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DISCUSSION

*On the Alleged Impossibility of Inductive Probability**

Karl Popper and David Miller [1983] have argued that there is no such thing as probabilistic inductive support, as conceived of, for example, in the Bayesian theory of evidence. A little more specifically, they argue that 'all probabilistic support is purely deductive' (p. 688), and that only probabilistic countersupport (disconfirmation) could be inductive in nature. Their interesting and striking argument, which they say is 'completely devastating to the inductive interpretation of probability' (p. 688), has received considerable attention, both in the way of criticism and in the way of defense: Richard Jeffrey [1984] and I. J. Good [1984] have criticised the argument, Isaac Levi [1984] attempts to improve it, Popper and Miller [1984] defend it from Jeffrey's and Good's criticisms, M. L. G. Redhead [1985] offers a new kind of criticism, and Donald Gillies [1986] defends the argument from Redhead's objection. I will not discuss these responses here. After a brief rehearsal of the Popper–Miller argument, I will criticize it in a way different from the criticisms just cited. I will argue that the Popper–Miller argument fails to establish its conclusion on any interesting way of understanding its conclusion. This will involve a little clarification of the structures of inductive and deductive support.

According to the Bayesian theory of probabilistic inductive support, the degree to which evidence e supports a hypothesis h is given by the measure:

$$s(h/e) = p(h/e) - p(h),$$

where p is an appropriate probability measure, $p(h)$ is the prior probability of h, and $p(h/e)$ is the posterior probability of h on the evidence e ($p(h/e) = p(h \& e)/p(e)$).[1] If $s(h/e)$ is positive, then e confirms h to the degree given by the difference; disconfirmation (countersupport) and its degree are indicated by a negative $s(h/e)$ and its magnitude; and evidential neutrality is indicated by $s(h/e) = 0$.

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[1] For some interpretations of probability, it is necessary to accommodate background knowledge, b, explicitly, yielding the measure $s(h/e,b) = p(h/b \& e) - p(h/b)$. But for a subjective interpretation of probability, assumed in Bayesian confirmation theory, background knowledge is already implicitly accommodated in the subjective probability function p: $p(b) = 1$, so that $p(-/b) = p(-)$.

Popper and Miller call s *probabilistic* support; what they question is whether it really measures *inductive* support.

Here is the gist of the Popper–Miller argument.[2] For any hypothesis h and evidence e, h is logically equivalent to the conjunction $(h \vee e)$ & $(h \vee -e)$. For simplicity, assume in what follows a probability p that assigns only nonextreme (not 0 and not 1) values to h, to e, and to nontautologous truth-functionally consistent truth-functional compounds of h and e. Then it is easy to see that:

$$(*) \quad s(h/e) = s(h \vee e/e) + s(h \vee -e/e).$$

It is also easy to see that $s(h \vee e/e)$ is necessarily positive and that $s(h \vee -e/e)$ is necessarily negative. Popper and Miller point out that the disjunction $h \vee e$ deductively follows from e and that it is the strongest part ('part' means *truth-functional implication*) of h that deductively follows from e. Thus, since $h \vee -e$ is the weakest part of h that in conjunction with $h \vee e$ is equivalent to h, they call $h \vee -e$ 'all of h that goes beyond e' ([1983], p. 687).[3]

Thus, when h is 'factored' into the strongest part of it that is deductively implied by e and the part (the rest) of it that 'goes beyond e', we see that e probabilistically supports only the former, and that it countersupports the latter. As Popper and Miller put it, 'that part of a hypothesis that is not deductively entailed by the evidence is always strongly countersupported by the evidence' ([1983], p. 688). The other part of the hypothesis, the part that is deductively entailed by the evidence, is the only part that is supported. They conclude that 'all probabilistic support is purely deductive' ([1983], p. 688).[4] And they point out that their argument is completely general. Indeed, if their argument is correct, 'it holds for every hypothesis h: and it holds for every evidence e, whether it [probabilistically] supports h, is independent of h, or countersupports h' ([1983], p. 688).

I wish to contest the inference from 'e probabilistically supports only that part of h that is deductively entailed by e' to 'all probabilistic support (as opposed to countersupport) is purely deductive.' I shall maintain that on a proper understanding of inductive support, even if an item X deductively entails an item Y, some aspects of X's suport of Y, may be purely *inductive* in nature. This can perhaps most readily be seen from the fact that, given that X implies Y, $s(Y/X)$ is a function wholely of $p(Y)$ (the function is $1 - p(Y)$)—*where $p(Y)$ is completely independent of deductive relations between X and Y.* But I shall first adduce some more or less independent motivation for my claim.

[2] As somewhat reformulated and simplified by Jeffrey [1984] and Gillies [1986].

[3] Jeffrey [1984] contests this characterization of $h \vee -e$, and Popper and Miller [1984] respond. But this issue is irrelevant to my main point here, which will focus on e's deductive and inductive significances for $h \vee e$.

