Consider the following constraint [1, 17] on a (prior/initial) credence function $\Pr(\cdot)$, over a language containing two factual atoms P and Q, and a third atom "$P \to Q$" (where "$P \to Q$" is interpreted *extra-systematically* as the indicative "if P, then Q").

The Equation (EQ). $\Pr(P \to Q) = \Pr(Q \mid P)$.

☞ Lewis [11] assumes that *if* **The Equation** is *rationally required*, *then* it must hold *resiliently* — *i.e.*, that **The Equation** must be a *structural* rational requirement [20]. To be more precise:

The Resilient Equation (REQ). For all x (where x is Boolean-definable in terms of P, Q) such that $\Pr(P \,\&\, x) > 0$,

$$\Pr(P \to Q \mid x) = \Pr(Q \mid P \,\&\, x).$$

Various triviality results have been derived from **The Resilient Equation**. The strongest possible such triviality result [5] is this.

(REQ)–Triviality. If $\Pr(P \,\&\, Q) > 0$ and $\Pr(P \,\&\, \neg Q) > 0$, then

$$\Pr(P \,\&\, (Q \equiv (P \to Q))) = 1.$$

The following table illustrates the relationships between **The Equation** (EQ) and **The Resilient Equation** (REQ) [5].

P	Q	$P \to Q$	$\Pr(\cdot)$	$\Pr(\cdot)$ + (EQ)	$\Pr(\cdot)$ + (REQ)
T	T	T	a	a	a
T	T	F	b	b	0
T	F	T	c	c	0
T	F	F	d	d	$1 - a$
F	T	T	e	e	0
F	T	F	f	f	0
F	F	T	g	$\frac{a+b}{a+b+c+d} - a - c - e$	0
F	F	F	$1 - \sum$	$1 - \sum$	0

(EQ) reduces the number of $\Pr(\cdot)$'s degrees of freedom (from 7) to 6, and (REQ) reduces the number of degrees of freedom to 1.

My reconstruction: Lewis (1976) assumed (for *reductio*) that **The Equation** is a rational requirement on $\Pr(\cdot)$. Then, he used the full resiliency requirement to complete his *reductio*.

(1) **The Equation** is a rational requirement for $\Pr(\cdot)$.
(2) $\therefore$ **The Equation** must hold in a (fully) *resilient* way.
(3) $\therefore$ **The Resilient Equation** is a rational requirement for $\Pr(\cdot)$.
(4) But, **(REQ)–Triviality** is *not* a rational requirement for $\Pr(\cdot)$.
(5) Contradiction. [Since (3) entails $\neg$(4).]
(6) $\therefore$ **The Equation** is *not* a rational requirement for $\Pr(\cdot)$. □

Lewis (1976) presupposes that (1) *implies* (2) (I call this **Presupposition #1**). I think the argument goes wrong here.

Premise (4), **Presupposition #2**, can be established both directly (*via* counterexamples) and indirectly (*via* independent arguments). I'll do both toward the end of the talk.

☞ Lewis (1980) is *not* moved by an analogous "*reductio* of the Principal Principle" as a rational constraint on priors.

Lewis [12] maintains that the Principal Principle (PP) is a rational requirement on initial/prior credence functions $\Pr(\cdot)$.

(PP) $\Pr(p \mid \mathrm{Ch}(p) = c) = c$.

Lewis knows that if we require (PP) to hold (fully) *resiliently*, then we get something *trivial*. To wit, consider the following schema:

(PP_x) $\Pr(p \mid x \,\&\, \mathrm{Ch}(p) = c) = c$.

Resilient PP (RPP) asserts that (PP_x) holds *for all x such that* (PP_x) *is well-defined*. *That* principle [$(\forall x)\ \mathrm{PP}_x$] is *trivial*. Let P, $\mathrm{Ch}(P) = 1$ and $\mathrm{Ch}(P) = 0$ be our three atoms. Then:

(RPP)–Triviality. $(\forall x)\mathrm{PP}_x$ implies *only two states can have non-zero probability*: $P \,\&\, \mathrm{Ch}(P) = 1$ and $\neg P \,\&\, \mathrm{Ch}(P) = 0$.

(RPP)–Triviality is very similar to **(REQ)–Triviality**. They both reduce the number of degrees of freedom in the class of models of the prior $\Pr(\cdot)$ *down to a single degree of freedom*.

