Probabilistic Measures of Causal Strength

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This is a companion *Mathematica* notebook for our paper “Probabilistic Measures of Causal Strength“, which can be downloaded from **http://fitelson.org/pmcs.pdf**.

■ Requires the PrSAT package, which can be downloaded from http://fitelson.org/PrSAT/.

```
<< PrSAT`
```

■ Defining the measures (Table 1)

In this section we give definitions of our measures of causal strength (CS) and preventative strength (PS), as described in Table 1 of the paper. As discussed in the beginning of the paper, we *define* PS(E,C) = −CS(¬E,C) for each of our CS-measures, and we introduce two rescalings for each of the CS-measures that are not already defined on [-1,1].

■ Eells

```
CS𝕖[e_, c_] := Pr[e | c] - Pr[e | ¬ c];
CS𝕖[e_, c1_, c2_] := Pr[e | c1 ⋀ c2] - Pr[e | ¬ c1 ⋀ c2];

PS𝕖[e_, c_] := -CS𝕖[¬ e, c];
PS𝕖[e_, c1_, c2_] := -CS𝕖[¬ e, c1, c2];
```

■ Suppes

```
CS𝕤[e_, c_] := Pr[e | c] - Pr[e];
CS𝕤[e_, c1_, c2_] := Pr[e | c1 ⋀ c2] - Pr[e | c2];

PS𝕤[e_, c_] := -CS𝕤[¬ e, c];
PS𝕤[e_, c1_, c2_] := -CS𝕤[¬ e, c1, c2];
```

■ Galton

```
CS𝕘[e_, c_] := 4 Pr[c] Pr[¬ c] (Pr[e | c] - Pr[e | ¬ c]);
CS𝕘[e_, c1_, c2_] := 4 Pr[c1 | c2] Pr[¬ c1 | c2] (Pr[e | c1 ⋀ c2] - Pr[e | ¬ c1 ⋀ c2]);

PS𝕘[e_, c_] := -CS𝕘[¬ e, c];
PS𝕘[e_, c1_, c2_] := -CS𝕘[¬ e, c1, c2];
```

■ Cheng

```
CS𝕔[e_, c_] := (Pr[e | c] - Pr[e | ¬ c]) / Pr[¬ e | ¬ c];
CS𝕔[e_, c1_, c2_] := (Pr[e | c1 ⋀ c2] - Pr[e | ¬ c1 ⋀ c2]) / Pr[¬ e | ¬ c1 ⋀ c2];

PS𝕔[e_, c_] := -CS𝕔[¬ e, c];
PS𝕔[e_, c1_, c2_] := -CS𝕔[¬ e, c1, c2];
```

■ Lewis Ratio

```
CS𝕝𝕣[e_, c_] := Pr[e | c] / Pr[e | ¬ c];
CS𝕝𝕣[e_, c1_, c2_] := Pr[e | c1 ⋀ c2] / Pr[e | ¬ c1 ⋀ c2];
```

- **First rescaling of Lewis Ratio**

```
CS𝕝𝕣1[e_, c_] := (Pr[e | c] - Pr[e | ¬ c]) / (Pr[e | c] + Pr[e | ¬ c]);
CS𝕝𝕣1[e_, c1_, c2_] := (Pr[e | c1 ⋀ c2] - Pr[e | ¬ c1 ⋀ c2]) / (Pr[e | c1 ⋀ c2] + Pr[e | ¬ c1 ⋀ c2]);

PS𝕝𝕣1[e_, c_] := -CS𝕝𝕣1[¬ e, c];
PS𝕝𝕣1[e_, c1_, c2_] := -CS𝕝𝕣1[¬ e, c1, c2];
```

- **Second rescaling of Lewis Ratio**

```
CS𝕝𝕣2[e_, c_] := 1 - (1 / CS𝕝𝕣[e, c]);
CS𝕝𝕣2[e_, c1_, c2_] := 1 - (1 / CS𝕝𝕣[e, c1, c2]);

PS𝕝𝕣2[e_, c_] := -CS𝕝𝕣2[¬ e, c];
PS𝕝𝕣2[e_, c1_, c2_] := -CS𝕝𝕣2[¬ e, c1, c2];
```

- **Good**

```
CS𝕚𝕛[e_, c_] := (Pr[¬ e | ¬ c]) / (Pr[¬ e | c]);
CS𝕚𝕛[e_, c1_, c2_] := (Pr[¬ e | ¬ c1 ⋀ c2]) / (Pr[¬ e | c1 ⋀ c2]);
```

