Some Abstract Properties of Confirmation Relations & Four Theories of Confirmation

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1 Some Properties of Confirmation Relations

Hempel (in his second installment) discusses various properties that confirmation relations might have. I will discuss a longer list of properties. Here are a bunch of properties that we'll discuss today. We will assume throughout most of our discussion that all of our E's, H's, and K's are logically contingent.

(M_E) If E confirms H relative to K, then $E \,\&\, E'$ confirms H relative to K (provided that E' does not contain any constant symbols not already contained in $\{H, E, K\}$).

(M_K) If E confirms H relative to K, then E confirms H relative to $K \,\&\, K'$ (provided that K' does not contain any constant symbols not already contained in $\{H, E, K\}$).

(NC) $\ulcorner \phi x \,\&\, \psi x \urcorner$ confirms $\ulcorner (\forall y)(\phi y \supset \psi y) \urcorner$ relative to (some/all/specific) K.

(SCC) If E confirms H relative to K and $H \vDash_K H'$, then E confirms H' relative to K.

(CCC) If E confirms H relative to K and $H' \vDash_K H$, then E confirms H' relative to K.

(CC) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash \sim(H \,\&\, H')$.

(CC′) If E confirms H relative to K and E confirms H' relative to K, then $K \nvDash \sim(H \equiv H')$.

(EC) If $E \vDash_K H$, then E confirms H relative to K.

(CEC) If $H \vDash_K E$, then E confirms H relative to K.

(EQC_E) If E confirms H relative to K and $K \vDash E \equiv E'$, then E' confirms H relative to K.

(EQC_H) If E confirms H relative to K and $K \vDash H \equiv H'$, then E confirms H' relative to K.

(EQC_K) If E confirms H relative to K and $K \Dashv\vDash K'$, then E confirms H relative to K'.

(NT) For some E, H, and K, E confirms H relative to K.
And, for every E/K, there exists an H such that E does *not* confirm H relative to K.

(ST) If E confirms H relative to K and E confirms H relative to $\sim K$, then E confirms H relative to $\top$.

As exercises, let's think about some subsets of this large set of conditions. Consider the following triples:

- (NT), (CEC), (SCC)
  - Inconsistent. Pick an E. Then, by (NT), E does not confirm (some) H relative to (some) K. But, by (CEC), E confirms $H \,\&\, E$, relative to K. Then, by (SCC), E confirms H, relative to K. Contradiction.
- (NT), (EC), (CCC)
  - Inconsistent. Pick an E. Then, by (NT), E does not confirm H relative to K. By (CCC), E does not confirm $H \vee E$ relative to K (if it did, then, by (CCC), it would also confirm the logically stronger H, contrary to our initial assumption). By (EC), E confirms $H \vee E$ relative to K. Contradiction.
- (NT), (CCC), (SCC)
  - Inconsistent. By (NT), (some) E confirms (some) H relative to (some) K. By (CCC), E confirms $H \,\&\, H'$ relative to K. By (SCC), E confirms H' relative to K. But, H' was arbitrary here. So, we have found an E/K such that, for all H', E confirms H' relative to K, which contradicts (NT).

As an exercise, it is useful to think about the consistency of other subsets of this large set of conditions.

2 Hempel's Theory of Confirmation

In order to understand Hempel's theory, we first need the concept of *the development of a (closed, first-order) statement H with respect to a set of individual constants I*, which I will write as $\mathrm{dev}_I(H)$. This is defined as:

> $\mathrm{dev}_I(H)$ is (*i*) the *conjunction* of the I-instances of H, if H is a *universal* ($\forall$) claim, (*ii*) the *disjunction* of the I-instances of H, if H is an *existential* ($\exists$) claim, and (iii) $\mathrm{dev}_I(H) = H$, if H is quantifier-free.

