Probabilistic Measures of Causal Strength

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This is a companion Mathematica notebook for our paper “Probabilistic Measures of Causal Strength”, which can be downloaded from http://fitelson.org/pmcs.pdf.

- Requires the PrSAT package, which can be downloaded from http://fitelson.org/PrSAT/.

<< PrSAT

Defining the measures (Table 1)

In this section we give definitions of our measures of causal strength (CS) and preventative strength (PS), as described in Table 1 of the paper. As discussed in the beginning of the paper, we define \( \text{PS}(E,C) = -\text{CS}(\neg E,C) \) for each of our CS-measures, and we introduce two rescalings for each of the CS-measures that are not already defined on \([-1,1]\).

Eells

\[
\begin{align*}
\text{CSe}[e_, c_] & := \Pr[e | c] - \Pr[e | \neg c]; \\
\text{CSe}[e_, c_1, c_2] & := \Pr[e | c_1 \land c_2] - \Pr[e | \neg c_1 \land c_2]; \\
PSe[e_, c_] & := -\text{CSe}[\neg e, c]; \\
PSe[e_, c_1, c_2] & := -\text{CSe}[\neg e, c_1, c_2]; \\
\end{align*}
\]

Suppes

\[
\begin{align*}
\text{CSs}[e_, c_] & := \Pr[e | c] - \Pr[e]; \\
\text{CSs}[e_, c_1, c_2] & := \Pr[e | c_1 \land c_2] - \Pr[e | c_2]; \\
Pss[e_, c_] & := -\text{CSs}[\neg e, c]; \\
Pss[e_, c_1, c_2] & := -\text{CSs}[\neg e, c_1, c_2]; \\
\end{align*}
\]

Galton

\[
\begin{align*}
\text{CSg}[e_, c_] & := 4 \Pr[c] \Pr[\neg c] (\Pr[e | c] - \Pr[e | \neg c]); \\
\text{CSg}[e_, c_1, c_2] & := 4 \Pr[c_1 | c_2] \Pr[\neg c_1 | c_2] (\Pr[e | c_1 \land c_2] - \Pr[e | \neg c_1 \land c_2]); \\
Psg[e_, c_] & := -\text{CSg}[\neg e, c]; \\
Psg[e_, c_1, c_2] & := -\text{CSg}[\neg e, c_1, c_2]; \\
\end{align*}
\]

Cheng

\[
\begin{align*}
\text{CSc}[e_, c_] & := \frac{\Pr[e | c] - \Pr[e | \neg c]}{\Pr[\neg e | \neg c]}; \\
\text{CSc}[e_, c_1, c_2] & := \frac{\Pr[e | c_1 \land c_2] - \Pr[e | \neg c_1 \land c_2]}{\Pr[\neg e | \neg c_1 \land c_2]}; \\
PSc[e_, c_] & := -\text{CSc}[\neg e, c]; \\
PSc[e_, c_1, c_2] & := -\text{CSc}[\neg e, c_1, c_2]; \\
\end{align*}
\]

Lewis Ratio

\[
\begin{align*}
\text{CSlr}[e_, c_] & := \Pr[e | c] / \Pr[e | \neg c]; \\
\text{CSlr}[e_, c_1, c_2] & := \Pr[e | c_1 \land c_2] / \Pr[e | \neg c_1 \land c_2]; \\
\end{align*}
\]
First rescaling of Lewis Ratio

\[
CSl1[e_, c_] := \frac{Pr[e | c] - Pr[e | c]}{Pr[e | c] + Pr[e | c]};
\]

\[
CSl1[e_, c1_, c2_] := \frac{Pr[e | c1 \land c2] - Pr[e | c1 \land c2]}{Pr[e | c1 \land c2] + Pr[e | c1 \land c2]};
\]

\[
PSl1[e_, c_] := -CSl1[\neg e, c];
\]

\[
PSl1[e_, c1_, c2_] := -CSl1[\neg e, c1, c2];
\]

Second rescaling of Lewis Ratio

\[
CSl2[e_, c_] := 1 - (1 / CSl1[e, c]);
\]

\[
CSl2[e_, c1_, c2_] := 1 - (1 / CSl1[e, c1, c2]);
\]

\[
PSl2[e_, c_] := -CSl2[\neg e, c];
\]

\[
PSl2[e_, c1_, c2_] := -CSl2[\neg e, c1, c2];
\]

Good

\[
CSi[j][e_, c_] := \frac{Pr[\neg e | \neg c]}{Pr[\neg e | c]};
\]

\[
CSi[j][e_, c1_, c2_] := \frac{Pr[\neg e | \neg c1 \land c2]}{Pr[\neg e | c1 \land c2]};
\]

First rescaling of Good

\[
CSi[j][e_, c_] := \frac{Pr[\neg e | \neg c] - Pr[\neg e | c]}{Pr[\neg e | \neg c] + Pr[\neg e | c]};
\]

\[
CSi[j][e_, c1_, c2_] := \frac{Pr[\neg e | \neg c1 \land c2] - Pr[\neg e | c1 \land c2]}{Pr[\neg e | c1 \land c2] + Pr[\neg e | c1 \land c2]};
\]

\[
PSi[j][e_, c_] := -CSi[j][\neg e, c];
\]

\[
PSi[j][e_, c1_, c2_] := -CSi[j][\neg e, c1, c2];
\]

Second rescaling of Good

\[
CSi[j][e_, c_] := 1 - (1 / CSi[j][e, c]);
\]

\[
CSi[j][e_, c1_, c2_] := 1 - (1 / CSi[j][e, c1, c2]);
\]

\[
PSi[j][e_, c_] := -CSi[j][\neg e, c];
\]

\[
PSi[j][e_, c1_, c2_] := -CSi[j][\neg e, c1, c2];
\]

Scale Verification

In this section, we verify that all the (possibly rescaled) measures we’ll examine below, are on a [-1,1] scale. We do this using the function \texttt{PrRange} (now part of the \texttt{PrSAT} package) which calculates the range of a probabilistic expression (first argument) subject to probabilistic constraints (second argument):

Eells

\[
PrRange[CSi[e, c], Pr[e | c] \geq Pr[E]]
\]

\{0, 1\}

\[
PrRange[PSi[e, c], Pr[e | c] \leq Pr[E]]
\]

\{-1, 0\}

Suppes
Here, we verify the inter-definability relations stated in Table 3 of the paper, using the function `PrReduce` from the `PrSAT` package:
Suppes

\[
\text{PrReduce}([\text{CSe}[E, C] = \text{Pr}[\sim C] \text{CSe}[E, C]])
\]

True

Galton

\[
\text{PrReduce}([\text{CSg}[E, C] = 4 \text{Pr}[C] \text{Pr}[\sim C] \text{CSe}[E, C] = 4 \text{Pr}[C] \text{CSs}[E, C]])
\]

True

Cheng

\[
\text{PrReduce}([\text{CSc}[E, C] = \frac{\text{CSs}[E, C]}{\text{Pr}[\sim E | \sim C]} = \frac{\text{CSs}[E, C]}{\text{Pr}[\sim E \land \sim C]} = \frac{\text{CSg}[E, C]}{4 \text{Pr}[C] \text{Pr}[\sim E \land \sim C]})
\]

True

Lewis Ratio

\[
\text{PrReduce}([\text{CSl}[1][E, C] = \frac{\text{CSl}[1][E, C] - 1}{\text{CSl}[1][E, C] + 1} = \frac{\text{CSs}[E, C]}{\text{Pr}[E \land \sim C] + \text{Pr}[E \land \sim C]})
\]

True

\[
\text{PrReduce}([\text{CSl}[2][E, C] = \frac{1}{\text{CSl}[1][E, C]} = \frac{\text{CSs}[E, C]}{\text{Pr}[E \land \sim C] + \text{Pr}[E \land \sim C]})
\]

True

Good

\[
\text{PrReduce}[\text{CSj}[E, C] = \text{CSl}[\sim E, \sim C]]
\]

True

\[
\text{PrReduce}([\text{CSj}[1][E, C] = \frac{\text{CSj}[1][E, C] - 1}{\text{CSj}[1][E, C] + 1} = \frac{\text{CSl}[1][\sim E, \sim C]}{\text{Pr}[\sim E \land \sim C] + \text{Pr}[\sim E \land \sim C]})
\]

True

\[
\text{PrReduce}([\text{CSj}[2][E, C] = \frac{1}{\text{CSj}[1][E, C]} = \frac{\text{CSs}[E, C]}{\text{Pr}[\sim E \land \sim C] + \text{Pr}[\sim E \land \sim C]})
\]

True

### Ordinal Relationship Verification (Table 4)

In this section, we verify all the ordinal relationships between all pairs of measures — as recorded in Table 4 of the paper (going from the first row, downward by rows). Here, we use PrSAT to search for models of the denials of the various ordinal relationships. If a model is found, this shows that the ordinal relationship in question does not hold (and the model given is a concrete counter-model to the ordinal relationship in question). If no model is found, then the ordinal relationship in question does hold.

### Eells & Suppes

\(\text{CSe}\) and \(\text{CSs}\) are not ordinally equivalent in general (they are not G-E):
PrSAT[$\{\text{CSe[E1, C1]} \supset \text{CSe[E2, C2]}, \text{CSs[E1, C1]} \subset \text{CSs[E2, C2]}\}, \text{Probabilities} \rightarrow \text{Regular}$]

\[
\begin{align*}
\{ \{ C_1 &\rightarrow \{ a_2, a_5, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16} \},
C_2 \rightarrow \{ a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16} \}, \\
E_1 &\rightarrow \{ a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16} \},
E_2 \rightarrow \{ a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16} \}, \\
\Omega &\rightarrow \{ a_1, a_2, a_4, a_5, a_7, a_8, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16} \} \}, \\
\{ a_1 &\rightarrow 5683323.220533.249.039, a_2 \rightarrow 20, a_3 \rightarrow 345, a_4 \rightarrow 125, a_5 \rightarrow 524, a_6 \rightarrow 41, a_7 \rightarrow 33, \\
a_8 &\rightarrow 1, a_9 \rightarrow 1, a_{10} \rightarrow 1, a_{11} \rightarrow 999, a_{12} \rightarrow 999, a_{13} \rightarrow 57, a_{14} \rightarrow 103, a_{15} \rightarrow 1, a_{16} \rightarrow 5 \} \\
\}
\]

CSe and CSs are not ordinally equivalent in the class of cases with two causes and a single effect (they are not I-E):

PrSAT[$\{\{\text{CSe[E, C]} \supset \text{CSe[E, C]}, \text{CSs[E, C]} \subset \text{CSs[E, C]}\}, \text{Probabilities} \rightarrow \text{Regular}$]

\[
\begin{align*}
\{ \{ C_1 &\rightarrow \{ a_2, a_5, a_6, a_8, a_{13}, a_{14}, a_{16} \},
C_2 \rightarrow \{ a_3, a_5, a_7, a_8, a_{13} \}, \\
E_1 &\rightarrow \{ a_4, a_5, a_7, a_8, a_{13} \},
E_2 \rightarrow \{ a_5, a_8, a_{13}, a_{14}, a_{15}, a_{16} \}, \\
\Omega &\rightarrow \{ a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \} \}, \\
\{ a_1 &\rightarrow 1400671.985, a_2 \rightarrow 981, a_3 \rightarrow 19, a_4 \rightarrow 58, a_5 \rightarrow 18, a_6 \rightarrow 43, a_7 \rightarrow 886, a_8 \rightarrow 20 \} \\
\}
\]

CSe and CSs are ordinally equivalent in the class of cases with two causes and a single case (they are I-II):

PrSAT[$\{\{\text{CSe[E1, C]} \supset \text{CSe[E2, C]}, \text{CSs[E1, C]} \subset \text{CSs[E2, C]}\}, \text{Probabilities} \rightarrow \text{Regular}$]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\[
\]

- Eells & Galton

CSe and CSg are not ordinally equivalent in general (they are not G-E):