- **First rescaling of Good**

```
CS𝕚𝕛1[e_, c_] := (Pr[¬ e | ¬ c] - Pr[¬ e | c]) / (Pr[¬ e | ¬ c] + Pr[¬ e | c]);
CS𝕚𝕛1[e_, c1_, c2_] := (Pr[¬ e | ¬ c1 ⋀ c2] - Pr[¬ e | c1 ⋀ c2]) / (Pr[¬ e | ¬ c1 ⋀ c2] + Pr[¬ e | c1 ⋀ c2]);

PS𝕚𝕛1[e_, c_] := -CS𝕚𝕛1[¬ e, c];
PS𝕚𝕛1[e_, c1_, c2_] := -CS𝕚𝕛1[¬ e, c1, c2];
```

- **Second rescaling of Good**

```
CS𝕚𝕛2[e_, c_] := 1 - (1 / CS𝕚𝕛[e, c]);
CS𝕚𝕛2[e_, c1_, c2_] := 1 - (1 / CS𝕚𝕛[e, c1, c2]);

PS𝕚𝕛2[e_, c_] := -CS𝕚𝕛2[¬ e, c];
PS𝕚𝕛2[e_, c1_, c2_] := -CS𝕚𝕛2[¬ e, c1, c2];
```

Scale Verification

In this section, we verify that all the (possibly rescaled) measures we‘ll examine below, are on a [-1,1] scale. We do this using the function **PrRange** (now part of the **PrSAT** package) which calculates the range of a probabilistic expression (first argument) subject to probabilistic constraints (second argument):

Eells

```
PrRange[CS𝕖[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]

{0, 1}

PrRange[PS𝕖[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]

{-1, 0}
```

Suppes

```
PrRange[CS𝕤[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]
{0, 1}

PrRange[PS𝕤[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]
{-1, 0}
```

Galton

```
PrRange[CS𝕘[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]
{0, 1}

PrRange[CS𝕘[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]
{-1, 0}
```

Cheng

```
PrRange[CS𝕔[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]
{0, 1}

PrRange[PS𝕔[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]
{-1, 0}
```

Lewis Ratio (two rescaled versions)

```
PrRange[CS𝕝𝕣1[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]
{0, 1}

PrRange[PS𝕝𝕣1[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]
{-1, 0}

PrRange[CS𝕝𝕣2[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]
{0, 1}

PrRange[PS𝕝𝕣2[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]
{-1, 0}
```

Good (two rescaled versions)

```
PrRange[CS𝕚𝕛1[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]
{0, 1}

PrRange[PS𝕚𝕛1[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]
{-1, 0}

PrRange[CS𝕚𝕛2[𝔼, ℂ], Pr[𝔼 | ℂ] ≥ Pr[𝔼]]
{0, 1}

PrRange[PS𝕚𝕛2[𝔼, ℂ], Pr[𝔼 | ℂ] ≤ Pr[𝔼]]
{-1, 0}
```

■ Inter-Definability Verification (Table 3)

Here, we verify the inter-definability relations stated in Table 3 of the paper, using the function **`PrReduce`** from the **`PrSAT`** package:

Suppes

$$\text{PrReduce}[\{\text{CS}\mathbb{s}[\mathbb{E},\mathbb{C}] == \text{Pr}[\neg\,\mathbb{C}]\ \text{CS}\mathbb{e}[\mathbb{E},\mathbb{C}]\}]$$

True

Galton

$$\text{PrReduce}[\{\text{CS}\mathbb{g}[\mathbb{E},\mathbb{C}] == 4\ \text{Pr}[\mathbb{C}]\ \text{Pr}[\neg\,\mathbb{C}]\ \text{CS}\mathbb{e}[\mathbb{E},\mathbb{C}] == 4\ \text{Pr}[\mathbb{C}]\ \text{CS}\mathbb{s}[\mathbb{E},\mathbb{C}]\}]$$

True

Cheng

$$\text{PrReduce}\left[\left\{\text{CS}\mathbb{c}[\mathbb{E},\mathbb{C}] == \frac{\text{CS}\mathbb{e}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\neg\,\mathbb{E}\mid\neg\,\mathbb{C}]} == \frac{\text{CS}\mathbb{s}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\neg\,\mathbb{E}\wedge\neg\,\mathbb{C}]} == \frac{\text{CS}\mathbb{g}[\mathbb{E},\mathbb{C}]}{4\ \text{Pr}[\mathbb{C}]\ \text{Pr}[\neg\,\mathbb{E}\wedge\neg\,\mathbb{C}]}\right\}\right]$$