[4] Alternative ways of factoring h into two components are discussed in Levi [1984], Jeffrey [1984], Good [1984], and Popper and Miller [1984]. This issue also is irrelevant to my main point here, since what I say will apply to the deductive and inductive significances of e for *any* factor of h that is a deductive implication of e.

The Popper–Miller critique of the theory of probabilistic inductive support shows that the degree of e's probabilistic support or countersupport for h ($s(h/e)$) is a function (addition) of (i) $s(h \vee e/e)$, which they seem to associate with e's purely deductive support for h, and (ii) $s(h \vee -e/e)$, the degree of e's probabilistic (and inductive) countersupport for h. To help motivate my claim of the previous paragraph, consider the following two numerical examples. In the two examples, the degree of the evidence's probabilistic inductive *countersupport* for the hypothesis (item (ii) above, that is, $s(h \vee -e/e)$) is the same (I'll use upper case 'H' and 'E' for the second example, so that $s(h \vee -e/e) = s(H \vee -E/E)$). However, in the first example, the evidence *supports* the hypothesis overall, and in the second, the evidence *countersupports* the hypothesis overall (so there is a difference in that $s(h/e) > 0 > s(H/E)$). The only difference therefore must lie in what Popper and Miller seem to identify with a piece of evidence's *deductive support* for a hypothesis (item (i) above, that is, $s(h \vee e/e)$ doesn't equal $s(H \vee E/E)$). What I dispute is that this difference is a 'purely deductive' difference between the two examples: part of the difference, I claim, is inductive in nature.

EXAMPLE 1:

$$p(h \,\&\, e) = \cdot 3 \quad p(-h \,\&\, e) = \cdot 2$$
$$p(h \,\&\, -e) = \cdot 2 \quad p(-h \,\&\, -e) = \cdot 3$$

So:

$$p(h/e) = \cdot 6 \quad p(h) = \cdot 5; \quad s(h/e) = \cdot 1$$
$$p(h \vee e/e) = 1 \quad p(h \vee e) = \cdot 7; \quad s(h \vee e/e) = \cdot 3$$
$$p(h \vee -e/e) = \cdot 6 \quad p(h \vee -e) = \cdot 8; \quad s(h \vee -e/e) = -\cdot 2$$

EXAMPLE 2:

$$p(H \,\&\, E) = \cdot 3 \quad p(-H \,\&\, E) = \cdot 2$$
$$p(H \,\&\, -E) = \cdot 4 \quad p(-H \,\&\, -E) = \cdot 1$$

So:

$$p(H/E) = \cdot 6 \quad p(H) = \cdot 7; \quad s(H/E) = -\cdot 1$$
$$p(H \vee E/E) = 1 \quad p(H \vee E) = \cdot 9; \quad s(H \vee E/E) = \cdot 1$$
$$p(H \vee -E/E) = \cdot 6 \quad p(H \vee -E) = \cdot 8; \quad s(H \vee -E/E) = -\cdot 2$$

In these two examples, *the relevant deductive relations among the relevant items are exactly the same.* In particular, the forms of the relevant deductive relations between e and $h \vee e$ on the one hand, and between E and $H \vee E$ on the other, are exactly the same. To make this point even more vivid, example 2 could have been described in terms of *the very same items h* and *e* in terms of which example 1 was described—but with different probability and support functions, P and S, corresponding, for example, to the subjective probabilities of a different agent. But the only items involved in the examples that are capable of standing in relevant deductive relations to each other are the pieces of evidence, the hypotheses, and their truth-functional compounds. Thus, it is clear that all the deductive relations among the items in example 1 that are capable of standing in relevant deductive relations (namely, h, e, and their truth-functional compounds) are *exactly the same* as the deductive relations among the corresponding items of example 2 that are capable of standing in relevant deductive relations (namely, H, E, and their truth-functional compounds). So it seems natural to say that e's deductive support for h (or e's deductive support for $h \vee e$) is exactly the same as E's deductive support for H (or E's deductive support for $H \vee E$). Also, as noted above, e's probabilistic inductive countersupport for h (or for $h \vee -e$) is exactly the same as E's probabilistic inductive countersupport for H (or for $H \vee -E$), for they have the same s-value. But e's overall probabilistic support for h is different from E's overall probabilistic support for H. It follows that the overall probabilistic support that evidence provides for a hypothesis cannot be any function of just the evidence's deductive support and its probabilistic inductive countersupport for the hypothesis—and there must be some difference between the two examples having to do with *nondeductive* support.

All of this is consistent with (*) and the fact that evidence only probabilistically supports the part of a hypothesis that it deductively implies and probabilistically countersupports the rest of it. I believe Popper and Miller are wrong in associating $s(h \vee e/e)$ (of (*)) with 'purely deductive support' of $h \vee e$ by e. The key to disentangling these two ideas is to be careful in distinguishing between *support that is purely deductive in nature* and *support that happens to be for a deductive implication of the evidence.* This perhaps is a subtle distinction, but, as clarified and argued below, the fact that a given piece of evidence supports an item deductively implied by the evidence doesn't mean that that support is purely deductive in nature.

