Overview Consider the following constraint [2, 14] on a (prior/initial) credence function \( Pr(\cdot) \), over a language containing two factual atoms \( P \) and \( Q \), and a third atom “\( P \rightarrow Q \)” (where “\( P \rightarrow Q \)” is interpreted extra-systematically as the indicative “if \( P \), then \( Q \)”).

The Equation (EQ). \( Pr(P \rightarrow Q) = Pr(Q \mid P) \).

Lewis [9] assumes that if The Equation is rationally required, then it must hold (fully) resiliently — i.e., that the following strengthening of The Equation must be a constraint on \( Pr(\cdot) \).

The Resilient Equation (REQ). For all \( x \) (where \( x \) is Boolean-definable in terms of \( P, Q \)) such that \( Pr(P \& x) > 0 \),

\[
Pr(P \rightarrow Q \mid x) = Pr(Q \mid P \& x).
\]

Various triviality results have been derived from The Resilient Equation. The strongest possible such triviality result [4] is this.

(REQ)-Triviality. If \( Pr(P \& Q) > 0 \) and \( Pr(P \& \neg Q) > 0 \), then

\[
Pr(P \& (Q \equiv (P \rightarrow Q))) = 1.
\]

In 1976, Lewis assumed that any rational requirement on initial (viz., prior) credence \( Pr(\cdot) \) must be fully resilient. What he says in connection with this doesn’t entail the desired (full) resiliency:

the … class of all those probability functions that represent possible systems of beliefs … is closed under conditionalizing. Rational change of belief never can take anyone to a subjective probability function outside the class; and … the change of belief that results from coming to know an item of new evidence should take place by conditionalizing on what was learned.

Even if we grant Lewis all of these claims, they don’t imply that all rational requirements on \( Pr(\cdot) \) must be fully resilient.

Of course, some constraints do satisfy even this very strong requirement. For instance, probabilism itself must satisfy it. For it is a theorem of the probability calculus that if an initial credence function is a probability function \( Pr(\cdot) \), then so is \( Pr(\cdot \mid x) \), provided only that \( Pr(\cdot \mid x) \) well-defined. [The above quotation from Lewis (1976) articulates something close to this truism.]

The following table illustrates the relationships between The Equation (EQ) and The Resilient Equation (REQ) [4].

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( P \rightarrow Q )</th>
<th>( Pr(\cdot) )</th>
<th>( Pr(\cdot) + (EQ) )</th>
<th>( Pr(\cdot) + (REQ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( T )</td>
<td>( T )</td>
<td>( a )</td>
<td>( a )</td>
<td>( a )</td>
</tr>
<tr>
<td>( T )</td>
<td>( T )</td>
<td>( F )</td>
<td>( b )</td>
<td>( b )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( T )</td>
<td>( F )</td>
<td>( T )</td>
<td>( c )</td>
<td>( c )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( T )</td>
<td>( F )</td>
<td>( F )</td>
<td>( d )</td>
<td>( d )</td>
<td>( 1 - a )</td>
</tr>
<tr>
<td>( F )</td>
<td>( T )</td>
<td>( T )</td>
<td>( e )</td>
<td>( e )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( F )</td>
<td>( T )</td>
<td>( F )</td>
<td>( f )</td>
<td>( f )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( F )</td>
<td>( F )</td>
<td>( T )</td>
<td>( g )</td>
<td>( \frac{a + b}{a + b + c + d} )</td>
<td>( a - c - e )</td>
</tr>
<tr>
<td>( F )</td>
<td>( F )</td>
<td>( F )</td>
<td>( 1 - \sum )</td>
<td>( 1 - \sum )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

(EQ) reduces the number of \( Pr(\cdot) \)’s degrees of freedom (from 7) to 6, and (REQ) reduces the number of degrees of freedom to 1.

My reconstruction: Lewis (1976) assumed (for reductio) that The Equation is a rational requirement on \( Pr(\cdot) \). Then, he used the full resiliency requirement to complete his reductio.

(1) The Equation is a rational requirement for \( Pr(\cdot) \).
(2) \( \therefore \) The Equation must hold in a (fully) resilient way.
(3) \( \therefore \) The Resilient Equation is a rational requirement for \( Pr(\cdot) \).
(4) But, (REQ)-Triviality is not a rational requirement for \( Pr(\cdot) \).
(5) Contradiction. [Since (3) entails ¬(4).]
(6) \( \therefore \) The Equation is not a rational requirement for \( Pr(\cdot) \). □

Lewis (1976) presupposes that (1) implies (2) (I call this Presupposition #1). This is where the argument goes wrong.