PrSAT[$\{\{\text{CSe[E1, C]} \supset \text{CSe[E2, C]}, \text{CSg[E1, C]} \subset \text{CSg[E2, C]}\}, \text{Probabilities} \rightarrow \text{Regular}$]

\[
\begin{align*}
\{ \{ C_1 &\rightarrow \{ a_2, a_5, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16} \},
C_2 \rightarrow \{ a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16} \}, \\
E_1 &\rightarrow \{ a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16} \},
E_2 \rightarrow \{ a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16} \}, \\
\Omega &\rightarrow \{ a_1, a_2, a_4, a_5, a_7, a_8, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16} \} \}, \\
\{ a_1 &\rightarrow 676101.601.782.601.735.273, a_2 \rightarrow 589, a_3 \rightarrow 25, a_4 \rightarrow 106, a_5 \rightarrow 56, a_6 \rightarrow 531, a_7 \rightarrow 107, \\
a_8 &\rightarrow 1, a_9 \rightarrow 2, a_{10} \rightarrow 97, a_{11} \rightarrow 952, a_{12} \rightarrow 23, a_{13} \rightarrow 999, a_{14} \rightarrow 54, a_{15} \rightarrow 179, a_{16} \rightarrow 34 \} \\
\}
\]

CSe and CSg are not ordinally equivalent in the class of cases with two causes and a single effect (they are not I-E):

PrSAT[$\{\{\text{CSe[E, C]} \supset \text{CSe[E, C]}, \text{CSg[E, C]} \subset \text{CSg[E, C]}\}, \text{Probabilities} \rightarrow \text{Regular}$]

\[
\begin{align*}
\{ \{ C_1 &\rightarrow \{ a_2, a_5, a_6, a_8, a_{13}, a_{14}, a_{16} \},
C_2 \rightarrow \{ a_3, a_5, a_7, a_8, a_{13} \}, \\
E_1 &\rightarrow \{ a_4, a_5, a_7, a_8, a_{13} \},
E_2 \rightarrow \{ a_5, a_8, a_{13}, a_{14}, a_{15}, a_{16} \}, \\
\Omega &\rightarrow \{ a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \} \}, \\
\{ a_1 &\rightarrow 488173, a_2 \rightarrow 70, a_3 \rightarrow 21, a_4 \rightarrow 48, a_5 \rightarrow 56, a_6 \rightarrow 152, a_7 \rightarrow 29, a_8 \rightarrow 21 \} \\
\}
\]

CSe and CSg are ordinally equivalent in the class of cases with two effects and a single cause (they are II-E):

PrSAT[$\{\{\text{CSe[E1, C]} \supset \text{CSe[E2, C]}, \text{CSg[E1, C]} \subset \text{CSg[E2, C]}\}, \text{Probabilities} \rightarrow \text{Regular}$]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\[
\]

- Eells & Cheng

CSe and CSs are not ordinally equivalent in general (they are not G-E):
\[
\text{PrSAT}\{\text{CSe}[E_1, C_1] \geq \text{CSe}[E_2, C_2], \text{CSc}[E_1, C_1] < \text{CSc}[E_2, C_2]\}, \text{Probabilities} \rightarrow \text{Regular}
\]

\[
\{\begin{array}{l}
\{C_1 \rightarrow \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, C_2 \rightarrow \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \\
E_1 \rightarrow \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, E_2 \rightarrow \{a_5, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \\
\Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}, \\
\{a_1 \rightarrow \frac{21008}{197882}, a_2 \rightarrow \frac{47874}{708747}, a_3 \rightarrow \frac{4}{5}, a_4 \rightarrow \frac{59}{45}, a_5 \rightarrow \frac{15}{65}, a_6 \rightarrow \frac{23}{10}, a_7 \rightarrow \frac{120}{7}, \\
a_8 \rightarrow \frac{1}{3}, a_9 \rightarrow \frac{2}{103}, a_{10} \rightarrow \frac{10}{118}, a_{11} \rightarrow \frac{1}{67}, a_{12} \rightarrow \frac{2}{31}, a_{13} \rightarrow \frac{3}{63}, a_{14} \rightarrow \frac{1}{320}, a_{15} \rightarrow \frac{1}{38}, a_{16} \rightarrow \frac{2}{65}\}\end{array}\}
\]

\text{CSe and CSc are not ordinally equivalent in the class of cases with two causes and a single effect (they are not I-E):}

\[
\text{PrSAT}\{\text{CSe}[E_1, C] \geq \text{CSe}[E_2, C], \text{CSc}[E_1, C] < \text{CSc}[E_2, C]\}, \text{Probabilities} \rightarrow \text{Regular}
\]

\[
\{\begin{array}{l}
\{C_1 \rightarrow \{a_2, a_5, a_6, a_8\}, C_2 \rightarrow \{a_3, a_5, a_7, a_8\}, E \rightarrow \{a_4, a_6, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 \rightarrow \frac{6552877}{19684665}, a_2 \rightarrow \frac{2}{23}, a_3 \rightarrow \frac{2}{39}, a_4 \rightarrow \frac{2}{63}, a_5 \rightarrow \frac{3}{105}, a_6 \rightarrow \frac{3}{46}, a_7 \rightarrow \frac{4}{33}\}\end{array}\}
\]

\text{CSe and CSc are not ordinally equivalent in the class of cases with two effects and a single cause (they are not II-E):}

\[
\text{PrSAT}\{\text{CSe}[E_1, C] \geq \text{CSe}[E_2, C], \text{CSc}[E_1, C] < \text{CSc}[E_2, C]\}, \text{Probabilities} \rightarrow \text{Regular}
\]

\[
\{\begin{array}{l}
\{C \rightarrow \{a_2, a_5, a_6, a_8\}, E_1 \rightarrow \{a_3, a_5, a_7, a_8\}, E_2 \rightarrow \{a_4, a_6, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 \rightarrow \frac{10764}{96005}, a_2 \rightarrow \frac{882}{820}, a_3 \rightarrow \frac{10}{882}, a_4 \rightarrow \frac{1}{57}, a_5 \rightarrow \frac{1}{693}, a_6 \rightarrow \frac{31}{31}, a_7 \rightarrow \frac{999}{999}, a_8 \rightarrow \frac{46}{123}\}\end{array}\}
\]

\text{Eells & Lewis Ratio}

\text{CSe and CS\text{\textsc{Ir}} are not ordinally equivalent in general (they are not G-E):}

\[
\text{PrSAT}\{\text{CSe}[E_1, C_1] \geq \text{CSe}[E_2, C_2], \text{CS\text{\textsc{Ir}}[E_1, C_1] < CS\text{\textsc{Ir}}[E_2, C_2]\}, \text{Probabilities} \rightarrow \text{Regular}
\]

\[
\{\begin{array}{l}
\{C_1 \rightarrow \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, C_2 \rightarrow \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \\
E_1 \rightarrow \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, E_2 \rightarrow \{a_5, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \\
\Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 \rightarrow \frac{8004879}{21360}, a_2 \rightarrow \frac{2}{5}, a_3 \rightarrow \frac{2}{5}, a_4 \rightarrow \frac{3}{70}, a_5 \rightarrow \frac{6}{53}, a_6 \rightarrow \frac{6}{37}, a_7 \rightarrow \frac{1}{60}, a_8 \rightarrow \frac{1}{11}, \\
a_9 \rightarrow \frac{1}{56}, a_{10} \rightarrow \frac{1}{469}, a_{11} \rightarrow \frac{1}{30}, a_{12} \rightarrow \frac{1}{999}, a_{13} \rightarrow \frac{1}{170}, a_{14} \rightarrow \frac{1}{69}, a_{15} \rightarrow \frac{1}{60}\}\end{array}\}
\]

\text{CSe and CS\text{\textsc{Ir}} are not ordinally equivalent in the class of cases with two causes and a single effect (they are not I-E):}

\[
\text{PrSAT}\{\text{CSe}[E_1, C_1] \geq \text{CSe}[E_2, C_2], \text{CS\text{\textsc{Ir}}[E_1, C_1] < CS\text{\textsc{Ir}}[E_2, C_2]\}, \text{Probabilities} \rightarrow \text{Regular}
\]

\[
\{\begin{array}{l}
\{C_1 \rightarrow \{a_2, a_5, a_6, a_8\}, C_2 \rightarrow \{a_3, a_5, a_7, a_8\}, E \rightarrow \{a_4, a_6, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 \rightarrow \frac{1656667}{1727710}, a_2 \rightarrow \frac{1}{999}, a_3 \rightarrow \frac{1}{20}, a_4 \rightarrow \frac{52}{5}, a_5 \rightarrow \frac{1}{999}, a_6 \rightarrow \frac{1}{798}, a_7 \rightarrow \frac{1}{18}, a_8 \rightarrow \frac{25}{57}\}\end{array}\}
\]

\text{CSe and CS\text{\textsc{Ir}} are not ordinally equivalent in the class of cases with two effects and a single cause (they are not II-E):}

\[
\text{PrSAT}\{\text{CSe}[E_1, C] \geq \text{CSe}[E_2, C], \text{CS\text{\textsc{Ir}}[E_1, C] < CS\text{\textsc{Ir}}[E_2, C]\}, \text{Probabilities} \rightarrow \text{Regular}
\]

\[
\{\begin{array}{l}
\{C \rightarrow \{a_2, a_5, a_6, a_8\}, E_1 \rightarrow \{a_3, a_5, a_7, a_8\}, E_2 \rightarrow \{a_4, a_6, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 \rightarrow \frac{18661411667}{75416368440}, a_2 \rightarrow \frac{2}{920}, a_3 \rightarrow \frac{2}{11}, a_4 \rightarrow \frac{339}{5}, a_5 \rightarrow \frac{2}{19}, a_7 \rightarrow \frac{267}{267}, a_8 \rightarrow \frac{3}{52}\}\end{array}\}
\]

\text{Eells & Good}

\text{CSe and CS\text{\textsc{I\textsc{I}}} are not ordinally equivalent in general (they are not G-E):}
PrSAT[\{CSE[E1, C1] \geq CSE[E2, C2], CStj[E1, C1] < CStj[E2, C2]\}, \text{Probabilities} \rightarrow \text{Regular}]

\begin{align*}
\{ & (C \rightarrow \{a_2, a_5, a_6, a_7\}, E \rightarrow \{a_4, a_5, a_7, a_8\}, \Omega \rightarrow \{a_1, a_3, a_4, a_5, a_6, a_7, a_8\}), \\
& \{a_1 \rightarrow 1410773791657, a_2 \rightarrow 21, a_3 \rightarrow 11, a_4 \rightarrow 3, a_5 \rightarrow 37, a_6 \rightarrow 2, a_7 \rightarrow 1, a_8 \rightarrow 7\} \}
\end{align*}

CSE and CS\text{S}\text{t}j are not ordinally equivalent in the class of cases with two causes and a single effect (they are not I-E):

PrSAT[\{CSE[E1, C1] \geq CSE[E2, C2], CStj[E1, C1] < CStj[E2, C2]\}, \text{Probabilities} \rightarrow \text{Regular}]

\begin{align*}
\{ & (C \rightarrow \{a_2, a_5, a_6, a_7, a_8\}, E \rightarrow \{a_4, a_5, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}), \\
& \{a_1 \rightarrow 3280116539, a_2 \rightarrow 12793606020, a_3 \rightarrow 13, a_4 \rightarrow 27, a_5 \rightarrow 59, a_6 \rightarrow 20, a_7 \rightarrow 17, a_8 \rightarrow 3\} \}
\end{align*}

Suppes & Galton

CSs and CSG are not ordinally equivalent in general (they are not G-E):

PrSAT[\{CSs[E1, C1] \geq CSs[E2, C2], CSG[E1, C1] < CSG[E2, C2]\}, \text{Probabilities} \rightarrow \text{Regular}]

\begin{align*}
\{ & (C \rightarrow \{a_2, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}, \\
& E \rightarrow \{a_4, a_5, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}, \\
& \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}), \\
& \{a_1 \rightarrow 6922763391162, a_2 \rightarrow 3, a_3 \rightarrow 1, a_4 \rightarrow 43, a_5 \rightarrow 43, a_6 \rightarrow 1, a_7 \rightarrow 58, a_8 \rightarrow 1\} \}
\end{align*}