The following table illustrates the basic algebraic relationships between (PP) and (RPP). Note the similarity to (EQ) *vs.* (REQ).

P	$\mathrm{Ch}(P)=0$	$\mathrm{Ch}(P)=1$	$\Pr(\cdot)$	$\Pr(\cdot)$ + (PP)	$\Pr(\cdot)$ + (RPP)
T	T	T	0	0	0
T	T	F	a	0	0
T	F	T	b	b	b
T	F	F	c	c	0
F	T	T	0	0	0
F	T	F	d	d	$1-b$
F	F	T	e	0	0
F	F	F	$1-\sum$	$1-\sum$	0

(PP) reduces the number of $\Pr(\cdot)$'s degrees of freedom (from 5) to 3, and (RPP) reduces the number of degrees of freedom to 1.

Interestingly, Lewis is *not* swayed by the following "reductio."

(1) (PP) is a rational requirement for $\Pr(\cdot)$.
(2) $\therefore$ (PP) must hold in a (fully) *resilient* way.
(3) $\therefore$ **Resilient (PP)** is a rational requirement for $\Pr(\cdot)$.
(4) But, **(RPP)-Triviality** is *not* a rational requirement for $\Pr(\cdot)$.
(5) Contradiction. [Since (3) entails $\neg$(4).]
(6) $\therefore$ (PP) is *not* a rational requirement for $\Pr(\cdot)$. $\square$

Here, Lewis *rejects* the presupposition that (1) *implies* (2). [2, 15]

He introduces the notion of "admissibility" to demarcate those x's for which $(\forall x)\mathrm{PP}_x$ *is* a *substantive* [20] rational requirement.

(RPP)-Triviality suggests p and $\neg p$ are *in*admissible (wrt PP). Analogously, **(REQ)-Triviality** suggests that *any* x *such that* $\ulcorner P \,\&\, x \urcorner$ *determines the truth-value of* Q is inadmissible (wrt EQ).

In both contexts, the logical restrictions inspired by *triviality* can be generalized, *via* notions of *conditional* Pr-*independence*.

Lewis requires *admissible* x's (with respect to PP) to satisfy **Chance Screening**, *i.e.*, x must be s.t., *for all* $c \in [0,1]$,

$$\Pr(p \mid x \,\&\, \mathrm{Ch}(p) = c) = \Pr(p \mid \mathrm{Ch}(p) = c).$$

Let's call the principle which restricts $\forall$ in (RPP) to *admissible* x's (in this sense) **The Quasi-Resilient Principal Principle** (QRPP).

☞ (QRPP) is not so interesting, because it is *equivalent to* (PP) (see Extras). But, analogous restrictions to **The Resilient Equation** (REQ) yield constraints that are *stronger* than **The Equation** (EQ).

The analogous quantifier restriction for **The Resilient Equation** is that (factual) x satisfy the following screening condition

> **Antecedent Screening.** The antecedent of the (simple) conditional $P \to Q$ *screens its consequent from* x.

There are various ways of precisifying "P screens Q from x." But, if we assume that x is Boolean-definable in terms of P and Q, then these different precisifications are all equivalent.

- **Antecedent Screening** — Three Equivalent Definitions
  - P screens Q from x iff $\ulcorner P \,\&\, x \urcorner \nvDash Q$ and $\ulcorner P \,\&\, x \urcorner \nvDash \neg Q$.
  - P screens Q from x iff *for all probability functions* $\Pr(\cdot)$
  $$\Pr(Q \mid P \,\&\, x) = \Pr(Q \mid P).$$
  - P screens Q from x iff *some* probability function $\Pr(\cdot)$ is s.t.
  $$0 < \Pr(Q \mid P \,\&\, x) = \Pr(Q \mid P) < 1.$$
- These definitions are equivalent, assuming x is Boolean-definable in terms of P, Q. They all imply that ***exactly 3*** *propositions are admissible*: $\{P, P \vee Q, P \vee \neg Q\}$.
- If we expand the language to include a fourth atomic sentence X, then these three definitions *diverge* (*viz.*, they allow different propositions to be counted as admissible).
- In this talk, I will keep things simple, by (mainly) only discussing x's that are Boolean-definable in terms of P, Q.

