CHARLES S. CHIHARA AND DONALD A. GILLIES

AN INTERCHANGE ON THE POPPER-MILLER ARGUMENT

(Received 16 October, 1986)

1. INTRODUCTION

In 1983, Popper and Miller published in *Nature* a new argument against inductive probability. There followed, the next year, a discussion of the validity of the argument in the pages of the same journal involving Good, Jeffrey, and Levi as well as Popper and Miller themselves. This has led in turn to still further discussions of the argument. Those who are critical include Redhead (1985) and Dunn and Hellman (1986); and those who are in favour Gillies (1986). The present interchange on the subject originated from a meeting of the two authors at a conference. The discussion which started there was continued by letter, and the two authors then decided to present their cases for and against the Popper-Miller argument in a kind of dialogue. Gillies begins in the next section by presenting one form of the argument. This is then criticized by Chihara in Section 3. Gillies replies in Section 4, and Chihara concludes the discussion in Section 5.

2. SUPPORT AND THE POPPER-MILLER ARGUMENT (GILLIES)

I shall assume that we have a notion of evidence (e) *confirming* or *corroborating* an hypothesis (h). I shall in fact use the terms 'confirmation' and 'corroboration' as synonyms, and write the degree of confirmation (or corroboration) of h given e as $c(h, e)$. Strictly speaking the evidence e will be in addition to some background knowledge b. So we ought really to write $c(h, e \wedge b)$. This will indeed be done when it is necessary to draw attention to the existence of background knowledge.

The Bayesian thesis can now be formulated as the claim that $c(h, e)$

Philosophical Studies **54** (1988) 1—8.

satisfies the standard axioms of the mathematical calculus of probability, or that, in symbols:

$$(1) \qquad c(h, e) = p(h, e)$$

We are now in a position to introduce the notion of *support*, which should not, so I claim, be identified with that of confirmation (or corroboration). The difference is this. $c(h, e \wedge b)$ stands for the total confirmation given to h by both e and b. Degree of support $s(h, e, b)$ is a 3-place function, and represents the contribution made to the total confirmation by the individual item of evidence e against a background b.[1]

From the way we have defined support, it is clear that it should have the following additive property:

$$(2) \qquad s(h, e_1 \wedge e_2, b) = s(h, e_1, e_2 \wedge b) + s(h, e_2, b)$$

It also seems to me that the following holds:

$$(3) \qquad c(h, e \wedge b) = s(h, e, b) + c(h, b)$$

For a Bayesian, we have from (1) and (3) that

$$(4) \qquad s(h, e, b) = p(h, e \wedge b) - p(h, b)$$

From now on I shall, for convenience of writing, omit the background knowledge b so that (4) becomes:

$$(5) \qquad s(h, e) = p(h, e) - p(h)$$

According to the Bayesians, scientific inference consists in going from the prior probability ($p(h)$) of an hypothesis h to its posterior probability ($p(h, e)$) in the light of the evidence collected. If $p(h, e) > p(h)$, then the evidence supports, h, whereas if $p(h, e) < p(h)$, then the evidence undermines h. On this picture, $s(h, e)$ as defined by (5) gives the simplest and most natural measure of support. Nonetheless it has been claimed that Bayesianism is compatible with other measures of support.

In his (1985), p. 190, Redhead suggests that a Bayesian could instead define support by

$$(6) \qquad s'(h, e) = p(h, e)/p(h)$$

However, in my (1986), I show that this measure has undesirable properties from the Bayesian point of view.

Good, developing some earlier work of Turing's, has for many years advocated the measure

(7) $s''(h, e) = \log(p(e, h)/p(e, -h))$

(See, for example, Good (1983) Ch. 15, Para. IV, pp. 159—162). This measure is an interesting one, but not, in my view, compatible with the other assumptions of Bayesianism. A Bayesian who adopted it would have to give up (3), which I regard as basic to the notion of support. I shall therefore assume that (5) gives the appropriate definition of support for a Bayesian, and proceed to state the Popper-Miller argument.

For simplicity, I shall assume from now on that e follows logically from h, and that any probability p cited is such that $0 < p < 1$. As can be checked by simple manipulations in the probability calculus, we have:

(8) $s(h, e) = s(h \vee e, e) + s(h \vee -e, e)$

Since $h \vee e$ follows logically from e, $s(h \vee e, e)$ must represent purely deductive support. So, if there is such a thing as inductive support in the Bayesian sense, it must be contained in the term $s(h \vee -e, e)$. However by Theorem 1 of Popper and Miller (1983), this term is always negative. It therefore follows that there cannot be inductive support of the kind that the Bayesians postulate.