CSs and CSG are not ordinally equivalent in the class of cases with two causes and a single effect (they are not I-E):

PrSAT[\{CSs[E1, C1] \geq CSs[E2, C2], CSG[E1, C1] < CSG[E2, C2]\}, \text{Probabilities} \rightarrow \text{Regular}]

\begin{align*}
\{ & (C \rightarrow \{a_2, a_5, a_6, a_7, a_8\}, E \rightarrow \{a_4, a_6, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}), \\
& \{a_1 \rightarrow 18490511, a_2 \rightarrow 2, a_3 \rightarrow 3, a_4 \rightarrow 1, a_5 \rightarrow 1, a_6 \rightarrow 1, a_7 \rightarrow 1, a_8 \rightarrow 1\} \}
\end{align*}

CSs and CSG are ordinally equivalent in the class of cases with two effects and a single cause (they are II-E):

PrSAT[\{CSs[E1, C1] \geq CSs[E2, C2], CSG[E1, C1] < CSG[E2, C2]\}, \text{Probabilities} \rightarrow \text{Regular}]

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}
PrSAT: [\{\text{CSs}[E_1, C_1] \geq \text{CSs}[E_2, C_2], \text{CSc}[E_1, C_1] < \text{CSc}[E_2, C_2]\}, \text{Probabilities} \rightarrow \text{Regular}]

\[
\{(C \rightarrow [a_2, a_5, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}], C \rightarrow [a_2, a_5, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}], E_1 \rightarrow [a_2, a_5, a_7, a_8, a_9, a_{10}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}], E_2 \rightarrow [a_2, a_5, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}], \Omega \rightarrow [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8] \}
\]
\[ \Pr\text{SAT}[\{\text{CS}_S[1, C], \text{CS}_S[2, C], \text{CS}_{ij}[1, C] < \text{CS}_{ij}[2, C]\}, \text{Probabilities} \rightarrow \text{Regular}] \]

\[ \left\{ \begin{array}{ll}
C & \rightarrow \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \\
E & \rightarrow \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \\
\Omega & \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 & \rightarrow \frac{1}{10538490604381}, a_2 \rightarrow \frac{5}{42}, a_3 \rightarrow \frac{999}{42}, a_4 \rightarrow \frac{42}{999}, a_5 \rightarrow \frac{1}{999}, a_6 \rightarrow \frac{1}{956}, a_7 \rightarrow \frac{624}{1972}, a_8 \rightarrow \frac{1}{37} \}
\end{array} \right. \]

\[ \Pr\text{SAT}[\{\text{CS}_S[1, C], \text{CS}_S[2, C], \text{CS}_{ij}[1, C] < \text{CS}_{ij}[2, C]\}, \text{Probabilities} \rightarrow \text{Regular}] \]

\[ \left\{ \begin{array}{ll}
C & \rightarrow \{a_2, a_5, a_6, a_8\}, \\
E & \rightarrow \{a_3, a_5, a_7, a_8\}, \\
\Omega & \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 & \rightarrow \frac{183180441981}{766827368230}, a_2 \rightarrow \frac{1}{211}, a_3 \rightarrow \frac{1}{19}, a_4 \rightarrow \frac{1}{22}, a_5 \rightarrow \frac{1}{35}, a_6 \rightarrow \frac{1}{109}, a_7 \rightarrow \frac{1}{43}, a_8 \rightarrow \frac{20}{53} \}
\end{array} \right. \]

\[ \Pr\text{SAT}[\{\text{CS}_S[1, C], \text{CS}_S[2, C], \text{CS}_{ij}[1, C] < \text{CS}_{ij}[2, C]\}, \text{Probabilities} \rightarrow \text{Regular}] \]

\[ \left\{ \begin{array}{ll}
C & \rightarrow \{a_2, a_5, a_6, a_8\}, \\
E & \rightarrow \{a_3, a_5, a_7, a_8\}, \\
\Omega & \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 & \rightarrow \frac{36642643319}{113226380280}, a_2 \rightarrow \frac{1}{65}, a_3 \rightarrow \frac{1}{999}, a_4 \rightarrow \frac{1}{54}, a_5 \rightarrow \frac{1}{158}, a_6 \rightarrow \frac{1}{89}, a_7 \rightarrow \frac{1}{120}, a_8 \rightarrow \frac{14}{31} \}
\end{array} \right. \]

- Galton & Cheng

\[ \text{CS}_G \text{ and } \text{CS}_C \text{ are not ordinally equivalent in general (they are not G-E):} \]

\[ \Pr\text{SAT}[\{\text{CS}_G[1, C], \text{CS}_G[2, C], \text{CS}_C[1, C] < \text{CS}_C[2, C]\}, \text{Probabilities} \rightarrow \text{Regular}] \]

\[ \left\{ \begin{array}{ll}
C & \rightarrow \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \\
E & \rightarrow \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \\
\Omega & \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 & \rightarrow \frac{16352691536717}{5625629461609}, a_2 \rightarrow \frac{1}{84}, a_3 \rightarrow \frac{1}{74}, a_4 \rightarrow \frac{1}{59}, a_5 \rightarrow \frac{1}{28}, a_6 \rightarrow \frac{1}{5}, a_7 \rightarrow \frac{1}{84}, a_8 \rightarrow \frac{1}{84}, a_9 \rightarrow \frac{1}{400}, a_{10} \rightarrow \frac{1}{423}, a_{11} \rightarrow \frac{1}{77}, a_{12} \rightarrow \frac{1}{23}, a_{13} \rightarrow \frac{1}{344}, a_{14} \rightarrow \frac{1}{77}, a_{15} \rightarrow \frac{1}{50} \}
\end{array} \right. \]

- Galton & Lewis Ratio

\[ \text{CS}_G \text{ and } \text{CS}_I \text{ are not ordinally equivalent in general (they are not G-E):} \]

\[ \Pr\text{SAT}[\{\text{CS}_G[1, C], \text{CS}_G[2, C], \text{CS}_I[1, C] < \text{CS}_I[2, C]\}, \text{Probabilities} \rightarrow \text{Regular}] \]

\[ \left\{ \begin{array}{ll}
C & \rightarrow \{a_2, a_5, a_6, a_8\}, \\
E & \rightarrow \{a_3, a_5, a_7, a_8\}, \\
\Omega & \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{a_1 & \rightarrow \frac{73624}{209715}, a_2 \rightarrow \frac{14}{41}, a_3 \rightarrow \frac{14}{41}, a_4 \rightarrow \frac{14}{55}, a_5 \rightarrow \frac{1}{124}, a_6 \rightarrow \frac{1}{41}, a_7 \rightarrow \frac{1}{31}, a_8 \rightarrow \frac{1}{60} \}
\end{array} \right. \]
PrSAT\([\{C \text{G}[E_1, C_1] \geq \text{CS}[E_2, C_2], \text{CS}[E_1, C_1] < \text{CS}[E_2, C_2]\}, \text{Probabilities \rightarrow Regular}\]

\[
\begin{align*}
\{C1 \rightarrow \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \\
E_1 \rightarrow \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \\
E_2 \rightarrow \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \\
\Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\begin{bmatrix}
\{1, 1, 7, 1, 13, 1, 30, 1, 827, 1, 36, 186, 1, 864, 1, 62\} \\
\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \\
\{1, 392, 1, 55, 1, 417, 1, 368, 1, 412, 1, 40\}
\end{bmatrix}
\end{align*}
\]

CSg and CSIr are not ordinarily equivalent in the class of cases with two causes and a single effect (they are not I-E):

PrSAT\([\{C \text{g}[E_1, C_1] \geq \text{CS}[E_2, C_2], \text{CS}[E_1, C_1] < \text{CS}[E_2, C_2]\}, \text{Probabilities \rightarrow Regular}\]

\[
\begin{align*}
\{C1 \rightarrow \{a_2, a_5, a_6, a_8\}, \\
E_1 \rightarrow \{a_3, a_5, a_7, a_8\}, \\
E_2 \rightarrow \{a_4, a_6, a_7, a_8\}, \\
\Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\begin{bmatrix}
\{1, 2, 3, 7, 7, 2\} \\
\{0, 0, 0, 0, 0, 0, 0, 0\} \\
\{1, 11, 1, 32, 1, 11\}
\end{bmatrix}
\end{align*}
\]

Galton & Good

CSg and CSij are not ordinarily equivalent in general (they are not G-E):

PrSAT\([\{C \text{g}[E_1, C_1] \geq \text{CS}[E_2, C_2], \text{CS}[E_1, C_1] < \text{CS}[E_2, C_2]\}, \text{Probabilities \rightarrow Regular}\]

\[
\begin{align*}
\{C \rightarrow \{a_2, a_5, a_6, a_8\}, \\
E_1 \rightarrow \{a_3, a_5, a_7, a_8\}, \\
E_2 \rightarrow \{a_4, a_6, a_7, a_8\}, \\
\Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\begin{bmatrix}
\{1, 5, 1, 1, 1, 2, 3, 1, 582, 1, 60, 1, 49, 1, 32, 1, 38\} \\
\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \\
\{2, 151, 3, 109, 3, 22, 1, 253, 1, 103, 1, 129\}
\end{bmatrix}
\end{align*}
\]

CSg and CSij are not ordinarily equivalent in the class of cases with two effects and a single cause (they are not I-E):

PrSAT\([\{C \text{g}[E_1, C_1] \geq \text{CS}[E_2, C_2], \text{CS}[E_1, C_1] < \text{CS}[E_2, C_2]\}, \text{Probabilities \rightarrow Regular}\]

\[
\begin{align*}
\{C \rightarrow \{a_2, a_5, a_6, a_8\}, \\
E_1 \rightarrow \{a_3, a_5, a_7, a_8\}, \\
E_2 \rightarrow \{a_4, a_6, a_7, a_8\}, \\
\Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\begin{bmatrix}
\{5, 129, 1, 632, 1, 233, 1, 17, 1, 47, 1, 47\} \\
\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \\
\{3, 4, 1, 2, 1, 129\}
\end{bmatrix}
\end{align*}
\]

CSg and CSij are not ordinarily equivalent in the class of cases with two effects and a single cause (they are not II-E):

PrSAT\([\{C \text{g}[E_1, C_1] \geq \text{CS}[E_2, C_2], \text{CS}[E_1, C_1] < \text{CS}[E_2, C_2]\}, \text{Probabilities \rightarrow Regular}\]

\[
\begin{align*}
\{C \rightarrow \{a_2, a_5, a_6, a_8\}, \\
E_1 \rightarrow \{a_3, a_5, a_7, a_8\}, \\
E_2 \rightarrow \{a_4, a_6, a_7, a_8\}, \\
\Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\begin{bmatrix}
\{156, 169, 1, 3, 1, 38, 1, 24, 1, 19\} \\
\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \\
\{1, 1, 1, 1, 1, 1\}
\end{bmatrix}
\end{align*}
\]

Cheng & Lewis Ratio

CSc and CSIr are not ordinarily equivalent in general (they are not G-E):
PrSAT[[CSc[E1, C1] ≥ CSc[E2, C2], CSlr[E1, C1] < CSlr[E2, C2]], Probabilities → Regular]

\{
\{C1 \to \{a2, a5, a6, a8\}, C2 \to \{a3, a6, a9, a10, a12, a13, a15, a16\},
E1 \to \{a4, a7, a9, a11, a12, a14, a15, a16\}, E2 \to \{a5, a8, a10, a11, a13, a14, a15, a16\},
Ω \to \{a1, a2, a3, a4, a5, a6, a7, a8\}\}
\[
\frac{71659}{73059} \frac{945}{028} \frac{737}{451} \frac{874}{2427}, \frac{a2}{999}, \frac{a3}{517}, \frac{a4}{25}, \frac{a5}{989}, \frac{a6}{999}, \frac{a7}{14},
\frac{a8}{999}, \frac{a9}{53}, \frac{a10}{94}, \frac{a11}{999}, \frac{a12}{5}, \frac{a13}{153}, \frac{a14}{5}, \frac{a15}{1}, \frac{a16}{47}\}
\}