Here are some examples to illustrate how $\mathrm{dev}_I(H)$ is computed. Let $I = \{a, b\}$. Then, we have:

- $\mathrm{dev}_I[(\forall x)Bx] = Ba \,\&\, Bb$.
- $\mathrm{dev}_I[(\exists x)Rx] = Ra \vee Rb$.
- $\mathrm{dev}_I[(\forall x)(Rx \supset Bx)] = (Ra \supset Ba) \,\&\, (Rb \supset Bb)$.
- $\mathrm{dev}_I[(\forall x)(\exists y)Lxy] = \mathrm{dev}_I[(\exists y)Lay \,\&\, (\exists y)Lby] = (Laa \vee Lab) \,\&\, (Lba \vee Lbb)$

It can be shown that logically equivalent hypotheses have logically equivalent developments. In fact, if $H \vDash H'$, then $\mathrm{dev}_I(H) \vDash \mathrm{dev}_I(H')$. Proving this is a useful exercise. We will just assume it in what follows.

With $\mathrm{dev}_I(H)$ understood, we are now ready for Hempel's definition of confirmation:

> **Definition**. *E directly Hempel-confirms H* relative to K iff $E \vDash_K \mathrm{dev}_{I(E\&K)}(H)$ with respect to the set $I(E \,\&\, K)$ of individual constants occurring in $E \,\&\, K$. *E Hempel-confirms H* relative to K iff E directly Hempel-confirms (relative to K) every member of some set S such that $S \vDash_K H$. [If E Hempel-confirms H relative to $\top$, then we just say E Hempel-confirms H, for short.]
>
> – Caveat #1: An "observation report" E is a closed, quantifier-free sentence. We will assume the same about "background evidence" K. This makes Hempel's theory unable to handle certain sorts of evidence (*e.g.*, statistical evidence, or even other sorts of evidential statements—although, as Hempel notes, the definition could be extended to general sentences in first-order languages).
>
> – Caveat #2: It is assumed that E contains all of its constant symbols *essentially*. For instance, while $E \overset{\text{def}}{=} Ra$ is logically equivalent to $E' \overset{\text{def}}{=} Ra \,\&\, (Qb \vee {\sim}Qb)$, they can partake in different confirmation relations, if we don't restrict $I(E')$ to the constant symbols E' contains essentially. So, strictly speaking, we need to think of $I(E \,\&\, K)$ as the smallest set of individual constants occurring in sentences E' that are logically equivalent to $E \,\&\, K$. This set can always be computed easily.

Here are some examples to illustrate the definition:

- $E \overset{\text{def}}{=} Ra$ and $E' \overset{\text{def}}{=} Ra \,\&\, (Qb \vee {\sim}Qb)$ both directly Hempel-confirm $(\forall x)Rx$. This is because of caveat #2. When we compute $I(E')$, we get $\{a\}$ and not $\{a, b\}$, since $E' \dashv\vDash E$. Thus, 'b' is *inessential* in E'.

- $Ra \,\&\, Ba$ does *not directly* Hempel-confirm $Rb \supset Bb$. This is because $Ra \,\&\, Ba \nvDash Rb \supset Bb$. But, $Ra \,\&\, Ba$ *does* Hempel-confirm $Rb \supset Bb$. Let $S \overset{\text{def}}{=} \{(\forall x)(Rx \supset Bx)\}$. Then, $Ra \,\&\, Ba$ directly Hempel-confirms the only member of S $[(\forall x)(Rx \supset Bx)]$, since $Ra \,\&\, Ba \vDash Ra \supset Ba = \mathrm{dev}_{I(Ra\&Ba)}[(\forall x)(Rx \supset Bx)]$. $\therefore$ We have (indirect) Hempel-confirmation, since $S \vDash Rb \supset Bb$. Does $Ra \,\&\, Ba$ Hempel-confirm $Rb \,\&\, Bb$?

- Let $I \overset{\text{def}}{=} \{a, b\}$, $H \overset{\text{def}}{=} (\forall x)(\forall y)Rxy$, $E \overset{\text{def}}{=} Raa \,\&\, Rab \,\&\, Rbb \,\&\, Rba$, and $E' \overset{\text{def}}{=} Raa \,\&\, Rab \,\&\, Rbb$. E Hempel confirms H, but E' does not. Moreover, Raa Hempel confirms H. This example shows that Hempel-confirmation is not *unrestrictedly* monotonic in E. But, subject to the caveat that E_2 does not contain any constant symbols that do not occur in $\{E_1, H\}$, we have that $E_1 \,\&\, E_2$ confirms H, if E_1 does. In other words, Hempel's theory entails (M_E) and (M_K) in the restricted forms stated above.