Premise (3), which I call Presupposition #2, can be established directly, via a knock-down counterexample to (REQ).

Lewis (1980) is not moved by an analogous “reductio of the Principal Principle” as a rational constraint on priors.
Overview Lewis 1976 Lewis 1980 Reconciliation Logic Extra References

Lewis [10] maintains that the Principal Principle (PP) is a rational requirement on initial/prior credence functions \( \Pr(\cdot) \).

\((PP) \quad \Pr(p \mid Ch(p) = c) = c.\)

Lewis knows that if we require (PP) to hold (fully) resilently, then we get something trivial. To wit, consider the following schema:

\((PP_x) \quad \Pr(p \mid x & Ch(p) = c) = c.\)

**Resilient PP** (RPP) asserts that (PPx) holds for all \( x \) such that (PPx) is well-defined. That principle \[ (\forall x) \text{PP}_x \] is trivial. Let \( P \), \( Ch(P) = 1 \) and \( Ch(P) = 0 \) be our three atoms. Then:

\[(RPP)-Triviality, \text{ (\forall x)PP}_x \text{ implies only two states can have non-zero probability: } P & Ch(P) = 1 \text{ and } \neg P & Ch(P) = 0.\]

\[(RPP)-Triviality\] is very similar to \((REQ)-Triviality\). They both reduce the number of degrees of freedom in the class of models of the prior \( \Pr(\cdot) \) down to a single degree of freedom.

Interestingly, Lewis is not swayed by the following “reductio.”

1. (PP) is a rational requirement for \( \Pr(\cdot) \).
2. \( (\forall) \text{ (PP)} \) must hold in a (fully) resilient way.
3. \( (\forall) \text { Resilient (PP)} \) is a rational requirement for \( \Pr(\cdot) \).
4. But, \((RPP)-Triviality\) is not a rational requirement for \( \Pr(\cdot) \).
5. Contradiction. [Since (3) entails \( \neg (4) \).]
6. \( (\forall) \text { (PP)} \) is not a rational requirement for \( \Pr(\cdot) \).

This time, Lewis rejects the presupposition that (1) implies (2).

He introduces the notion of “admissibility” with the aim of demarcating those \( x \)’s for which \( (\forall x)\text{PP}_x \) is a requirement.

\((RPP)-Triviality\) suggests \( p \) and \( \neg p \) are inadmissible (wrt PP). Analogously, \((REQ)-Triviality\) suggests \( \neg Q, P \supset Q \), and their negations are inadmissible (with respect to The Equation) [4].

In both of these cases, the simple restrictions inspired by triviality merely scratch the surface of (in)admissibility.

The following table illustrates the basic algebraic relationships between (PP) and (RPP). Note the similarity to (EQ) vs. (REQ).

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Ch(P) = 0 )</th>
<th>( Ch(P) = 1 )</th>
<th>( \Pr(\cdot) )</th>
<th>( \Pr(\cdot) + (PP) )</th>
<th>( \Pr(\cdot) + (RPP) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( T )</td>
<td>( T )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( T )</td>
<td>( T )</td>
<td>( F )</td>
<td>( a )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( T )</td>
<td>( F )</td>
<td>( T )</td>
<td>( b )</td>
<td>( b )</td>
<td>( b )</td>
</tr>
<tr>
<td>( F )</td>
<td>( T )</td>
<td>( F )</td>
<td>( c )</td>
<td>( c )</td>
<td>0</td>
</tr>
<tr>
<td>( F )</td>
<td>( F )</td>
<td>( T )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( F )</td>
<td>( F )</td>
<td>( F )</td>
<td>( 1 - \sum )</td>
<td>( 1 - \sum )</td>
<td>0</td>
</tr>
</tbody>
</table>

(PP) reduces the number of \( \Pr(\cdot) \)’s degrees of freedom (from 5) to 3, and (RPP) reduces the number of degrees of freedom to 1.

Lewis requires admissible \( x \)’s (with respect to PP) to satisfy **Chance Screening**, i.e., \( x \) must be s.t., for all \( c \in [0, 1] \),

\[ \Pr(p \mid x & Ch(p) = c) = \Pr(p \mid Ch(p) = c). \]

Let’s call the principle which restricts \( \forall \) in (RPP) to admissible \( x \)’s (in this sense) **The Quasi-Resilient Principal Principle** (QRPP).