CSc and CSlr are not ordinarily equivalent in the class of cases with two effects and a single cause (they are not I-E):

PrSAT[[CSc[E, C] ≥ CSc[E, C2], CSlr[E, C] < CSlr[E2, C]], Probabilities → Regular]

\{
\{C1 \to \{a2, a5, a6, a8\}, C2 \to \{a3, a5, a7, a8\}, E \to \{a4, a6, a7, a8\}, Ω \to \{a1, a2, a3, a4, a5, a6, a7, a8\}\},
\[
\frac{3229}{11888} \frac{633}{100}, \frac{a2}{72}, \frac{a3}{11}, \frac{a4}{840}, \frac{a5}{999}, \frac{a6}{153}, \frac{a7}{5}, \frac{a8}{2}\}
\}

CSc and CSlr are not ordinarily equivalent in the class of cases with two effects and a single cause (they are not II-E):

PrSAT[[CSc[E1, C] ≥ CSc[E2, C], CSlr[E1, C] < CSlr[E2, C]], Probabilities → Regular]

\{
\{C \to \{a2, a5, a6, a8\}, E1 \to \{a3, a5, a7, a8\}, E2 \to \{a4, a6, a7, a8\}, Ω \to \{a1, a2, a3, a4, a5, a6, a7, a8\}\},
\[
\frac{1417}{4609} \frac{850}{269280}, \frac{a2}{29}, \frac{a3}{11}, \frac{a4}{582}, \frac{a5}{60}, \frac{a6}{49}, \frac{a7}{32}, \frac{a8}{38}\}
\}

- Cheng & Good

CSc and CSlr are ordinarily equivalent in general (they are G-E):

PrSAT[[CSc[E1, C1] ≥ CSc[E2, C2], CSlr[E1, C1] < CSlr[E2, C2]], Probabilities → Regular]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\{
\}

- Lewis Ratio & Good

CSlr and CSlr are not ordinarily equivalent in general (they are not G-E):

PrSAT[[CSlr[E1, C1] ≥ CSlr[E2, C2], CSlr[E1, C1] < CSlr[E2, C2]], Probabilities → Regular]

\{
\{C1 \to \{a2, a5, a6, a8\}, C2 \to \{a3, a6, a9, a10, a12, a13, a15, a16\},
E1 \to \{a4, a7, a9, a11, a12, a14, a15, a16\}, E2 \to \{a5, a8, a10, a11, a13, a14, a15, a16\},
Ω \to \{a1, a2, a3, a4, a5, a6, a7, a8\}\},
\[
\frac{1927}{6545} \frac{705}{193} \frac{715}{215} \frac{645}{139} \frac{624}{594} \frac{223}{505} \frac{873}{40}, \frac{a2}{68}, \frac{a3}{986}, \frac{a4}{640}, \frac{a5}{33}, \frac{a6}{139}, \frac{a7}{35},
\frac{a8}{56}, \frac{a9}{985}, \frac{a10}{71}, \frac{a11}{998}, \frac{a12}{521}, \frac{a13}{52}, \frac{a14}{56}, \frac{a15}{132}, \frac{a16}{82}\}
\}

CSlr and CSlr are not ordinarily equivalent in the class of cases with two causes and a single effect (they are not I-E):

PrSAT[[CSlr[E, C] ≥ CSlr[E, C2], CSlr[E, C] < CSlr[E2, C]], Probabilities → Regular]

\{
\{C1 \to \{a2, a5, a6, a8\}, C2 \to \{a3, a5, a7, a8\}, E \to \{a4, a6, a7, a8\}, Ω \to \{a1, a2, a3, a4, a5, a6, a7, a8\}\},
\[
\frac{50912}{162069} \frac{868509}{276936}, \frac{a2}{97}, \frac{a3}{24}, \frac{a4}{86}, \frac{a5}{47}, \frac{a6}{38}, \frac{a7}{111}, \frac{a8}{49}\}
\}

CSlr and CSlr are not ordinarily equivalent in the class of cases with two effects and a single cause (they are not II-E):
Continuity Property Verification (Table 5)

In this section, we verify the continuity properties of all the measures, as reported in Table 5 of the paper (again, using PrSAT, as above — so if a model is found, then it is a counterexample to the salient continuity property, and if no models are found, then the salient continuity property holds generally in that case).

Causation-Prevention Continuity (CPC)

Eells

PrSAT\{[CSe[\neg Y, X] \neq -CSe[\neg \neg Y, \neg X]]\}

PrSAT::srchfail: Search phase failed; attempting FindInstance

Suppes

PrSAT\{[CSs[\neg Y, X] \neq -CSe[\neg \neg Y, \neg X]]\}

PrSAT::srchfail: Search phase failed; attempting FindInstance

Galton

PrSAT\{[CSg[\neg Y, X] \neq -CSe[\neg \neg Y, \neg X]]\}

PrSAT::srchfail: Search phase failed; attempting FindInstance

Cheng

PrSAT\{[CSc[\neg Y, X] \neq -CSc[\neg \neg Y, \neg X]], Probabilities \rightarrow Regular\}

\{\{X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4\}\}, \{a_1 \rightarrow \frac{7510271}{14910716}, a_2 \rightarrow \frac{1}{943}, a_3 \rightarrow \frac{1}{268}, a_4 \rightarrow \frac{29}{59}\}\}

Lewis Ratio (first rescaling)

PrSAT\{[CSIr1[\neg Y, X] \neq -CSIr1[\neg \neg Y, \neg X]], Probabilities \rightarrow Regular\}

\{\{X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4\}\}, \{a_1 \rightarrow \frac{997}{1998}, a_2 \rightarrow \frac{1}{4}, a_3 \rightarrow \frac{1}{999}, a_4 \rightarrow \frac{1}{4}\}\}

Lewis Ratio (second rescaling)

PrSAT\{[CSIr2[\neg Y, X] \neq -CSIr2[\neg \neg Y, \neg X]], Probabilities \rightarrow Regular\}

\{\{X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4\}\}, \{a_1 \rightarrow \frac{4031}{4091020}, a_2 \rightarrow \frac{108}{215}, a_3 \rightarrow \frac{35}{71}, a_4 \rightarrow \frac{1}{268}\}\}

Good (first rescaling)

PrSAT\{[CSij1[\neg Y, X] \neq -CSij1[\neg \neg Y, \neg X]], Probabilities \rightarrow Regular\}

\{\{X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4\}\}, \{a_1 \rightarrow \frac{505481}{1010988}, a_2 \rightarrow \frac{63}{253}, a_3 \rightarrow \frac{1}{999}, a_4 \rightarrow \frac{1}{4}\}\}
Good (second rescaling)

PrSAT[\{CSij2[Y, X] \neq -CSij2[\sim Y, X]\}, Probabilities \rightarrow Regular]

\{(X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \sim \rightarrow \{a_1, a_2, a_3, a_4\}), \{a_1 \rightarrow \frac{119333}{244790}, a_2 \rightarrow \frac{1}{910}, a_3 \rightarrow \frac{1}{269}, a_4 \rightarrow \frac{33}{65}\}\}

- Causation-Omission Continuity (COC)

Eells

PrSAT[\{CSe[Y, X] \neq -CSe[\sim Y, \sim X]\}, Probabilities \rightarrow Regular]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Suppes

PrSAT[\{CSs[Y, X] \neq -CSs[\sim Y, \sim X]\}, Probabilities \rightarrow Regular]

\{(X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \sim \rightarrow \{a_1, a_2, a_3, a_4\}), \{a_1 \rightarrow \frac{9896}{46953}, a_2 \rightarrow \frac{1}{999}, a_3 \rightarrow \frac{1}{999}, a_4 \rightarrow \frac{37}{47}\}\}

Galton

PrSAT[\{CSg[Y, X] \neq -CSg[\sim Y, \sim X]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Cheng

PrSAT[\{CSc[Y, X] \neq -CSc[\sim Y, \sim X]\}, Probabilities \rightarrow Regular]

\{(X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \sim \rightarrow \{a_1, a_2, a_3, a_4\}), \{a_1 \rightarrow \frac{1}{300}, a_2 \rightarrow \frac{29}{60}, a_3 \rightarrow \frac{12}{25}, a_4 \rightarrow \frac{1}{30}\}\}

Lewis Ratio (first rescaling)

PrSAT[\{CSl1[Y, X] \neq -CSl1[\sim Y, \sim X]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Lewis Ratio (second rescaling)

PrSAT[\{CSl2[Y, X] \neq -CSl2[\sim Y, \sim X]\}, Probabilities \rightarrow Regular]

\{(X \rightarrow \{a_2, a_4\}, Y \rightarrow \{a_3, a_4\}, \sim \rightarrow \{a_1, a_2, a_3, a_4\}), \{a_1 \rightarrow \frac{2266503}{44695430}, a_2 \rightarrow \frac{1}{998}, a_3 \rightarrow \frac{1}{265}, a_4 \rightarrow \frac{84}{169}\}\}

Good (first rescaling)

PrSAT[\{CSij1[Y, X] \neq -CSij1[\sim Y, \sim X]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Good (second rescaling)
\[
\text{PrSAT}[[\text{CSIj2}[Y, X] \neq \text{CSIj2}[Y, \neg X]], \text{Probabilities} \rightarrow \text{Regular}]
\]
\[
\{[X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \emptyset \to \{a_1, a_2, a_3, a_4\}], \{a_1 \rightarrow \frac{3304601}{6618375}, a_2 \rightarrow \frac{1}{265}, a_3 \rightarrow \frac{1}{999}, a_4 \rightarrow \frac{62}{125}\}\}
\]

- **Causation = Prevention By Omission Continuity (CPO)**

**Eells**

\[
\text{PrSAT}[[\text{CSE}[Y, X] \neq \text{CSE}[\neg Y, \neg X]]]
\]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\[
\{
\}
\]

**Suppes**

\[
\text{PrSAT}[[\text{CSs}[Y, X] \neq \text{CSs}[\neg Y, \neg X]], \text{Probabilities} \rightarrow \text{Regular}]
\]
\[
\{[X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \emptyset \to \{a_1, a_2, a_3, a_4\}], \{a_1 \rightarrow \frac{1105}{892107}, a_2 \rightarrow \frac{37}{47}, a_3 \rightarrow \frac{4}{19}, a_4 \rightarrow \frac{1}{999}\}\}
\]

**Galton**

\[
\text{PrSAT}[[\text{CSg}[Y, X] \neq \text{CSg}[\neg Y, \neg X]]]
\]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\[
\{
\}
\]

**Cheng**

\[
\text{PrSAT}[[\text{CSc}[Y, X] \neq \text{CSc}[\neg Y, \neg X]], \text{Probabilities} \rightarrow \text{Regular}]
\]
\[
\{[X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \emptyset \to \{a_1, a_2, a_3, a_4\}], \{a_1 \rightarrow \frac{995}{3996}, a_2 \rightarrow \frac{1}{999}, a_3 \rightarrow \frac{1}{4}, a_4 \rightarrow \frac{1}{2}\}\}
\]

**Lewis Ratio (first rescaling)**

\[
\text{PrSAT}[[\text{CSlr1}[Y, X] \neq \text{CSlr1}[\neg Y, \neg X]]]
\]
\[
\{[X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \emptyset \to \{a_1, a_2, a_3, a_4\}], \{a_1 \rightarrow \frac{1}{4}, a_2 \rightarrow 0, a_3 \rightarrow \frac{1}{4}, a_4 \rightarrow \frac{1}{2}\}\}
\]

**Lewis Ratio (second rescaling)**

\[
\text{PrSAT}[[\text{CSlr2}[Y, X] \neq \text{CSlr2}[\neg Y, \neg X]], \text{Probabilities} \rightarrow \text{Regular}]
\]
\[
\{[X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \emptyset \to \{a_1, a_2, a_3, a_4\}], \{a_1 \rightarrow \frac{505481}{1010988}, a_2 \rightarrow \frac{1}{4}, a_3 \rightarrow \frac{1}{999}, a_4 \rightarrow \frac{63}{253}\}\}
\]