True

Lewis Ratio

$$\text{PrReduce}\left[\left\{\text{CS}\mathbb{lr}1[\mathbb{E},\mathbb{C}] == \frac{\text{CS}\mathbb{lr}[\mathbb{E},\mathbb{C}]-1}{\text{CS}\mathbb{lr}[\mathbb{E},\mathbb{C}]+1} == \frac{\text{CS}\mathbb{e}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\mathbb{E}\mid\mathbb{C}]+\text{Pr}[\mathbb{E}\mid\neg\,\mathbb{C}]}\right\}\right]$$

True

$$\text{PrReduce}\left[\left\{\text{CS}\mathbb{lr}2[\mathbb{E},\mathbb{C}] == 1-\frac{1}{\text{CS}\mathbb{lr}[\mathbb{E},\mathbb{C}]} == \frac{\text{CS}\mathbb{e}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\mathbb{E}\mid\mathbb{C}]} == \frac{\text{CS}\mathbb{s}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\mathbb{E}\mid\mathbb{C}]\ \text{Pr}[\neg\,\mathbb{C}]} == \text{CS}\mathbb{c}[\mathbb{E},\mathbb{C}]\ \frac{\text{Pr}[\neg\,\mathbb{E}\mid\neg\,\mathbb{C}]}{\text{Pr}[\mathbb{E}\mid\mathbb{C}]} == \frac{\text{CS}\mathbb{g}[\mathbb{E},\mathbb{C}]}{4\ \text{Pr}[\mathbb{E}\wedge\mathbb{C}]\ \text{Pr}[\neg\,\mathbb{C}]}\right\}\right]$$

True

Good

$$\text{PrReduce}[\text{CS}\mathbb{ij}[\mathbb{E},\mathbb{C}] == \text{CS}\mathbb{lr}[\neg\,\mathbb{E},\neg\,\mathbb{C}]]$$

True

$$\text{PrReduce}\left[\left\{\text{CS}\mathbb{ij}1[\mathbb{E},\mathbb{C}] == \frac{\text{CS}\mathbb{ij}[\mathbb{E},\mathbb{C}]-1}{\text{CS}\mathbb{ij}[\mathbb{E},\mathbb{C}]+1} == \text{CS}\mathbb{lr}1[\neg\,\mathbb{E},\neg\,\mathbb{C}] == \frac{\text{CS}\mathbb{e}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\neg\,\mathbb{E}\mid\mathbb{C}]+\text{Pr}[\neg\,\mathbb{E}\mid\neg\,\mathbb{C}]}\right\}\right]$$

True

$$\text{PrReduce}\left[\left\{\text{CS}\mathbb{ij}2[\mathbb{E},\mathbb{C}] == 1-\frac{1}{\text{CS}\mathbb{ij}[\mathbb{E},\mathbb{C}]} == \frac{\text{CS}\mathbb{e}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\neg\,\mathbb{E}\mid\neg\,\mathbb{C}]} == \frac{\text{CS}\mathbb{s}[\mathbb{E},\mathbb{C}]}{\text{Pr}[\neg\,\mathbb{E}\wedge\neg\,\mathbb{C}]} == \frac{\text{CS}\mathbb{g}[\mathbb{E},\mathbb{C}]}{4\ \text{Pr}[\neg\,\mathbb{E}\wedge\neg\,\mathbb{C}]\ \text{Pr}[\mathbb{C}]} == \text{CS}\mathbb{lr}2[\neg\,\mathbb{E},\neg\,\mathbb{C}]\right\}\right]$$

True

■ Ordinal Relationship Verification (Table 4)

In this section, we verify all the ordinal relationships between all pairs of measures — as recorded in Table 4 of the paper (going from the first row, downward by rows). Here, we use **PrSAT** to search for models of the *denials* of the various ordinal relationships. If a model *is* found, this shows that the ordinal relationship in question does *not* hold (and the model given is a concrete *counter*-model to the ordinal relationship in question). If *no* model is found, then the ordinal relationship in question *does* hold.