Alas, the (QRPP) is not so interesting, because it is equivalent to (PP) — see Extras. However, an analogous quantifier restriction for (REQ) yields a plausible constraint that is stronger than (EQ).

The analogous quantifier restriction for The Resilient Equation is that \( x \) satisfy **Antecedent Screening**, i.e., that \( x \) be s.t. both

\[ \Pr(Q \mid x & P) = \Pr(Q \mid P) \text{ and } \Pr(Q \mid x & \neg P) = \Pr(Q \mid \neg P). \]

Why? Consider indicative conditionals with chance antecedents, e.g., \( P := Ch(Q) = 1/2 \). Then, The Equation & (PP) jointly imply

\[ \Pr(P \rightarrow Q) = \Pr(Q \mid P) = 1/2 \]
Now, in order to bring (REQ) into alignment with (QRPP), we must impose **Antecedent Screening** (viz., \(Q \equiv x \mid P\)) as a \(\forall\)-restriction.

When we restrict (REQ) to admissible \(x\)'s, we get more plausible (non-trivial) constraint on priors, which is stronger than (EQ).

**The Quasi-Resilient Equation** (QREQ). All rational initial credence functions \(Pr(-)\) should satisfy the following **restricted** version of The Resilient Equation.

For all factual propositions \(p\), \(q\), and \(x\):

\[
Pr(p \rightarrow q \mid x) = Pr(q \mid p \& x),
\]

provided \(x\) satisfies **Antecedent Screening** (viz., \(q \equiv x \mid p\)).

(QREQ) restricts (REQ) to \(x\)'s which don't trump the (a priori) informational connection between antecedent and consequent. This seems (to me) to be an intuitive restriction on (REQ).

Moreover, this restriction not only avoids triviality, but it also avoids all of the (known) counterexamples to (REQ).

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Stalnaker [15] shows that The Equation is incompatible with his preferred conditional logic (C2). His argument can be generalized to all Lewis-Stalnaker conditional logics.

Logics validating import-export (in general) are a no-go [4, 5]. My approach begins with a **Minimal Probabilistic Logic** (MPL).

The basic idea behind MPL is to start with a single-premise entailment relation (\(\vdash\)), which I define in the following way.

\((\text{MPL}_0)\) \(p \vdash q \equiv Pr(p) \leq Pr(q), \forall Pr\)-functions satisfying (QREQ).

Compare (MPL\(_0\)) with Adams’s approach to entailment [1].

\(p \vdash q \equiv Pr(p) \leq Pr(q), \forall Pr\)'s satisfying The Equation.

Interestingly, \(p \vdash q \iff p \vdash q\). Thus, **MPL is coextensional with Adams’s (single-premise) logic for the indicative conditional** [1].

Therefore, the following chain of MPL validites holds

\((\text{MPL}_1)\) \(p \& q \vdash p \rightarrow q \vdash p \vdash q\).

---

Here is a **knock-down** counterexample to The Resilient Equation (thanks to Paolo Santorio). A fair die (Die) was tossed.

\[
\begin{array}{c|c|c}
\text{(P)} & \text{(Q)} & \text{Pr(-)} \\
\hline
T & T & 1/6 \\
T & F & 1/2 \\
F & T & 0 \\
F & F & 1/3 \\
\end{array}
\]

**The Equation** \(\Rightarrow Pr(P \rightarrow Q) = Pr(Q \mid P) = 1/4 \Rightarrow Pr((P \rightarrow Q) \& X) \leq 1/4\).

\(\therefore Pr(P \rightarrow Q \mid X) = \frac{Pr((P \rightarrow Q) \& X)}{Pr(X)} \leq \frac{1/4}{1/2} = 1/2 < 1 = Pr(Q \mid P \& X)\).

Note that \(X\) (viz., \(P \vdash Q\)) is **inadmissible** (in our sense), since

\(Pr(Q \mid X & P) = Pr(Q \mid (P \& Q)) = 1 \neq 1/4 = Pr(Q \mid P)\).

Every (known, knock-down) counterexample to (REQ) involves inadmissible \(x\)'s in our sense. My conjecture is that all counterexamples to (REQ) violate **Antecedent Screening**.