**Good (first rescaling)**

\[
\text{PrSAT}[[\text{CSIj1}[Y, X] \neq \text{CSIj1}[\neg Y, \neg X]]]
\]
\[
\{[X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \emptyset \to \{a_1, a_2, a_3, a_4\}], \{a_1 \rightarrow \frac{1}{4}, a_2 \rightarrow \frac{1}{2}, a_3 \rightarrow \frac{1}{4}, a_4 \rightarrow 0\}\}
\]

**Good (second rescaling)**

\[
\text{PrSAT}[[\text{CSIj2}[Y, X] \neq \text{CSIj2}[\neg Y, \neg X]], \text{Probabilities} \rightarrow \text{Regular}]
\]
\[
\{[X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \emptyset \to \{a_1, a_2, a_3, a_4\}], \{a_1 \rightarrow \frac{995}{3996}, a_2 \rightarrow \frac{1}{2}, a_3 \rightarrow \frac{1}{4}, a_4 \rightarrow \frac{1}{999}\}\}
\]
Causal Independence (definitions, and two fundamental properties)

First, we define the various causal independence relations, for the various measures of causal strength:

\[
\text{ICSe}[E_\gamma, C_\gamma, C_\gamma'] := \text{CS}[E, C_\gamma, C_\gamma] = \text{CS}[E, C_\gamma', C_\gamma]
\]

\[
\text{ICS}[E_\gamma, C_\gamma, C_\gamma'] := \text{CS}[E, C_\gamma, C_\gamma] = \text{CS}[E, C_\gamma', C_\gamma]
\]

\[
\text{ICSg}[E_\gamma, C_\gamma, C_\gamma'] := \text{CSg}[E, C_\gamma, C_\gamma] = \text{CSg}[E, C_\gamma', C_\gamma]
\]

\[
\text{ICSc}[E_\gamma, C_\gamma, C_\gamma'] := \text{CSc}[E, C_\gamma, C_\gamma] = \text{CSc}[E, C_\gamma', C_\gamma]
\]

\[
\text{ICSlr}[E_\gamma, C_\gamma, C_\gamma'] := \text{CSlr}[E, C_\gamma, C_\gamma] = \text{CSlr}[E, C_\gamma', C_\gamma]
\]

Then, we set-up our background conditions (**BACK**), which include the following: (i) that \(C_1\) and \(C_2\) are unconditionally probabilistically independent, (ii) that \(C_1\) and \(C_2\) are both positively causally relevant to \(E\), i.e., that \(\text{Pr}[E|C_1]\) > \(\text{Pr}[E]\) and \(\text{Pr}[E|C_2]\) > \(\text{Pr}[E]\). Finally, to simplify the searches, we will also assume (as part of **BACK**) — without loss of generality in this context — (iii) that \(\text{Pr}[E|C_1] = 1/2\) and \(\text{Pr}[E] = 1/4\) and that \(\text{Pr}[E|C_2] = 1/2\) and \(\text{Pr}[E] = 1/4\). This last assumption [which is just a more precise way of asserting (ii)] could be relaxed, but the searches would take much longer to complete.

\[
\text{BACK} := \{ \text{Pr}[C_1 | C_2] = \text{Pr}[C_1], \text{Pr}[E | C_1] > \text{Pr}[E], \text{Pr}[E | C_2] > \text{Pr}[E],
\]

\[
\text{Pr}[E | C_1] = 1/2, \text{Pr}[E] = 1/4, \text{Pr}[E | C_2] = 1/2, \text{Pr}[E] = 1/4\}
\]

The following two fundamental properties involving causal Independence judgments are satisfied by all of our measures, given **BACK**:

- \(\text{ICS}(E, C_1, C_2)\) iff \(\text{ICS}(E, C_2, C_1)\) [Symmetry of **ICS** in \(C_1, C_2\)]
- \(\text{ICS}(E, C_1, C_2)\) iff \(\text{ICS}(E, C_1, C_2) = \text{ICS}(E, C_1)\) [Equivalence of conditional/unconditional definitions of **ICS**]

Here are **PrSAT**-verifications of these fundamental properties (given **BACK**), for each of our measures of causal strength. First, we define a non-equivalence relation \(\#\), to make it easier to assert that a logical equivalence fails to hold:

\[
p_\# q_\# := (p \equiv q) & (q \equiv p);
\]

- Eells

Symmetry of **ICSe** in \(C_1, C_2\), given **BACK**:

\[
\text{PrSAT}[\text{BACK} \cup \{ \text{ICSe}[E, C_1, C_2] \neq \text{ICSe}[E, C_2, C_1] \}]
\]

**PrSAT::srchfail**: Search phase failed; attempting FindInstance

\[
\}
\]

Equivalence of conditional/unconditional definitions of **ICSe**, given **BACK**:

\[
\text{PrSAT}[\text{BACK} \cup \{ \text{ICSe}[E, C_1, C_2] \neq \text{CS}[E, C_1, C_2] \}]
\]

**PrSAT::srchfail**: Search phase failed; attempting FindInstance

\[
\}
\]

- Suppes

Symmetry of **ICScs** in \(C_1, C_2\), given **BACK**:

\[
\text{PrSAT}[\text{BACK} \cup \{ \text{ICScs}[E, C_1, C_2] \neq \text{ICScs}[E, C_2, C_1] \}]
\]

**PrSAT::srchfail**: Search phase failed; attempting FindInstance

\[
\}
\]

Equivalence of conditional/unconditional definitions of **ICScs**, given **BACK**:

\[
\text{PrSAT}[\text{BACK} \cup \{ \text{ICScs}[E, C_1, C_2] \neq \text{CS}[E, C_1, C_2] \}]
\]

**PrSAT::srchfail**: Search phase failed; attempting FindInstance

\[
\}
Galton

Symmetry of $IC_{Sg}$ in $c_1, c_2$, given $BACK$:

$$PrSAT[BACK \cup \{IC_{Sg}[E, c_1, c_2] \neq IC_{Sg}[E, c_2, c_1]\}]$$

$PrSAT::srchfail$ : Search phase failed; attempting FindInstance

{} 

Equivalence of conditional/unconditional definitions of $IC_{Sg}$, given $BACK$:

$$PrSAT[BACK \cup \{IC_{Sg}[E, c_1, c_2] \neq (CS_{g}[E, c_1, c_2] == CS_{g}[E, c_1])\}]$$

$PrSAT::srchfail$ : Search phase failed; attempting FindInstance

{}

Cheng

Symmetry of $IC_{Sc}$ in $c_1, c_2$, given $BACK$:

$$PrSAT[BACK \cup \{IC_{Sc}[E, c_1, c_2] \neq IC_{Sc}[E, c_2, c_1]\}]$$

$PrSAT::srchfail$ : Search phase failed; attempting FindInstance

{}

Equivalence of conditional/unconditional definitions of $IC_{Sg}$, given $BACK$:

$$PrSAT[BACK \cup \{IC_{Sc}[E, c_1, c_2] \neq (CS_{c}[E, c_1, c_2] == CS_{c}[E, c_1])\}]$$

$PrSAT::srchfail$ : Search phase failed; attempting FindInstance

{}

Lewis Ratio

Symmetry of $IC_{Sr}$ in $c_1, c_2$, given $BACK$:

$$PrSAT[BACK \cup \{IC_{Sr}[E, c_1, c_2] \neq IC_{Sr}[E, c_2, c_1]\}]$$

$PrSAT::srchfail$ : Search phase failed; attempting FindInstance

{}

Equivalence of conditional/unconditional definitions of $IC_{Sr}$, given $BACK$:

$$PrSAT[BACK \cup \{IC_{Sr}[E, c_1, c_2] \neq (CS_{r}[E, c_1, c_2] == CS_{r}[E, c_1])\}]$$

$PrSAT::srchfail$ : Search phase failed; attempting FindInstance

{}

Good

Symmetry of $IC_{Si}$ in $c_1, c_2$, given $BACK$:

$$PrSAT[BACK \cup \{IC_{Si}[E, c_1, c_2] \neq IC_{Si}[E, c_2, c_1]\}]$$

$PrSAT::srchfail$ : Search phase failed; attempting FindInstance

{}

Equivalence of conditional/unconditional definitions of $IC_{Si}$, given $BACK$:
Agreement on Causal Independence Judgments (Table 6)

In this section, we verify the claims about agreement on independence judgments reported in Table 6 of the paper (using PrSAT, as above).

Eells & Suppes

CSe and CSs agree on all independence judgments (assuming BACK). First, we show that ICS[e, c1, c2] \lneq ICS[e, c1, c2], given BACK:

PrSAT[BACK \cup \{ICS[e, c1, c2], \lneq ICS[e, c1, c2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Then, we show that ICS[e, c1, c2] \lneq ICS[e, c1, c2], given BACK:

PrSAT[BACK \cup \{\lneq ICS[e, c1, c2], ICS[e, c1, c2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Finally, we show that there are some cases (satisfying BACK) in which CSe and CSs agree that c1 and c2 are independent causes of E (non-triviality):

PrSAT[BACK \cup \{ICS[e, c1, c2], ICS[e, c1, c2], Pr[c1] = \frac{1}{3}, \text{Probabilities } \rightarrow \text{Regular}\}]

\{(c1 \rightarrow \{a2, a5, a6, a8\}, c2 \rightarrow \{a3, a5, a7, a8\}, e \rightarrow \{a4, a6, a7, a8\}, o \rightarrow \{a1, a2, a3, a4, a5, a6, a7, a8\}!,

\{a1 \rightarrow \frac{391}{876}, a2 \rightarrow \frac{61}{438}, a3 \rightarrow \frac{10}{73}, a4 \rightarrow \frac{1}{876}, a5 \rightarrow \frac{2}{73}, a6 \rightarrow \frac{37}{438}, a7 \rightarrow \frac{6}{73}, a8 \rightarrow \frac{6}{73}\}\}

Eells & Galton

CSe and CSg agree on all independence judgments (assuming BACK). First, we show that ICS[e, c1, c2] \lneq ICS[e, c1, c2], given BACK:

PrSAT[BACK \cup \{ICS[e, c1, c2], \lneq ICS[e, c1, c2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Then, we show that ICS[e, c1, c2] \lneq ICS[e, c1, c2], given BACK:

PrSAT[BACK \cup \{\lneq ICS[e, c1, c2], ICS[e, c1, c2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

[]

Finally, we show that there are some cases (satisfying BACK) in which CSe and CSg agree that c1 and c2 are independent causes of E (non-triviality):
PrSAT\[BACK \cup \{ \text{ICS}e[E, C1, C2], \text{ICS}g[E, C1, C2], Pr[C1] = \frac{1}{3} \}, \text{Probabilities \to Regular} \]

\{ (C1 \to \{ a_2, a_5, a_6, a_8 \}, C2 \to \{ a_3, a_5, a_7, a_8 \}, E \to \{ a_4, a_6, a_7, a_8 \} , \Omega \to \{ a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \} \},
\{(a_1 \to 949, a_2 \to 74, a_3 \to 145, a_4 \to 1, a_5 \to 29, a_6 \to 5, a_7 \to 29, a_8 \to 29) \}

- **Eells & Cheng**

\text{CS}e \text{ and } \text{CS}c \text{ agree on no independence judgments (assuming BACK).}

PrSAT\[BACK \cup \{ \text{ICS}e[E, C1, C2], \text{ICS}c[E, C1, C2] \} \]

PrSAT::\text{srchfail}: Search phase failed; attempting \text{FindInstance}  
{}

- **Eells & Lewis Ratio**

\text{CS}e \text{ and } \text{CS}lr \text{ agree on no independence judgments (assuming BACK).}

PrSAT\[BACK \cup \{ \text{ICS}e[E, C1, C2], \text{ICS}lr[E, C1, C2] \} \]