Quasi-Resilient Equation (**QREQ**). All rational initial credence functions $\Pr(\cdot)$ should satisfy the following *restricted* version of **The Resilient Equation.**

For all x that are Boolean-definable in terms of P and Q,

$$\Pr(P \to Q \mid x) = \Pr(Q \mid P \mathbin{\&} x),$$

provided x satisfies **Antecedent Screening**.

It can be shown that **QREQ** is equivalent to the following.

Quasi-Resilient Equation (**QREQ**). All rational initial credence functions $\Pr(\cdot)$ should be such that

$\Pr(P \to Q \mid P) = \Pr(Q \mid P)$,
$\Pr(P \to Q \mid P \vee Q) = \Pr(Q \mid P)$, and
$\Pr(P \to Q \mid P \vee \neg Q) = \Pr(Q \mid P)$.

Unlike (QRPP), which is equivalent to (PP), **QREQ** is *strictly logically stronger* than **The Equation** (see Extras).

On the simplest $\{P, Q, P \to Q\}$-algebra, there is strong agreement between our approach and and McGee's [13].

McGee also seeks to restrict **The Resilient Equation**. But, he does so *indirectly*, *via* the following principle.

Independence (IND). For all factual propositions x, if $x \vDash \neg P$, then $\Pr(x \mathbin{\&} (P \to Q)) = \Pr(x) \cdot \Pr(P \to Q)$.

McGee also assumes

Modus Ponens (MP). For all factual propositions x, if $x \vDash P$, then $\Pr(x \mathbin{\&} (P \to Q)) = \Pr(x \mathbin{\&} P \mathbin{\&} Q)$.

☞ The following theorem reveals that *McGee's indirect restriction on **The Resilient Equation** is equivalent to our direct one.*

Theorem.[1] Assuming **Modus Ponens**, McGee's **Independence** is equivalent to our **QREQ.**

[1] I am indebted to Snow Zhang for her wise and expert counsel. She [21] has also shown that McGee's theory is equivalent to the recent theory of Goldstein & Santorio [9]. Hence, *all three theories converge on our* **QREQ**.

P	Q	$P \to Q$	$\Pr(\cdot)$	$\Pr(\cdot)$ + (MP) + (IND) = $\Pr(\cdot)$ + (MP) + **QREQ**
T	T	T	a	a
T	T	F	b	0
T	F	T	c	0
T	F	F	d	$\frac{a \cdot (1-(a+e+g))}{a+e+g}$
F	T	T	e	e
F	T	F	f	$e \cdot \left(\frac{1}{a+e+g} - 1\right)$
F	F	T	g	g
F	F	F	$1-\sum$	$1-\sum$

Assuming (MP) — which explains the two zeros — McGee's theory and mine (**QREQ**) both have 3 degrees of freedom.

Here is a *knock-down* counterexample to **The Resilient Equation** (thanks to Paolo Santorio). A fair die (Die) was tossed.

(P) Die landed on either 1, 3, 5, or 6.
(Q) Die landed on 6.
(X) Die landed even.

P	Q	$\Pr(\cdot)$
T	**T**	$1/6$
T	**F**	$1/2$
F	**T**	0
F	**F**	$1/3$

The Equation $\Rightarrow \Pr(P \to Q) = \Pr(Q \mid P) = 1/4$. $\therefore \Pr((P \to Q) \mathbin{\&} X) \leqslant 1/4$.

$$\therefore \Pr(P \to Q \mid X) = \frac{\Pr((P \to Q) \mathbin{\&} X)}{\Pr(X)} \leqslant \frac{1/4}{1/2} = 1/2 < 1 = \Pr(Q \mid P \mathbin{\&} X).$$

Note that X (*viz.*, $P \supset Q$) is *inadmissible* (in our sense), since

$$\Pr(Q \mid X \mathbin{\&} P) = \Pr(Q \mid (P \supset Q) \mathbin{\&} P) = 1 \neq 1/4 = \Pr(Q \mid P).$$

☞ *Every* (known, knock-down) counterexample to (REQ) involves *inadmissible* x's in our sense. My conjecture is that *all* counterexamples to (REQ) violate **Antecedent Screening**.

Here's what McGee's Theory ($\therefore$ **MP** + **QREQ**) says about Die.