Popper and Miller associate a *degree* with the component of an evidence's support of a hypothesis that they call purely deductive support: $s(h \vee e/e)$ (and $s(H \vee E/E)$). But, properly understood, it seems that support that is purely deductive in nature is an 'all or nothing' affair; either the evidence fully guarantees the truth of the hypothesis (deductively implies it) or it does not fully guarantee the truth of the hypothesis (does not deductively imply it). Purely deductive support does not come in degrees.

What, then, is the significance of $s(h \vee e/e)$ (and of $s(H \vee E/E)$)? It is the *difference* between the posterior and prior probability of $h \vee e$ on e. The fact that the posterior probability is 1 is a consequence of the fact that e deductively implies $h \vee e$, *but it is this fact alone about* $s(h \vee e/e)$*—along with consequences of this fact, such as the measure's necessarily being nonnegative—that has anything to do with e's deductive support of* $h \vee e$. The *particular magnitude* of $s(h \vee e/e)$, being a *degree* and a *function partly of* $p(h \vee e)$, clearly 'goes beyond' the deductive relations between e and $h \vee e$. This naturally suggests *inductive* support. The question therefore arises of whether this aspect of $s(h \vee e/e)$ could, for additional reasons, be correctly described as representing *inductive* support of $h \vee e$ by e.

The answer quite clearly is 'yes.' Induction has to do with the proper way of learning from experience. Stated in (Bayesian) terms of subjective probability, it is the process of properly changing one's degrees of belief when confronted with evidence. Evidence confirms (supports) or disconfirms (countersupports) a hypothesis (for the relevant person) according to whether the (subjective) probability of the hypothesis is (or would be) properly changed to a higher or lower value upon learning the evidence. And the magnitude of the change naturally measures the degree of confirmation or disconfirmation. The measure $s(h \vee e/e)$ is just that magnitude in the case in question.

Thus, equation (*) should be interpreted as: the degree of e's probabilistic *(and, I say, inductive) support for h is the sum of (i) the degree of e's probabilistic inductive* support for $h \vee e$ and (ii) the degree of e's probabilistic inductive countersupport for $h \vee -e$. Item (i) here is represented in (*) as the value $s(h \vee e/e)$. This value is a *degree*; it is a *function in part of* $p(h \vee e)$; and, as this and the two examples above show, it is *not a function just of the deductive relations between e and* $h \vee e$—hence, item (i) cannot represent purely deductive support, as Popper and Miller claim. And since it measures our change in degree of belief in $h \vee e$ upon learning e, it is natural to take it as representing the probabilistic inductive aspect of e's support of $h \vee e$.

Of course, given any hypothesis h and evidence e, deductive relations between e and h can impose restrictions on $s(h/e)$. In particular, if e deductively implies h, then $s(h/e)$ has to be nonnegative. But there is more to say about the inductive significance of e for h than simply that $s(h/e) \geqslant 0$). Conversely, attending to deductive relations between evidence and hypothesis can sometimes tell us things about their probabilistic relations to one another that attending to inductive relations alone doesn't. For example, if e deductively implies h, this tells us that $p(h/e) = 1$, whereas knowing only the value of $s(h/e)$ tells us no such thing. But again, there is more to the inductive significance of e for h than that $p(h/e) = 1$: there is also the *change* in probability. And to describe this change, it is not enough to say where the probability of the hypothesis *ends up*, which is all that can be gathered from the deductive relations between evidence and hypothesis in the cases in question where the evidence implies the hypothesis. When the evidence implies the hypothesis, it is easy to overlook the purely inductive aspects of the evidence's support of the hypothesis. For if the evidence is learned, the deductive bearing of the evidence on the hypothesis settles the question of the *truth* of the hypothesis—which is usually the important question—whereas the purely inductive bearing (i.e., the value of s) does not.

Thus, I believe that Popper and Miller have failed to establish the conclusion that all probabilistic (positive) support is purely deductive support and all probabilistic inductive support is countersupport. They *have* shown that a hypothesis can be divided into two parts such that the evidence only probabilistically supports the part of the hypothesis that the evidence deductively implies, and probabilistically countersupports the rest. But if we are careful in distinguishing between the ideas of *support that is purely deductive in nature* and *support of a deductively implied hypothesis*, it is easy to see that their argument fails to establish the conclusion that all probabilistic support is purely deductive in nature.

Also, the conclusion that *is* established by Popper and Miller's argument (noted just above) is hardly 'completely devastating to the inductive interpretation of the calculus of probability.' For, as shown above, this 'weaker' (or less interesting) conclusion is perfectly compatible with the idea that whether any evidence e confirms, disconfirms, or is neutral for any hypothesis h turns on *nondeductive*—indeed *probabilistic inductive*—features of the bearing of e on h, indeed, on *both* of the two components into which Popper and Miller divide h: $h \vee e$ (which e probabilistically inductively supports to the degree $s(h \vee e/e)$) *and* $h \vee -e$ (which e probabilistically inductively countersupports to the degree $s(h \vee -e/e)$).

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