PrSAT::\text{srchfail}: Search phase failed; attempting \text{FindInstance}  
{}

- **Eells & Good**

\text{CS}e \text{ and } \text{CS}ij \text{ agree on no independence judgments (assuming BACK).}

PrSAT\[BACK \cup \{ \text{ICS}e[E, C1, C2], \text{ICS}ij[E, C1, C2] \} \]

PrSAT::\text{srchfail}: Search phase failed; attempting \text{FindInstance}  
{}

- **Suppes & Galton**

\text{CS}s \text{ and } \text{CS}g \text{ agree on all independence judgments (assuming BACK). First, we show that ICS}s[E, C1, C2] \Rightarrow ICS}g[E, C1, C2], given BACK:}

PrSAT\[BACK \cup \{ \text{ICS}s[E, C1, C2], \neg ICS}g[E, C1, C2] \} \]

PrSAT::\text{srchfail}: Search phase failed; attempting \text{FindInstance}  
{}

Then, we show that ICS}g[E, C1, C2] \Rightarrow ICS}s[E, C1, C2], given BACK:

PrSAT\[BACK \cup \{ \neg ICS}s[E, C1, C2], ICS}g[E, C1, C2] \} \]

PrSAT::\text{srchfail}: Search phase failed; attempting \text{FindInstance}  
{}

Finally, we show that there are some cases (satisfying BACK) in which \text{CS}s \text{ and } \text{CS}g \text{ agree that C1 and C2 are independent causes of E (non-triviality):}
PrSAT[BACK \cup \{ICSs[E, C1, C2], ICSc[E, C1, C2], Pr[C1] = \frac{1}{3}\}, Probabilities \rightarrow Regular]

\{(C1 \rightarrow \{a_2, a_5, a_6, a_8\}, C2 \rightarrow \{a_3, a_5, a_7, a_8\}, E \rightarrow \{a_4, a_6, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}

\{a_1 \rightarrow \frac{949}{2124}, a_2 \rightarrow \frac{74}{531}, a_3 \rightarrow \frac{145}{1062}, a_4 \rightarrow \frac{1}{708}, a_5 \rightarrow \frac{29}{1062}, a_6 \rightarrow \frac{5}{59}, a_7 \rightarrow \frac{29}{354}, a_8 \rightarrow \frac{29}{354}\}

- Suppes & Cheng

CSs and CSc agree on no independence judgments (assuming BACK).

PrSAT[BACK \cup \{ICSs[E, C1, C2], ICSc[E, C1, C2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\{
\}

- Suppes & Lewis Ratio

CSs and CSlr agree on no independence judgments (assuming BACK).

PrSAT[BACK \cup \{ICSs[E, C1, C2], ICSc[E, C1, C2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\{
\}

- Suppes & Good

CSs and CSi\ j agree on no independence judgments (assuming BACK).

PrSAT[Union[BACK, \{ICSs[E, C1, C2], ICSc[E, C1, C2]\}]]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\{
\}

- Galton & Cheng

CSg and CSc agree on no independence judgments (assuming BACK).

PrSAT[BACK \cup \{ICSg[E, C1, C2], ICSc[E, C1, C2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\{
\}

- Galton & Lewis Ratio

CSg and CSlr agree on no independence judgments (assuming BACK).

PrSAT[BACK \cup \{ICSg[E, C1, C2], ICSc[E, C1, C2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\{
\}

- Galton & Good

CSg and CSi\ j agree on no independence judgments (assuming BACK).

PrSAT[BACK \cup \{ICSg[E, C1, C2], ICSc[E, C1, C2]\}]

PrSAT::srchfail: Search phase failed; attempting FindInstance

\{
\}
PrSAT[BACK ∪ \{ICSg[E, C1, C2], ICSi[j[E, C1, C2]\}]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\[
\]

- **Cheng & Lewis Ratio**

CSc and CSIr agree on \emph{no} independence judgments (assuming BACK).

PrSAT[BACK ∪ \{ICS [E, C1, C2], ICSi[j[E, C1, C2]\}]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\[
\]

- **Cheng & Good**

CSc and CSij agree on \emph{all} independence judgments (assuming BACK). First, we show that ICS [E, C1, C2] ⇒ ICSi[j[E, C1, C2]. given BACK:

PrSAT[BACK ∪ \{-ICS [E, C1, C2], ICSi[j[E, C1, C2]\}]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\[
\]

Then, we show that ICSi[j[E, C1, C2] ⇒ ICS [E, C1, C2]. given BACK:

PrSAT[BACK ∪ \{-ICS [E, C1, C2], ICSi[j[E, C1, C2]\}]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\[
\]

Finally, we show that there \emph{are} some cases (satisfying BACK) in which CSc and CSij agree that C1 and C2 are independent causes of E (non-triviality):

PrSAT[BACK ∪ \{ICS [E, C1, C2], ICSi[j[E, C1, C2], Pr[C1] = \frac{1}{3}, Probabilities → Regular\]

\[
\]

- **Lewis Ratio & Good**

CSIr and CSij agree on \emph{no} independence judgments (assuming BACK).

PrSAT[BACK ∪ \{ICSir[E, C1, C2], ICSi[j[E, C1, C2]\}]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\[
\]
Constraint (†) on the values of independent causal strengths

In this section, we will show that some of our measures m are such that:

(†) \( ICS_m[E, c_1, c_2] \Rightarrow CS_m[E, c_1] + CS_m[E, c_2] \leq 1. \)

That is, for some of our measures m, if \( c_1 \) and \( c_2 \) are independent causes of \( E \) according to m, then the individual m-causal-strengths of \( c_1 \) and \( c_2 \) cannot sum to more than 1. We will also show that some measures do not imply any such constraint (†) on independent individual causal strengths.

- **Eells**

  \( CS_e \) does entail (†):

  \[
  \text{PrSAT}[\{ICS_e[E, c_1, c_2], \Pr[c_1 | c_2] = \Pr[c_1], \CS_e[E, c_1] + \CS_e[E, c_2] > 1, \CS_e[E, c_1] > 0, \CS_e[E, c_2] > 0\}]
  \]

  PrSAT::srchfail : Search phase failed; attempting FindInstance

  

- **Suppes**

  \( CS_s \) does entail (†):

  \[
  \text{PrSAT}[\{ICS_s[E, c_1, c_2], \Pr[c_1 | c_2] = \Pr[c_1], \CS_s[E, c_1] + \CS_s[E, c_2] > 1, \CS_s[E, c_1] > 0, \CS_s[E, c_2] > 0\}]
  \]

  PrSAT::srchfail : Search phase failed; attempting FindInstance

- **Galton**

  \( CS_g \) does entail (†):

  \[
  \text{PrSAT}[\{ICS_g[E, c_1, c_2], \Pr[c_1 | c_2] = \Pr[c_1], \CS_g[E, c_1] + \CS_g[E, c_2] > 1, \CS_g[E, c_1] > 0, \CS_g[E, c_2] > 0\}]
  \]

  PrSAT::srchfail : Search phase failed; attempting FindInstance

- **Cheng**

  \( CS_c \) does not entail (†):

  \[
  \text{PrSAT}[\{ICS_c[E, c_1, c_2], \Pr[c_1 | c_2] = \Pr[c_1], \CS_c[E, c_1] + \CS_c[E, c_2] > 1, \CS_c[E, c_1] > 0, \CS_c[E, c_2] > 0, \Pr[c_1] = 1/2, \Pr[c_2] = 1/2, \text{Probabilities} \to \text{Regular}, \text{BypassSearch} \to \text{True}\}]
  \]

  \[
  \{c_1 \to \{a_2, a_5, a_6, a_8\}, c_2 \to \{a_3, a_5, a_7, a_8\}, E \to \{a_4, a_6, a_7, a_8\}, \emptyset \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},
  \]

  \[
  \{a_1 \to \frac{1}{6}, a_2 \to \frac{1}{6}, a_3 \to \frac{1}{16}, a_4 \to \frac{1}{16}, a_5 \to \frac{3}{128}, a_6 \to \frac{3}{16}, a_7 \to \frac{3}{16}, a_8 \to \frac{29}{128}\}\}
  \]

- **Lewis Ratio (first rescaling)**

  \( CS_{lr}1 \) does not entail (†):

  \[
  \]

  \[
  \]
PrSAT\{$\{\text{ICS}r[E, C1, C2], \text{Pr}[C1 \mid C2] = \text{Pr}[C1], \text{CS}r1[E, C1] + \text{CS}r1[E, C2] > 1, \text{CS}r1[E, C1] > 0, \text{CS}r1[E, C2] > 0, \text{Pr}[C1] = 1/2, \text{Pr}[C2] = 1/2\}$, Probabilities $\rightarrow$ Regular, BypassSearch $\rightarrow$ True

\[
\begin{align*}
\{C1 \rightarrow \{a2, a5, a6, a8\}, C2 \rightarrow \{a3, a5, a7, a8\}, E \rightarrow \{a4, a6, a7, a8\}, \Omega \rightarrow \{a1, a2, a3, a4, a5, a6, a7, a8\}\}, \\
\{a1 \rightarrow 31, a2 \rightarrow 15, a3 \rightarrow 3, a4 \rightarrow 1, a5 \rightarrow 1, a6 \rightarrow 1, a7 \rightarrow 1, a8 \rightarrow 1\}
\end{align*}
\]

- Lewis Ratio (second rescaling)

CSI\text{r}2 does not entail ($\uparrow$):

PrSAT\{$\{\text{ICS}r[E, C1, C2], \text{Pr}[C1 \mid C2] = \text{Pr}[C1], \text{CS}r2[E, C1] + \text{CS}r2[E, C2] > 1, \text{CS}r2[E, C1] > 0, \text{CS}r2[E, C2] > 0, \text{Pr}[C1] = 1/2, \text{Pr}[C2] = 1/2\}$, Probabilities $\rightarrow$ Regular, BypassSearch $\rightarrow$ True

\[
\begin{align*}
\{C1 \rightarrow \{a2, a5, a6, a8\}, C2 \rightarrow \{a3, a5, a7, a8\}, E \rightarrow \{a4, a6, a7, a8\}, \Omega \rightarrow \{a1, a2, a3, a4, a5, a6, a7, a8\}\}, \\
\{a1 \rightarrow 21, a2 \rightarrow 3, a3 \rightarrow 3, a4 \rightarrow 1, a5 \rightarrow 9, a6 \rightarrow 1, a7 \rightarrow 23, a8 \rightarrow 1\}
\end{align*}
\]

- Good (first rescaling)

CS\text{i}j1 does not entail ($\uparrow$):

PrSAT\{$\{\text{ICS}i[j[E, C1, C2], \text{Pr}[C1 \mid C2] = \text{Pr}[C1], \text{CS}i[j[E, C1] + \text{CS}i[j[E, C2] > 1, \text{CS}i[j[E, C1] > 0, \text{CS}i[j[E, C2] > 0, \text{Pr}[C1] = 1/2, \text{Pr}[C2] = 1/2\}$, Probabilities $\rightarrow$ Regular, BypassSearch $\rightarrow$ True

\[
\begin{align*}
\{C1 \rightarrow \{a2, a5, a6, a8\}, C2 \rightarrow \{a3, a5, a7, a8\}, E \rightarrow \{a4, a6, a7, a8\}, \Omega \rightarrow \{a1, a2, a3, a4, a5, a6, a7, a8\}\}, \\
\{a1 \rightarrow 1, a2 \rightarrow 1, a3 \rightarrow 1, a4 \rightarrow 1, a5 \rightarrow 1, a6 \rightarrow 1, a7 \rightarrow 5, a8 \rightarrow 15\}
\end{align*}
\]

- Good (second rescaling)

CS\text{i}j2 does not entail ($\uparrow$):

PrSAT\{$\{\text{ICS}i[j[E, C1, C2], \text{Pr}[C1 \mid C2] = \text{Pr}[C1], \text{CS}i[j[E, C1] + \text{CS}i[j[E, C2] > 1, \text{CS}i[j[E, C1] > 0, \text{CS}i[j[E, C2] > 0, \text{Pr}[C1] = 1/2, \text{Pr}[C2] = 1/2\}$, Probabilities $\rightarrow$ Regular, BypassSearch $\rightarrow$ True

\[
\begin{align*}
\{C1 \rightarrow \{a2, a5, a6, a8\}, C2 \rightarrow \{a3, a5, a7, a8\}, E \rightarrow \{a4, a6, a7, a8\}, \Omega \rightarrow \{a1, a2, a3, a4, a5, a6, a7, a8\}\}, \\
\{a1 \rightarrow 3, a2 \rightarrow 1, a3 \rightarrow 1, a4 \rightarrow 1, a5 \rightarrow 3, a6 \rightarrow 3, a7 \rightarrow 3, a8 \rightarrow 29\}
\end{align*}
\]

- Causal Independence and The Causal Strength of Conjunctive Factors

In this section, we show that some of our measures $m$ appear to violate the following “independence synergy property”:

(S) \[ \text{ICS}m[E, C1, C2] \Rightarrow (\text{CS}m[E, C1 \land C2] > \text{CS}m[E, C1] \land \text{CS}m[E, C1 \land C2] > \text{CS}m[E, C2]) \]

But, that the appearance of the failure of (S) for (all but one of) these measures $m$ depends on an incorrect way of calculating “$\text{CS}m[E, C1 \land C2]". Once this is corrected, we see that — on a proper understanding of "$\text{CS}m[E, C1 \land C2]"; all but one of our measures $m$ do satisfy (S). There is but one “recalcitrant” measure — the Galton measure $CSg$.