P	Q	$P \to Q$	$\Pr(\cdot)$	$\Pr(\cdot)$ + (MP) + (IND) = $\Pr(\cdot)$ + (MP) + **QREQ**
T	T	T	a	$1/6$
T	T	F	b	0
T	F	T	c	0
T	F	F	d	$1/2$
F	T	T	e	0
F	T	F	f	0
F	F	T	g	$1/12$
F	F	F	$1 - \sum$	$1/4$

$\therefore \Pr(P \to Q \mid X) = 1/2 \neq 1 = \Pr(Q \mid P \,\&\, X)$

If we add $X \to (P \to Q)$ to the language, then **QREQ** seems to break down. On *any* screening-off notion of admissibility, *all tautologies* $\top$ *will be admissible*. So, naïvely, we would have

$$\Pr(X \to (P \to Q) \mid \top) = \Pr(P \to Q \mid X).$$

And, in the Die case, $\Pr(P \to Q \mid X) = 1/2$. But, intuitively, $X \to (P \to Q)$ should be *certain* in the Die case.

It seems that the only way — in the spirit of (**QREQ**) — to get this answer for $\Pr(X \to (P \to Q))$ is to *revise the* ***Ratio*** *formula for the indicative suppositional probability* $\Pr(p \to q \parallel x)$.

One such revision is based on the following (factual p, q, x).[2]

(†) $$\Pr(p \to q \parallel x) = \Pr[(x \,\&\, p) \to q \parallel x]$$

That is: the conditionals $p \to q$ and $(x \,\&\, p) \to q$ *should be equally probable — on the indicative supposition that* x.

[2] This was inspired by discussions with Wes Holliday [10] and Jim Joyce.

Given (†), this revision of **Ratio** for $\Pr(p \to q \parallel x)$ makes sense.

Ratio*. $\Pr(p \to q \parallel x) = \Pr[(x \,\&\, p) \to q \mid x]$.[3]

Because $x \,\&\, p$ screens-off x from q, (**QREQ**) applies to $\Pr[(x \,\&\, p) \to q \mid x]$. Hence, **Ratio*** + (**QREQ**) yields

QREQ*. All rational (indicative) *suppositional* credence functions $\Pr(\cdot \parallel \cdot)$ should be such that, for all factual p, q, and x,

$$\Pr(p \to q \parallel x) = \Pr(q \mid p \,\&\, x).$$

QREQ* seems sensible in both admissible and inadmissible cases, *e.g.*, in Die: $\Pr(P \to Q \parallel X) = \Pr(Q \mid P \,\&\, X) = 1$.

Finally, if we require $\Pr[x \to (p \to q)] = \Pr(p \to q \parallel x)$ [18], then *import-export* follows from **QREQ***, *i.e.*,

$$\Pr[x \to (p \to q)] = \Pr(p \to q \parallel x) = \Pr(q|p\&x) = \Pr[(x\&p) \to q].$$

[3] Holliday [10] proposes $\Pr(p \to q \parallel x) = \Pr[x \to (p \to q) \mid x]$, which is equivalent to **Ratio*** — if one assumes import-export as a resilient law. Goldstein & Santorio endorse **QREQ*** — they call it the **Update Thesis** [9].

Fact. (QRPP) $\Leftrightarrow$ (PP).

Proof.

($\Rightarrow$) Let x be a tautology ($\top$). Then, **Chance Screening** obtains — $\top$ is screened from *every* q by *every* p, on *every* $\Pr(\cdot)$. $\therefore$ (QRPP) $\Rightarrow$ (PP).

($\Leftarrow$) Let $\alpha \stackrel{\text{def}}{=} \Pr(p \mid \mathrm{Ch}(p) = c)$, and $\beta \stackrel{\text{def}}{=} \Pr(p \mid x \,\&\, \mathrm{Ch}(p) = c)$. Then, **Chance Screening** is $\alpha = \beta$, (PP) is $\alpha = c$, and (QRPP) is $\alpha = \beta \Rightarrow \beta = c$ (where all three claims are $\forall$-quantified over p, x, c). By logic, (QRPP) is equivalent to $\alpha = \beta \Rightarrow \alpha = c$, which (by logic) is implied by $\alpha = c$. $\square$

Fact. (QREQ) $\Rightarrow$ (EQ). But, (QREQ) $\not\Leftarrow$ (EQ).

Proof.

