fed analysis of leading economic indicators put *the chances of a recession in the next 6 months even lower, at between 10 and 20 percent.*

(c) *I wouldn't bet the ranch on such statistical measures.*

5 *Clones* (from *Nature*, one of the major weekly science journals): Researchers have identified a gene linked with Marfan syndrome—which involves a wide variety of problems, eyesight defects, heart disease and abnormally long limb bones . . . Francesco Ramirez has now cloned the gene for fibrillin and mapped it to a segment of chromosome 15. After studying families with the disorder, he concluded that *the probability of exhibiting Marfan syndrome, given the defective gene, is over 0.7.*

KEY WORDS FOR REVIEW

Frequency	Interpersonal/evidential
Belief	Personal probability
Single case	Propensity
Frequency principle	Relevant subset