- Eells

$CSe$ appears to violate (S), as the existence of the following model indicates:
PrSAT[
  {  
    Pr[C1 \ C2] = Pr[C1] Pr[C2],
    Pr[E | C2] > Pr[E],
    Pr[E | C1] > Pr[E],
    CSe[E, C1, C2] = CSe[E, C1, ¬ C2],
    CSe[E, C1 \ C2] < CSe[E, C1]
  },
  Probabilities \rightarrow\ Regular
]

\{  
  (C1 \rightarrow [a_2, a_5, a_6, a_8], C2 \rightarrow [a_1, a_5, a_7, a_8], E \rightarrow [a_4, a_6, a_7, a_8], \Omega \rightarrow [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]),
  a_1 \rightarrow 10 736 022 587,  
  a_2 \rightarrow 36 278 969,  
  a_3 \rightarrow 2,  
  a_4 \rightarrow 1 465 641 97,  
  a_5 \rightarrow 41,  
  a_6 \rightarrow 101,  
  a_7 \rightarrow 1 31,  
  a_8 \rightarrow 21 \}

But, on the following proper reformulation of CSe[E, C1 \ C2]

CSe[E, C1 \ C2] = Pr[E | C1 \ C2] - Pr[E | ¬ C1 \ ¬ C2].

which compares Pr[E | C1 \ C2] and Pr[E | ¬ C1 \ ¬ C2] rather than Pr[E | C1 \ C2] and Pr[E | ¬(C1 \ C2)], such examples do not exist. So (S) is satisfied by CSe — once it is properly understood.

PrSAT[
  {  
    Pr[C1 \ C2] = Pr[C1] Pr[C2],
    Pr[E | C2] > Pr[E],
    Pr[E | C1] > Pr[E],
    CSe[E, C1, C2] = CSe[E, C1, ¬ C2],
    Pr[E | C1 \ C2] - Pr[E | ¬ C1 \ ¬ C2] ≤ CSe[E, C1]
  }
]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\{
\}

This is to be expected, in light of the fact that CSe (assuming a proper reformulation of CSe[E, C1 \ C2]) admits of the following (additive) "decomposition" of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing E:

ICSe[E, C1, C2] ⇒ CSe[E, C1 \ C2] = Pr[E | C1 \ C2] - Pr[E | ¬ C1 \ ¬ C2] = CSe[E, C1] + CSe[E, C1]

This can be verified using PrSAT, as follows:

PrSAT[
  {  
    Pr[C1 \ C2] = Pr[C1] Pr[C2],
    Pr[E | C2] > Pr[E],
    Pr[E | C1] > Pr[E],
    CSe[E, C1, C2] = CSe[E, C1, ¬ C2],
    Pr[E | C1 \ C2] - Pr[E | ¬ C1 \ ¬ C2] ≤ CSe[E, C1] + CSe[E, C2]
  }
]

PrSAT::srchfail : Search phase failed; attempting FindInstance

\{
\}

- Suppes

CSe does not even appear to violate (S):
PrSAT[
    { Pr[C1 \cap C2] = Pr[C1] Pr[C2],
      Pr[E \mid C2] > Pr[E],
      Pr[E \mid C1] > Pr[E],
      CSs[E, C1, C2] = CSs[E, C1, \neg C2],
      CSs[E, C1 \cap C2] < CSs[E, C1]
    },
    Probabilities \rightarrow Regular
]

PrSAT::srchfail : Search phase failed; attempting FindInstance

Thus, CSs satisfies (S) — even on naïve application. This is to be expected, in light of the following (formal) additivity property of CSs:

ICSs[E, C1, C2] = CSs[E, C1 \cap C2] = CSs[E, C1] + CSs[E, C2]

which can be verified using PrSAT, as follows:

PrSAT[
    { Pr[C1 \cap C2] = Pr[C1] Pr[C2],
      Pr[E \mid C2] > Pr[E],
      Pr[E \mid C1] > Pr[E],
      CSs[E, C1, C2] = CSs[E, C1, \neg C2],
      CSs[E, C1 \cap C2] < CSs[E, C1] + CSs[E, C2]
    },
    Probabilities \rightarrow Regular
]

PrSAT::srchfail : Search phase failed; attempting FindInstance

Since the Suppes measure does not involve conditioning on \neg C, there is no need to consider reformulations of CSs[E, C1 \cap C2].

Galton

CSg appears to violate (S), as the existence of the following model indicates:

PrSAT[
    { Pr[C1 \cap C2] = Pr[C1] Pr[C2],
      Pr[E \mid C2] > Pr[E],
      Pr[E \mid C1] > Pr[E],
      CSg[E, C1, C2] = CSg[E, C1, \neg C2],
      CSg[E, C1 \cap C2] < CSg[E, C1],
      Pr[C1] = 1/3
    },
    Probabilities \rightarrow Regular
]

\{ \{ C1 \rightarrow \{ a_2, a_5, a_6, a_8 \}, C2 \rightarrow \{ a_1, a_5, a_7, a_8 \}, E \rightarrow \{ a_4, a_6, a_7, a_8 \}, \Omega \rightarrow \{ a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \} \} \}

\{ a_1 \rightarrow 668143271, a_2 \rightarrow 1096, a_3 \rightarrow 151739, a_4 \rightarrow 506617, a_5 \rightarrow 1, a_6 \rightarrow 3, a_7 \rightarrow 17, a_8 \rightarrow 5 \}

\{ a_1 \rightarrow 1860173964, a_2 \rightarrow 10989, a_3 \rightarrow 1109889, a_4 \rightarrow 206685996, a_5 \rightarrow 999, a_6 \rightarrow 37, a_7 \rightarrow 101, a_8 \rightarrow 33 \}

Surprisingly, even on a proper reformulation of CSg[E, C1 \cap C2], which (presumably) would be given by the following:

CSg[E, C1 \cap C2] = 4 (Pr[E \wedge (C1 \wedge C2)] - Pr[E]Pr[C1 \wedge C2])

such examples still exist. So (S) seems to be violated by CSg — even once CSg[E, C1 \cap C2] is properly reformulated. Here’s a “recalcitrant” model:
PrSAT
{
    Pr[C1 \land C2] = Pr[C1] Pr[C2],
    Pr[E \mid C2] > Pr[E],
    Pr[E \mid C1] > Pr[E],
    CSg[E, C1, C2] = CSg[E, C1, \neg C2],
    CSg[E, C1 \land C2] < CSg[E, C1],
    4 (Pr[E \land (C1 \land C2)] - Pr[E] Pr[C1 \land C2]) < CSg[E, C1],
}
Pr[C1] = 1/4,
Pr[C2] = 1/4,
Pr[E] = 1/2
}
Probabilities \rightarrow \text{Regular},
BypassSearch \rightarrow \text{True}

\begin{align*}
(C1 \rightarrow (a_2, a_5, a_6, a_8), \quad C2 \rightarrow (a_3, a_5, a_7, a_8), \quad E \rightarrow (a_4, a_6, a_7, a_8), \quad \Omega \rightarrow (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8))
\end{align*}

\begin{align*}
\{ a_1 \rightarrow 45, \quad a_2 \rightarrow 3, \quad a_3 \rightarrow 3, \quad a_4 \rightarrow 27, \quad a_5 \rightarrow 1, \quad a_6 \rightarrow 4, \quad a_7 \rightarrow 3, \quad a_8 \rightarrow 7 \} 
\end{align*}

\text{Cheng}

CSc appears to violate (S), as the existence of the following model indicates:

PrSAT
{
    Pr[C1 \land C2] = Pr[C1] Pr[C2],
    Pr[E \mid C2] > Pr[E],
    Pr[E \mid C1] > Pr[E],
    CSc[E, C1, C2] = CSc[E, C1, \neg C2] = 1/2,
    CSc[E, C1 \land C2] < CSc[E, C1]
}
Probabilities \rightarrow \text{Regular}

\begin{align*}
(C1 \rightarrow (a_2, a_5, a_6, a_8), \quad C2 \rightarrow (a_3, a_5, a_7, a_8), \quad E \rightarrow (a_4, a_6, a_7, a_8), \quad \Omega \rightarrow (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8))
\end{align*}

\begin{align*}
\{ a_1 \rightarrow 1312788, \quad a_2 \rightarrow 2, \quad a_3 \rightarrow 656394, \\
733167407, \quad a_4 \rightarrow 19, \quad a_5 \rightarrow 6234475, \\
4922955, \quad a_6 \rightarrow 7, \quad a_7 \rightarrow 35117079, \quad a_8 \rightarrow 25, \\
1956877013, \quad a_9 \rightarrow 25, \quad a_10 \rightarrow 329458050, \quad a_11 \rightarrow 74 \} 
\end{align*}

But, on a proper reformulation of CSc[E, C1 \land C2], i.e.:

\begin{align*}
\text{CSc}[E, C1 \land C2] = \frac{\text{Pr}[E \mid C1 \land C2] - \text{Pr}[E \mid \neg C1 \land \neg C2]}{1 - \text{Pr}[E \mid \neg C1 \land \neg C2}}
\end{align*}

which compares Pr[E \mid C1 \land C2] and Pr[E \mid \neg C1 \land \neg C2] rather than Pr[E \mid C1 \land C2] and Pr[E \mid \neg(C1 \land C2)], such examples do not exist. So (S) really is satisfied by CSc — once it is properly understood.
This is to be expected, in light of the fact that $C_{Sc}$ (assuming a proper reformulation of $C_{Sc}[E, C_1 \land C_2]$) admits of the following (multiplicative) "decomposition" of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing $E$:

$$IC_{Sc}[E, C_1, C_2] = C_{Sc}[E, C_1 \land C_2] = \frac{Pr[E | C_1 \land C_2] - Pr[E | \neg C_1 \land \neg C_2]}{1 - Pr[E | \neg C_1 \land \neg C_2]} = 1 - (1 - C_{Sc}[E, C_1]) (1 - C_{Sc}[E, C_2])$$

This can be verified using $PrSAT$, as follows:

```plaintext
PrSAT[
{
Pr[C_1 \land C_2] = Pr[C_1] Pr[C_2],
Pr[E | C_2] > Pr[E],
Pr[E | C_1] > Pr[E],
C_{Sc}[E, C_1, C_2] = C_{Sc}[E, C_1, \neg C_2],

Pr[E | C_1 \land C_2] - Pr[E | \neg C_1 \land \neg C_2]
1 - Pr[E | \neg C_1 \land \neg C_2]

]}
```