- *No consistent* E can confirm the following hypothesis (H), which is *true* on $\mathbb{N}$:

  (H) $\qquad (\forall x)(\exists y)(x < y) \,\&\, (\forall x)(x \nless x) \,\&\, (\forall x)(\forall y)(\forall z)[(x < y \,\&\, y < z) \supset x < z]$

  since $\mathrm{dev}_I(H)$ *is inconsistent, for any finite I*. That is an interesting consequence of Hempel's definition.

- Let $H \overset{\text{def}}{=} (\forall x)(Rx \supset Bx)$. Which of the following six propositions Hempel-confirm H?

E_1: $Ra \,\&\, Ba$	E_2: $\sim Ra$	E_3: Ba
E_4: $\sim Ra \,\&\, \sim Ba$	E_5: $\sim Ra \,\&\, Ba$	E_6: $Ra \,\&\, \sim Ba$

  Answer: *All but E_6 Hempel-confirm H*. Indeed, all but E_6 *directly* Hempel-confirm H.

- Consider the following argument (given by Hempel, and endorsed also by Goodman):

  > If E consists *only* of one nonraven [$\sim Ra$], then E confirms that all objects are nonravens [$(\forall x)\sim Rx$], and *a fortiori* E supports the weaker assertion that all nonblack objects are nonravens [$(\forall x)(\sim Bx \supset \sim Rx)$]. [Therefore, "one nonblack nonraven" $\sim Ba \,\&\, \sim Ra$ also confirms that all nonblack objects are nonravens, and hence that all ravens are black.]

(i) $\sim Ra$ confirms $(\forall x)\sim Rx$. (NC)

(ii) $(\forall x)\sim Rx \vDash (\forall x)(\sim Bx \supset \sim Rx)$ (Logic)

(iii) $\sim Ra$ confirms $(\forall x)(\sim Bx \supset \sim Rx)$ (i), (ii), (SCC)

(iv) $(\forall x)(\sim Bx \supset \sim Rx) \dashv\vDash (\forall x)(Rx \supset Bx)$ (Logic)

(v) $\sim Ra$ confirms $(\forall x)(Rx \supset Bx)$ (iii), (iv), (EQC_H)

(PC) $\sim Ba \,\&\, \sim Ra$ confirms $(\forall x)(Rx \supset Bx)$ (v), (M_E)

This "independent argument" for (PC) depends essentially on (NC), (SCC), *and* (M_E). Hempel and Goodman don't even *state* the final conclusion! This shows how subtle (M_E) is. They make this error because they talk about the *object* "one nonraven" doing the confirming. And, "one nonblack nonraven" *is* "one nonraven"!

3 Hypothetico–Deductive Confirmation ("The Prediction Criterion")

The hypothetico-deductive (HD) account of confirmation is typically not defended by anyone who is serious about confirmation theory.[1] But, it is often used as a foil in contemporary discussions. Here it is (Hempel calls this "the prediction criterion" — his definition is a bit more complicated, but I'll ignore that):

> **Definition**. E *HD-confirms* H relative to K iff $H \vDash_K E$.

While Hempel and HD confirmation are both defined in terms of $\vDash_K$, it's a pretty significant difference that Hempel has E on the left hand side of $\vDash_K$, whereas HD has E on the right-hand side of $\vDash_K$.

4 Two Probabilistic Confirmation Concepts

As I have mentioned before, Carnap discusses two distinct probabilistic confirmation concepts.

4.1 Confirmation as Firmness ("Absolute" Confirmation)

Confirmation as firmness is a conditional-probability-threshold concept:

> **Definition**. E *confirms$_f$* H relative to K iff $\Pr(H \mid E \,\&\, K) > t$, for some threshold value t.