($\Rightarrow$) Let x be a tautology ($\top$). **Antecedent Screening** obtains —$\top$ is screened from *every* q by *every* p, on *every* $\Pr(\cdot)$. $\therefore$ (QREQ) $\Rightarrow$ (EQ).

($\not\Leftarrow$) There exist regular probability functions $\Pr(\cdot)$ over the $\{P, Q, P \to Q\}$ algebra such that (1) $\Pr(P \to Q) = \Pr(Q \mid P)$, and (2) $\Pr(Q \mid P \,\&\, x) = \Pr(Q \mid P)$; *but*, (3) $\Pr(P \to Q \mid x) \neq \Pr(Q \mid P \,\&\, x)$, and where x is Boolean-definable in terms of P and Q (*e.g.*, $x := P$). $\square$

Consider the following three fundamental (structural) principles of the classical theory of conditional probability.

(C1) $0 \leqslant \Pr(p \mid q) \leqslant 1$

(C2) $\Pr(p \mid p \,\&\, q) = 1$

Ratio. $\Pr(p \,\&\, q \mid r) = \Pr(p \mid q \,\&\, r) \cdot \Pr(q \mid r)$

If we add **The Resilient Equation** to these three fundamental principles of the classical theory of conditional probability, then we can derive the following confirmational absurdity [14, 19].

No Disconfirmation. For all factual propositions e, h, k,

$$\Pr(h \mid k \,\&\, e) \geqslant \Pr(h \mid k).$$

In order to avoid **No Disconfirmation**, we must reject at least one of these four: {(C1), (C2), **Ratio**, **The Resilient Equation**}.

☞ Since (C1) and (C2) are non-negotiable, **The Resilient Equation** conflicts *directly* with **Ratio** (see [9] for a similar conclusion).

One key consequence of our approach — in addition to many other recent approaches [13, 9, 4] — is the following.

No Antecedent Confirmation (NAC). For all factual p, q,

$$\Pr(p \to q \parallel p) \leqslant \Pr(p \to q).$$

Some claim that there *are* cases in which p *confirms* (is *positively relevant to*) $p \to q$ [3]. If this is true, then one of the following three principles must be rejected (for some factual p, q).

(1) $\Pr(p \,\&\, (p \to q)) \leqslant \Pr(p \,\&\, q)$.

(2) The **Ratio** formula for $\Pr(p \to q \parallel p)$.

(3) **The Equation**: $\Pr(p \to q) = \Pr(q \mid p)$.

(1) seems non-negotiable. And, it's hard to see how rejecting (2) can help. The problem (above) with **Ratio** for $\Pr(p \to q \parallel x)$ *does not arise* when $x := p$, since $\Pr[(p \,\&\, p) \to q \mid p] = \Pr(p \to q \mid p)$.

It is also worth noting that some ways of rejecting (3) won't help. *E.g.*, the material conditional analysis of $\to$ also entails (NAC).

Wait — didn't Gibbard [8] show that any theory which accepts **Import-Export** must *reject* **The Equation**, since such theories entail the equivalence of $\to$ and $\supset$; and, $\Pr(p \supset q) \neq \Pr(q \mid p)$.

Well — not quite. First, Gibbard's argument only (essentially) entails collapse of $\to$ to the *intuitionistic* conditional [6].

Second, and more importantly, Gibbard's argument assumes that $p \,\&\, (p \to q) \vDash q$, for *all* conditionals — *even nested* ones.

Assuming import-export (and **The Equation**), the **Die** case is a *counterexample* to $p \,\&\, (p \to q) \vDash q$. To see why, consider

(1) $p \,\&\, (p \to q) \vDash q$ *only if* $\Pr[p \,\&\, (p \to q)] \leqslant \Pr(q)$.

(2) In **Die**, let $p := X$, and $q := P \to Q$.

Import-Export entails $\Pr[X \to (P \to Q)] = \Pr[(X \,\&\, P) \to Q] = 1$.
$\therefore$ (1) + **The Equation** $\Longrightarrow$ **Die** is such that $p \,\&\, (p \to q) \nvDash q$, since

$$\Pr[p \& (p \to q)] = \Pr[X \& (X \to (P \to Q))] = 1/2 > 1/4 = \Pr(Q|P) = \Pr(q).$$

Seen from this perspective, Gibbard's argument is *unsound*.

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