$PrSAT::srchfail : Search phase failed; attempting FindInstance$

{}
Thus, \texttt{CSlr} satisfies (S) — even on \textit{naive} application. This is \textit{despite} the fact that \texttt{CSlr}[E, c_1 \land c_2] is not properly formulated (on \textit{naive} application). What’s more important here is that \texttt{CSlr} satisfies (S), once \texttt{CSlr}[E, c_1 \land c_2] is properly reformulated, as follows:

\[
\text{CSlr}[E, c_1 \land c_2] = \frac{\Pr[E | c_1 \land c_2]}{\Pr[E | \neg c_1 \land \neg c_2]}
\]

This can be verified using \texttt{PrSAT}, as follows:

\[
\text{PrSAT[}
\{
\Pr[c_1 \land c_2] = \Pr[c_1] \Pr[c_2],
\Pr[E | c_2] > \Pr[E],
\Pr[E | c_1] > \Pr[E],
\text{CSlr}[E, c_1, c_2] = \text{CSlr}[E, c_1, \neg c_2],
\frac{\Pr[E | c_1 \land c_2]}{\Pr[E | \neg c_1 \land \neg c_2]} \leq \text{CSlr}[E, c_1]
\},
\text{Probabilities } \rightarrow \text{ Regular}
\]

\text{PrSAT::srchfail : Search phase failed; attempting FindInstance}

\[
\}
\]

This is to be expected, in light of the fact that \texttt{CSlr} (assuming a proper reformulation of \texttt{CSlr}[E, c_1 \land c_2]) admits of the following (multiplicative) “decomposition” of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing \(E\):

\[
\text{CSlr}[E, c_1, c_2] = \text{CSlr}[E, c_1 \land c_2] = \frac{\Pr[E | c_1 \land c_2]}{\Pr[E | \neg c_1 \land \neg c_2]} = \text{CSlr}[E, c_1] \ast \text{CSlr}[E, c_2]
\]

This can be verified using \texttt{PrSAT}, as follows:

\[
\text{PrSAT[}
\{
\Pr[c_1 \land c_2] = \Pr[c_1] \Pr[c_2],
\Pr[E | c_2] > \Pr[E],
\Pr[E | c_1] > \Pr[E],
\text{CSlr}[E, c_1, c_2] = \text{CSlr}[E, c_1, \neg c_2],
\frac{\Pr[E | c_1 \land c_2]}{\Pr[E | \neg c_1 \land \neg c_2]} \leq \text{CSlr}[E, c_1] \ast \text{CSlr}[E, c_2]
\},
\text{Probabilities } \rightarrow \text{ Regular}
\]

\text{PrSAT::srchfail : Search phase failed; attempting FindInstance}

\[
\}
\]

- Lewis Ratio (second rescaling)

\texttt{CSlr} \textit{appears} to violate (S), as the existence of the following model indicates:
\textbf{PrSAT[}
\begin{align*}
&\text{Pr}[C_1 \land C_2] = \text{Pr}[C_1] \text{ Pr}[C_2], \\
&\text{Pr}[E \mid C_2] > \text{Pr}[E], \\
&\text{Pr}[E \mid C_1] > \text{Pr}[E], \\
&\text{CSlr2}[E, C_1, C_2] = \text{CSlr2}[E, C_1, \neg C_2], \\
&\text{CSlr2}[E, C_1 \land C_2] < \text{CSlr2}[E, C_1]
\end{align*}
\text{]}

, \text{Probabilities } \rightarrow \text{ Regular}

\text{\{ }
\begin{align*}
(C_1 \rightarrow \{a_2, a_5, a_6, a_8\}, C_2 \rightarrow \{a_1, a_5, a_7, a_8\}, E \rightarrow \{a_4, a_6, a_7, a_8\}, \Omega \rightarrow \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}, \\
\{ a_1 \rightarrow \frac{426722843}{7206311529}, a_2 \rightarrow \frac{632350171}{52668350928}, a_3 \rightarrow 2, a_4 \rightarrow \frac{64}{9}, a_5 \rightarrow \frac{1}{63423}, a_6 \rightarrow \frac{4}{999}, a_7 \rightarrow \frac{1}{29}, a_8 \rightarrow \frac{9}{243}\}
\end{align*}
\text{\}}

But, on a proper reformulation of \text{CSlr2}[E, C_1 \land C_2], \text{i.e.}:

\text{CSlr2}[E, C_1 \land C_2] = 1 - \frac{\text{Pr}[E \mid \neg C_1 \land \neg C_2]}{\text{Pr}[E \mid C_1 \land C_2]}

which involves \text{Pr}[E \mid C_1 \land C_2] and \text{Pr}[E \mid \neg C_1 \land \neg C_2] rather than \text{Pr}[E \mid C_1 \land C_2] and \text{Pr}[E \mid \neg(C_1 \land C_2)], such examples do not exist. So (S) really is satisfied by \text{CSlr2} — once it is properly understood.

\textbf{PrSAT[}
\begin{align*}
&\text{Pr}[C_1 \land C_2] = \text{Pr}[C_1] \text{ Pr}[C_2], \\
&\text{Pr}[E \mid C_2] > \text{Pr}[E], \\
&\text{Pr}[E \mid C_1] > \text{Pr}[E], \\
&\text{CSlr2}[E, C_1, C_2] = \text{CSlr2}[E, C_1, \neg C_2], \\
&1 - \frac{\text{Pr}[E \mid \neg C_1 \land \neg C_2]}{\text{Pr}[E \mid C_1 \land C_2]} \leq \text{CSlr2}[E, C_1]
\end{align*}
\text{]}

, \text{Probabilities } \rightarrow \text{ Regular}

\text{PrSAT::srchfail: Search phase failed; attempting FindInstance}

\text{\{}

This is to be expected, in light of the fact that \text{CSlr2} (assuming a proper reformulation of \text{CSlr2}[E, C_1 \land C_2]) admits of the following (multiplicative) “decomposition” of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing \text{E}:

\text{CSlr2}[E, C_1, C_2] = \text{CSlr2}[E, C_1 \land C_2] = 1 - \frac{\text{Pr}[E \mid \neg C_1 \land \neg C_2]}{\text{Pr}[E \mid C_1 \land C_2]} = 1 - (\text{CSlr2}[E, C_1]) (1 - \text{CSlr2}[E, C_2])

This can be verified using \text{PrSAT}, as follows:
\textbf{PrSAT}

\begin{verbatim}
{ 
Pr[C1 \land C2] = Pr[C1] Pr[C2],
Pr[E \mid C2] > Pr[E],
Pr[E \mid C1] > Pr[E],
CSlr2[E, C1, C2] = CSlr2[E, C1, \neg C2],
1 - \frac{Pr[E \mid \neg C1 \land \neg C2]}{Pr[E \mid C1 \land C2]} \neq 1 - (1 - CSlr2[E, C1]) (1 - CSlr2[E, C2])
},
Probabilities \to \text{Regular}
}
\end{verbatim}

\texttt{PrSAT::srchfail : Search phase failed; attempting FindInstance}

\{
\}

\textbf{Good}

\textbf{CSij} does not even appear to violate (S):

\textbf{PrSAT}

\begin{verbatim}
{ 
Pr[C1 \land C2] = Pr[C1] Pr[C2],
Pr[E \mid C2] > Pr[E],
Pr[E \mid C1] > Pr[E],
CSij[E, C1, C2] = CSij[E, C1, \neg C2] = 1/2,
CSij[E, C1 \land C2] < CSij[E, C1]
},
Probabilities \to \text{Regular}
}
\end{verbatim}

\texttt{PrSAT::srchfail : Search phase failed; attempting FindInstance}

\{
\}

Thus, \textbf{CSij} satisfies (S) — even on naive application. This is despite the fact that \textbf{CSij[E, C1 \land C2]} is not properly formulated (on naive application). What’s more important here is that \textbf{CSij} satisfies (S), once \textbf{CSij[E, C1 \land C2]} is properly reformulated, as follows:

\[
\textbf{CSij[E, C1 \land C2]} = \frac{Pr[\neg E \mid \neg C1 \land \neg C2]}{Pr[\neg E \mid C1 \land C2]}
\]

This can be verified using \texttt{PrSAT}, as follows:

\textbf{PrSAT}

\begin{verbatim}
{ 
Pr[C1 \land C2] = Pr[C1] Pr[C2],
Pr[E \mid C2] > Pr[E],
Pr[E \mid C1] > Pr[E],
CSij[E, C1, C2] = CSij[E, C1, \neg C2] = 1/2,
Pr[\neg E \mid \neg C1 \land \neg C2] < CSij[E, C1]
},
Probabilities \to \text{Regular}
}
\end{verbatim}

\texttt{PrSAT::srchfail : Search phase failed; attempting FindInstance}

\{
\}
This is to be expected, in light of the fact that $CSij$ (assuming a proper reformulation of $CSij[E, c1 ∧ c2]$) admits of the following (multiplicative) “decomposition” of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing $E$:

$$CSij[E, c1, c2] = CSij[E, c1 ∧ c2] = \frac{Pr[\neg E | \neg c1 \land \neg c2]}{Pr[\neg E | c1 \land c2]} = 1 - (1 - CSij[E, c1])(1 - CSij[E, c2])$$

This can be verified using PrSAT, as follows:

```
PrSAT[
{
Pr[c1 ∧ c2] = Pr[c1] Pr[c2],
Pr[E | c2] > Pr[E],
Pr[E | c1] > Pr[E],
CSij[E, c1, c2] = CSij[E, c1, ¬ c2] = 1 / 2,
Pr[¬ E | ¬ c1 ∧ ¬ c2]
Pr[¬ E | c1 ∧ c2] ≠ 1 - (1 - CSij[E, c1]) (1 - CSij[E, c2])
},
Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

## Boolean Representations of Cheng, Eells, Suppes, and Lewis Ratio (second rescaling)

### Cheng

Assume that (1) $A$, $Q$, and $C$ are pairwise independent and jointly independent, and (2) $E = A ∨ (Q ∧ C)$. Then, we have the following Boolean representation of $CSc$:

$$CSc[E, c] = Pr[Q]$$

Here is a verification:

$$ASSc = \{Pr[A ∧ Q] = Pr[A] Pr[Q], Pr[A ∧ C] = Pr[A] Pr[C],
Pr[Q ∧ C] = Pr[Q] Pr[C], Pr[A ∧ (Q ∧ C)] = Pr[A] Pr[Q ∧ C];
E = A ∨ (Q ∧ C);
PrSAT[ASSc ∪ (CSc[E, c] ≠ Pr[Q])]
PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

### Eells

Assume that (1) $A$ and $Q$ are mutually exclusive, (2) $A$ and $C$ are independent, (3) $Q$ and $C$ are independent, and (4) $E = A ∨ (Q ∧ C)$. Then, we have the following Boolean representation of $CSe$:

$$CS[e[E, c] = Pr[Q]$$

Here is a verification:

$$ASSe = \{Pr[A ∧ Q] = 0, Pr[A ∧ C] = Pr[A] Pr[C], Pr[Q ∧ C] = Pr[Q] Pr[C];
$$
PrSAT[ASSe ∪ (CSe[E, C] ≠ Pr[Q])]

PrSAT::srchfail : Search phase failed; attempting FindInstance

()  

- Suppes

The same assumptions used for the Eells measure above yield the following Boolean representation of CSs:

\[ CSs[E, C] = Pr[Q ∧ ¬C] \]

Here is a verification:

PrSAT[ASSe ∪ (CSs[E, C] ≠ Pr[Q ∧ ¬C])]

PrSAT::srchfail : Search phase failed; attempting FindInstance

()

- Lewis Ratio (second rescaling)

The same assumptions used for the Eells and Suppes measures above yield the following Boolean representation of CSlr2:

\[ CSlr2[E, C] = Pr[Q | C ∧ E] \]

Here is a verification:

PrSAT[ASSe ∪ (CSlr2[E, C] ≠ Pr[Q | C ∧ E])]

PrSAT::srchfail : Search phase failed; attempting FindInstance

()}