3. CRITICISM OF THE POPPER-MILLER ARGUMENT (Chihara)

I think it will be helpful to regard the argument not as an attack on some position or other (e.g. Bayesianism), but rather as a sort of attempted proof of something, i.e. as something with premises and a conclusion.

Now review the argument. Well, everything goes through as it is presented ... up to the last step. I agree that inductive support can be represented as a sum of two functions, f and g; that f is a function that represents deductive support; that g is always negative; and that neither f nor g can represent inductive support. But that last step that goes '. . . therefore . . . there cannot be inductive support . . .' seems to me to be unjustified. The question to ask here is: 'What principle of reasoning (or rule of inference) allows one to take that last step?' Consider an analogous line of reasoning. I can show, using what seems to be an analogous rule of inference, that there is no such thing as probability. Thus, suppose that $p(x)$ is a probability function. Then for all x, $p(x)$ belongs to $[0, 1]$. Now

$$\begin{aligned} p(x) &= p(x)/2 + p(x)/2 \\ &= (2 + p(x)/2) + (p(x)/2 - 2) \end{aligned}$$

So, letting

$$(9) \qquad h(x) = 2 + p(x)/2 \text{ and } j(x) = p(x)/2 - 2,$$

$$p(x) = h(x) + j(x)$$

But, for all x, $h(x)$ is always greater than 1. So if there is such a thing as probability, it must be $j(x)$. But for all x, $j(x)$ is always negative. Hence $p(x)$ is not a probability function.

I thus don't see how one can draw the conclusion of the argument from what is given.

4. REPLY (GILLIES)

First as regards the question: 'What principle of reasoning (or rule of inference) allows one to take that last step?', I find this difficult to answer. The argument is an informal, plausibility kind of argument, and would be difficult to formalize. I certainly don't think it could be reduced to formal logic.

Second as regards the example of $p(x) = h(x) + j(x)$, I found this very ingenious, but, of course, I would say that it is not really analogous. This is how I see the situation. We can suppose, for the sake of argument, both sides agree on (i) a support function defined by (7) i.e. $s(h, e) = p(h, e) - p(h)$, and (ii) that deductive support exists and is unproblematic. The dispute now centers on whether inductive support in the Bayesian sense exists as well as deductive support. Since one side accepts inductive support and the other doesn't, inductive support must be regarded as a more problematic notion than deductive support. But now the Popper-Miller decomposition shows that support in general can always be reduced to the less problematic notion of deductive support. There is therefore no need to postulate the more problematic inductive support.

Consider, by contrast, your decomposition $p(x) = h(x) + j(x)$. If $h(x)$ were a less problematic notion in some way than $p(x)$, then indeed this decomposition might be used in a way analogous to the Popper-Miller case. However there is no sense in which $h(x)$ is less problematic than $p(x)$, and so the analogy breaks down.

5. REPLY TO REPLY (CHIHARA)

The support function $s(h, e)$ satisfies the relationship (8) i.e.

$$s(h, e) = s(h \vee e, e) + s(h \vee -e, e)$$

For purposes of simplicity and perspicuity, I use the notation

$$(10) \qquad s = s_1 + s_2$$

to express (8). Now s_1 can be shown to represent purely deductive support, and s_2 can be proved to be always negative. Hence neither s_1 nor s_2 can represent inductive support. The last step in the Popper-Miller argument is the controversial one. The Popperian believes that one can infer, from the fact that neither s_1 nor s_2 can represent inductive support, that s cannot either and that there is no such thing as inductive support. But why? It is claimed in the reply to my objection that since the Popper-Miller argument is an informal one, there is no need to articulate or justify the rule of inference being used. In response I wish to emphasize two things:

(1) *The argument is supposed to be a 'proof'*. The title of the original Popper-Miller paper is 'A proof of the impossibility of inductive probability', and the argument is called a 'proof' in the paper. In a more recent paper, Popper claims to have 'proved' that probabilistic support can never be inductive support, and then goes on to say: 'I am here restating our case, adding some further *proofs*' ((1985), p. 303, italics mine). One of the 'proofs' he adds in this paper is essentially the one being discussed in this interchange.

(2) An *informal proof*, i.e. a proof expressed in a natural language, must have acceptable premises and proceed according to rules of inference that can be seen to be valid; and the fact that one is proposing an informal proof does not free one from proceeding according to inference rules that can be seen to be valid. For example, a mathematician cannot shield a proposed proof from the demands of other mathematicians to justify a step by noting that the proposed proof is informal. (Cf. the controversy over Zermelo's proof of the well-ordering theorem). In short, if the Popperians have really produced a *proof* of their thesis about the impossibility of inductive support, then we ought to be able to articulate a rule of inference according to which the last step proceeds, and we should be able to see that this rule of inference is indeed valid. If, for example, the only people who believe that this rule of inference is valid turn out to be Popperians, then that would be a reason for non-Popperians to be dubious about the supposed proof. Indeed, if the whole argument reduces, in the end, to an 'intuition' of validity that only Popperians have, then why call it a proof?