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This chain holds for \(\models\) because \(Pr(p \vdash q) \geq Pr(q \mid p) \geq Pr(p \& q)\) is a theorem of Pr-calculus. So, \(\rightarrow\) is “intermediate” between \& and \(\vdash\), in terms of logical strength (according to MPL).

The key entailment in any logic of this sort is (v) \(p \rightarrow q \vdash p \vdash q\). Logics of \(\rightarrow\) validating (v) are bound to run into cases like Paolo’s Die. To wit: the following generalization of Gibbard’s collapse theorem for \(\rightarrow\), which simplifies an earlier result of mine [5].

\(i\) \(\vdash (p \& q) \rightarrow q\).

\(ii\) If \(p \vdash q\) and \(p \vdash q\), then \(p \vdash q\).

\(iii\) If \(p \vdash q\) and \(p \vdash q\), then \(p \vdash q\).

\(iv\) If \(p \vdash (p \& q)\), then \(p \vdash (x \& p) \vdash q\).

\(v\) \(p \vdash q \vdash p \vdash q\).

\(vi\) If \(x \& p \vdash q\), then \(x \vdash (p \rightarrow q)\).

**Theorem.** (i)–(vi) jointly entail \(\vdash\)-**Collapse** \(p \vdash q \vdash p \rightarrow q\).

MPL satisfies (i)–(v). So, on pain of \(\vdash\)-collapse, **MPL must reject** (vi). **Ergo** the inevitability of cases like Paolo’s Die example.
I’ll close with another way to think about my approach, and its relation to others (e.g., van Fraassen’s [16] and McGee’s [11]).

Consider the following three probabilistic constraints (where $X$, $P$, and $Q$ are Boolean/atomic — i.e., they contain no “→”s).

(I) $\Pr((P \land X) \rightarrow Q) = \Pr(Q \ | \ P \land X)$

(II) $\Pr(X \rightarrow (P \rightarrow Q)) = \Pr(P \rightarrow Q \ | \ X)$

(III) $\Pr(X \rightarrow (P \rightarrow Q)) = \Pr((P \land X) \rightarrow Q)$

Anyone who accepts The Equation will accept (I). Specifically, van Fraassen [16], McGee [11], and myself all accept (I).

Nobody can reasonably accept all three of (I)–(III) — without restriction — since that would imply all instances of The Resilient Equation (including, e.g., Paolo’s Die case instance).

van Fraassen [16] accepts (II) and rejects (III); whereas McGee [11] accepts (II) and rejects (II). On my approach, (II) and (III) are equivalent — provided that $P$ screens-off $Q$ from $X$ ($Q \perp X \mid P$).

Fact. $(\text{QRPP}) \iff (\text{PP})$.

Proof.

($\Rightarrow$) Let $x$ be a tautology. Then, Chance Screening obtains ($\top$ is screened-off from every $p$ by every $q$), and $(\text{QRPP}) \Rightarrow (\text{PP})$.

($\Leftarrow$) Let $\alpha \equiv \Pr(p \mid \text{Ch}(p) = c)$, and $\beta \equiv \Pr(p \mid x \land \text{Ch}(p) = c)$. Then, Chance Screening is $\alpha = \beta$, $(\text{PP})$ is $\alpha = c$, and $(\text{QRPP})$ is $\alpha = \beta = c$ (where all three claims are $\forall$-quantified over $p, x, c$). By logic, $(\text{QRPP})$ is equivalent to $\alpha = \beta = c$ and implies $\alpha = c$.

Fact. $(\text{QREQ}) \Rightarrow (\text{EQ})$. But, $(\text{QREQ}) \not\iff (\text{EQ})$.

Proof.

($\Rightarrow$) Let $x$ be a tautology. Then, Antecedent Screening obtains ($\top$ is screened-off from every $p$ by every $q$), and $(\text{QREQ}) \Rightarrow (\text{EQ})$.

($\not\Rightarrow$) There exist probability distributions $\Pr(\cdot)$ over the Boolean algebra generated by the four atomic sentences $\{P, Q, X, P \rightarrow Q\}$ (where $P$, $Q$, and $X$ are all factual claims) such that (1) $\Pr(P \rightarrow Q) = \Pr(Q \mid P)$,

(2) $Q \perp X \mid P$, but (3) $\Pr(P \rightarrow Q \mid X) \neq \Pr(Q \mid P \land X)$.