Philosophy 148 — Assignment #2

02/28/08

This assignment is due Thursday, March 13 at (at the end of class). If you work in a group, list your group members at the top of your submitted work.

1 Playing the Rôle of a Dutch Bookie

Bob assigns the following betting quotients in a situation where propositions A and B have no special logical relationships:

(*i*) $q(A \,\&\, B) = \frac{1}{4}$ (*ii*) $q(A) = \frac{1}{8}$ (*iii*) $q(B) = \frac{1}{2}$ (*iv*) $q(A \vee B) = \frac{3}{4}$

- First, demonstrate that Bob's q is not a probability function (*i.e.*, that q violates some theorem or axiom of the probability calculus).
- Second, describe a bet or collection of bets which would result in a sure-loss for Bob (*i.e.*, make a Dutch Book against Bob). If you work in a group, each group member must submit a different answer. [**NOTE**: there may be *multiple* ways in which Bob violates the probability calculus, and, as a result, there may also be *many* Dutch Books that can be made against him (some easier than others)! If you have extra time, try to see *how many* violations you can spot resulting from (*i*)–(*iv*), and how many *different kinds* of Dutch Books you can make against Bob.]
- Third, does your Dutch book require assuming a "package principle"? Is it *necessary* to use a "package principle" in order to make book against Bob? Justify your answer.

2 Comparing the "Distance" Measures $\rho^\dagger$ and ρ^*

2.1 Weak Convexity, Symmetry, and $\rho^\dagger$

In Joyce's framework, the inaccuracy of a credence function q (over a set of propositions $\mathcal{B}$[1]) in a world w is defined as follows:

$$I(q, w) = \sum_{X \in \mathcal{B}} \rho(q(X), w(X))$$

where $q(X)$ is the agent's credence in proposition X, $w(X)$ is the truth-value of X in world w ($w(x)$ is 0 if X is false in w, and 1 if X is true in w), and $\rho(q(X), w(X))$ is some measure of "distance" between $q(X)$ and $w(X)$. Joyce gives six "axioms" for I, which place strong constraints on the ρ function. The two most controversial of these "axioms" are the following:

[1]Note: Joyce does not assume that $\mathcal{B}$ is a Boolean algebra – it can be any set of propositions.

Weak Convexity (WC). If $I(q_1, w) = I(q_2, w)$ and $q_3 = \frac{1}{2}q_1 + \frac{1}{2}q_2$, then $I(q_1, w) = I(q_3, w)$ implies $q_1 = q_2$.

Symmetry (S). If $I(q_1, w) = I(q_2, w)$ then, for any $\lambda \in [0, 1]$, $I(\lambda \cdot q_1 + (1 - \lambda) \cdot q_2, w) = I((1 - \lambda) \cdot q_1 + \lambda \cdot q_2, w)$.

(Here, q_1, q_2, and q_3 are each credence functions defined over $\mathcal{B}$. Adding two functions is shorthand for adding their components, e.g. $q_4 = q_1 + q_2$ iff for all X in $\mathcal{B}$, $q_4(X) = q_1(X) + q_2(X)$. Similarly, multiplying a function by a constant is shorthand for multiplying each of its components by that constant, e.g. $q_5 = \lambda \cdot q_1$ iff for all X in $\mathcal{B}$, $q_5(X) = \lambda \cdot q_1(X)$.)

The measures $\rho^\dagger$ and ρ^* are defined as: $\rho^\dagger(q(X), w(X)) = (q(X) - w(X))^2$, and $\rho^*(q(X), w(X)) = |q(X) - w(X)|$.

I showed in class (and Maher shows less simply) that ρ^* violates both (WC) and (S). Here are two exercises:

1. Prove that $\rho^\dagger$ satisfies (WC), in the case where $\mathcal{B}$ contains only 2 propositions A and B.
2. Prove that $\rho^\dagger$ satisfies (S), in the case where $\mathcal{B}$ contains only 2 propositions A and B.

Your proofs for this problem do not need to be formal, two-column proofs. Most of the work will be algebra. Be sure to show enough algebraic steps that a reader can follow your work, and justify each non-algebraic step.

Extra Credit [10% worth, for a complete answer]. Generalize your proofs of (1) and (2) to the case in which $\mathcal{B}$ contains n propositions $X_1, \ldots, X_n$. Each correct generalization is worth 5% extra-credit.

2.2 The Ordinal Non-Equivalence of I's Based on $\rho^\dagger$, ρ^*

As we have now shown, $\rho^\dagger$ and ρ^* disagree on (WC) and (S). However, it's not intuitively clear that this means they lead to inaccuracy measures $I^\dagger$ and I^* that measure different quantities. How might we show that $I^\dagger$ and I^* measure different quantities?

We say that two inaccuracy measures I_1 and I_2 are *ordinally equivalent* just in case, for all q, q', $\mathcal{B}$, and w: $I_1(q, w) \geq I_1(q', w)$ if and only if $I_2(q, w) \geq I_2(q', w)$. If two measures are ordinally equivalent, then it is reasonable to suspect that they are measuring the same quantity (just on different scales). For instance, the Celsius scale and the Farneheit scale are both scales of *temperature* because if x is warmer than y in Celsius, then x is warmer than y in Farenheit, and *vice versa*.

Question: do $\rho^\dagger$ and ρ^* lead to inaccuracy measures $I^\dagger$ and I^* (as defined above) that are ordinally equivalent? Answer: No. Exercise: Prove this. That is, show that there exists a q, q', $\mathcal{B}$, and w, such that $I^\dagger(q, w) \geq I^\dagger(q', w)$, but $I^*(q, w) < I^*(q', w)$ (or *vice versa*). (**Hint**: you will need a $\mathcal{B}$ with at least 2 propositions. I recommend looking for a 2-proposition counterexample.) If you work in a group, each group member must submit a different answer.