The rule of inference according to which the last step of the argument seems to proceed is something like the following:

(A) If function p is thought to represent F, and if $p = f + g$, where neither f nor g can represent F, then p cannot represent F.

I produced a sort of counter-example to this rule, i.e. an example to show the unacceptable consequences of taking such a rule to be valid. The reply to my objection takes the form of bringing out a difference between the inductive support example and my counter-example: the 'F' of the inductive support inference is problematic or controversial, whereas the 'F' of my counter-example (probability) is not. Of course, any counter-example will of necessity be different in some respects from the example in question, and if we are to have a clear counter-example (one that everyone would accept), the 'F' will have to be non-controversial. The point to be examined is whether the difference is *relevant to the rule of inference being presupposed.* In claiming that it is, the defender of the Popper-Miller 'proof' is in effect arguing that the rule of inference being presupposed is not (A) above, but rather something like:

(B) If function p is thought to represent F, and if F is problematic or controversial, and if $p = f + g$, where neither f nor g can represent F, then p cannot represent F.

But what possible reason could one have for accepting (B) as valid and not (A)? A valid rule of inference must carry one from truths to truths, for all 'instances'. If (A) does not, how can (B)? The only difference between the two is the clause in the antecedent about the controversial nature of F. But it is hard to see how that clause can change an invalid rule into a valid one, especially when one notes that whether a notion or relationship is controversial or not is context dependent — what is controversial in one society at some particular time may not be controversial at another time or in another society. And this doubt may be reinforced by the following considerations. What is a (total) function? Nowadays, it is usual to regard a function as a set of ordered pairs satisfying a certain condition (which need not concern us here). Suppose that a certain function S represents some notion or relationship F. And suppose that there are two other functions S_1 and S_2 which are such that S is the same set of ordered pairs as the addition function $S_1 + S_2$. Would it be at all surprising if neither S_1 nor S_2 represented the notion or relationship F? Not at all. Yet the Popperians think that when F is the relationship of inductive support, then either S_1 or S_2 must represent this relationship. The burden of proof is on the Popperians to explain why.

Finally, I sense a subtle shift in position taking place. Whereas before Popper and Miller spoke of *proving* that there is no such thing as inductive support, Gillies now talks of *reducing* inductive support to deductive support, i.e. instead of claiming to prove that inductive support is impossible, he now claims that 'there is therefore no need to postulate the more problematic inductive support'. The question of whether what has been given amounts to an acceptable and genuine reduction is one that I do not wish to pursue here. It is enough, for my purposes, to have cast doubt on the claim that inductive support has been *proven* to be impossible.

NOTE

[1] The fact that support (or *weight of evidence*, as he calls it) is a 3-place function is rightly stressed by Good (cf. his (1983) Ch. 15, para. IV, pp. 159—160). I am grateful to I. J. Good for persuading me in correspondence that this is indeed the case.

BIBLIOGRAPHY

Dunn, J. Michael and Geoffrey Hellman: 1986, 'Dualling: A critique of an argument of Popper and Miller', British Journal for the Philosophy of Science 37(2), pp. 220—223.

Gillies, D. A.: 1986, 'In defence of the Popper-Miller argument', Philosophy of Science 53, pp. 110—113.

Good, I. J.: 1983, Good Thinking. The Foundations of Probability and Its applications, University of Minnesota Press.

Good, I. J.: 1984, 'Letter concerning Popper and Miller (1983)', Nature 310, August 2 1984, p. 433.

Jeffrey, R. C.: 1984, 'Letter concerning Popper and Miller (1983)', Nature 310 August 2 1984, p. 433.

Levi, I.: 1984, 'Letter concerning Popper and Miller (1983)', Nature 310, August 2 1984, p. 433.

Popper, K. R.: 1985, 'The non-existence of probabilistic inductive support', in G. Dorn and P. Weingartner (eds.), Foundations of Logic and Linguistics, Plenum.

Popper, K. R. and D. W. Miller: 1983, 'A proof of the impossibility of inductive probability', Nature 302, April 21 1983, pp. 687—688.

Popper, K. R. and D. W. Miller: 1984, 'Letter in reply to Good (1984), Jeffrey (1984), and Levi (1984)', Nature 310, August 2 1984, p. 434.

Redhead, M. L. G.: 1985, 'On the impossibility of inductive probability', British Journal for the Philosophy of Science 36(2), pp. 185—191.

Department of Philosophy,
University of California at Berkeley,
Berkeley, CA 94720,
U.S.A.

Department of Philosophy,
King's College, University of London,
Strand, London WC2 R2LS,
England.