Here, one is usually also offered an *interpretation* of $\Pr(\cdot \mid \cdot)$. But, I will ignore this for now. We'll return to that issue later in the seemster (I will argue that logical confirmation concepts do not require any interpretation of Pr, although their epistemic correlates will involve some interpretation of Pr).

[1] Quine (in his "Natural Kinds" paper, which we'll discuss when we get to "grue") occasionally talks about (HD) confirmation, but he seems to be confused about the distinction between (HD) and Hempelian confirmation. In fact, they differ radically.

4.2 Confirmation as Increase in Firmness ("Incremental" Confirmation)

Confirmation as increase in firmness is a probabilistic relevance concept:

> **Definition**. E *confirms*$_i$ H relative to K iff $\Pr(E \mid H \,\&\, K) > \Pr(E \mid {\sim}H \,\&\, K)$.

As above, I will ignore the interpretive questions about Pr for now.

5 A Summary of the Properties of our Four Confirmation Concepts

So far, we have seen four confirmation concepts: Hempelian, HD, Firmness, and Increase in Firmness. These four concepts differ in important ways with respect to various properties, including our list of properties above. The following table summarizes these properties, for each of the four concepts.

	Does Concept Satisfy Condition?										
Concept	$EQC_{E/H/K}$	EC	CC	NT	SCC	CCC	CEC	$M_{E/K}$	NC	CC′	ST
Hempelian	Yes	Yes	Yes[1]	Yes	Yes	No	No	Yes	Yes	Yes[1]	Yes
HD	Yes	No	No	Yes	No	Yes	Yes	No	No	Yes[5]	Yes
Confirms$_f$	Yes	Yes[2]	No	Yes	Yes	No	No	No	No	Yes[4]	Yes
Confirms$_i$	Yes	Yes[3]	No	Yes	No	No	Yes[3]	No	No	Yes	No

It is a useful exercise to go through and convince yourself of all these answers. I won't dwell on these here, but I will briefly discuss one important subset of properties of Hempel-confirmation: (SCC), (NC), (M_E). These three properties are involved in both paradoxes of confirmation. We discussed the raven paradox last week [and again above in connection with the "independent argument" for (PC)]. When we get to "grue" in a few weeks, we'll see these properties playing a crucial role again. Here's an example. Let $Ox \overset{\text{def}}{=} x$ is examined (for the first time) prior to t (for some t in the distant future). Let any statement $\mathcal{E}$ of the form $\ulcorner Oa \,\&\, \psi a \urcorner$ be called an "observation statement". Now, in Hempel's theory of confirmation, we can reason as follows:

(a) Oa confirms $(\forall x)Ox$. (NC)

(b) $\ulcorner Oa \,\&\, \psi a \urcorner$ confirms $(\forall x)Ox$, for any ψ. (a), (M_E)

(c) $\ulcorner Oa \,\&\, \psi a \urcorner$ confirms $\ulcorner (\forall x)({\sim}Ox \supset \phi x) \urcorner$, for any ψ and any ϕ. (b), (SCC)

$\therefore$ *Any observation statement $\mathcal{E}$ confirms any universal generalization H about unexamined objects.*

As we'll see when we get to Goodman, this is one of the main "odd consequences" of Hempel's theory that Goodman complains about. Goodman does lots of fancy syntactic fiddling with antecedent and consequent predicate expressions of universal claims (*e.g.*, "emerose"). But, all that fancy footwork unnecessary. All one needs is to recognize that Hempel's theory entails (SCC), (NC), and (M_E). Apparently, Hempel and Goodman were unaware that Hempel's theory implies (M_E). I discussed this last week (see also my "ravens" paper).

[1] Assuming that $E \,\&\, K$ is not self-contradictory.
[2] Assuming that $\Pr(E \mid K) \neq 0$.
[3] Assuming that $\Pr(H \mid K) \in (0,1)$, and $\Pr(E \mid K) \in (0,1)$.
[4] Assuming that $t \geq \frac{1}{2}$.
[5] Assuming that $K \nvDash E$.