■ Eells & Suppes

CS𝕖 and **CS𝕤** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CS𝕖[𝔼1, ℂ1] ≥ CS𝕖[𝔼2, ℂ2], CS𝕤[𝔼1, ℂ1] < CS𝕤[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \Big\{a_1 \to \frac{5\,683\,233\,420\,533\,249\,039}{16\,245\,548\,056\,506\,210\,750}, a_2 \to \frac{3}{20}, a_3 \to \frac{1}{345}, a_4 \to \frac{1}{125}, a_5 \to \frac{1}{524}, a_6 \to \frac{5}{41}, a_7 \to \frac{1}{33}, a_8 \to \frac{1}{121}, a_9 \to \frac{1}{999}, a_{10} \to \frac{1}{999}, a_{11} \to \frac{1}{999}, a_{12} \to \frac{1}{7}, a_{13} \to \frac{2}{57}, a_{14} \to \frac{1}{103}, a_{15} \to \frac{1}{953}, a_{16} \to \frac{5}{37}\Big\}\Big\}$$

CS𝕖 and **CS𝕤** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CS𝕖[𝔼, ℂ1] ≥ CS𝕖[𝔼, ℂ2], CS𝕤[𝔼, ℂ1] < CS𝕤[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \Big\{a_1 \to \frac{1\,400\,671\,985}{2\,148\,190\,703\,964}, a_2 \to \frac{1}{981}, a_3 \to \frac{19}{58}, a_4 \to \frac{1}{5}, a_5 \to \frac{18}{43}, a_6 \to \frac{1}{886}, a_7 \to \frac{1}{20}, a_8 \to \frac{1}{991}\Big\}\Big\}$$

CS𝕖 and **CS𝕤** *are* ordinally equivalent in the class of cases with two effects and a single cause (they *are* II-E):

```
PrSAT[{CS𝕖[𝔼1, ℂ] ≥ CS𝕖[𝔼2, ℂ], CS𝕤[𝔼1, ℂ] < CS𝕤[𝔼2, ℂ]}, Probabilities → Regular]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

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- **Eells & Galton**

CS𝕖 and **CS𝕘** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CS𝕖[𝔼1, ℂ1] ≥ CS𝕖[𝔼2, ℂ2], CS𝕘[𝔼1, ℂ1] < CS𝕘[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \Big\{a_1 \to \frac{676\,101\,601\,782\,601\,735\,273}{1\,256\,644\,399\,321\,100\,793\,150}, a_2 \to \frac{1}{589}, a_3 \to \frac{2}{25}, a_4 \to \frac{1}{106}, a_5 \to \frac{1}{56}, a_6 \to \frac{1}{531}, a_7 \to \frac{1}{107}, a_8 \to \frac{1}{548}, a_9 \to \frac{2}{53}, a_{10} \to \frac{14}{97}, a_{11} \to \frac{1}{952}, a_{12} \to \frac{1}{23}, a_{13} \to \frac{1}{999}, a_{14} \to \frac{1}{54}, a_{15} \to \frac{1}{179}, a_{16} \to \frac{3}{34}\Big\}\Big\}$$

CS𝕖 and **CS𝕘** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CS𝕖[𝔼, ℂ1] ≥ CS𝕖[𝔼, ℂ2], CS𝕘[𝔼, ℂ1] < CS𝕘[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \Big\{a_1 \to \frac{488\,713}{828\,240}, a_2 \to \frac{1}{70}, a_3 \to \frac{4}{21}, a_4 \to \frac{1}{48}, a_5 \to \frac{3}{56}, a_6 \to \frac{1}{952}, a_7 \to \frac{1}{29}, a_8 \to \frac{2}{21}\Big\}\Big\}$$

CS𝕖 and **CS𝕘** *are* ordinally equivalent in the class of cases with two effects and a single cause (they *are* II-E):

```
PrSAT[{CS𝕖[𝔼1, ℂ] ≥ CS𝕖[𝔼2, ℂ], CS𝕘[𝔼1, ℂ] < CS𝕘[𝔼2, ℂ]}, Probabilities → Regular]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Eells & Cheng**

CS𝕖 and **CS𝕔** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSe[E1, C1] ≥ CSe[E2, C2], CSc[E1, C1] < CSc[E2, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \{a_1 \to \frac{91\,068\,197\,882\,708\,747}{270\,322\,787\,092\,438\,080}, a_2 \to \frac{5}{51}, a_3 \to \frac{3}{52}, a_4 \to \frac{1}{59}, a_5 \to \frac{1}{15}, a_6 \to \frac{1}{23}, a_7 \to \frac{7}{120}, a_8 \to \frac{1}{33}, a_9 \to \frac{1}{103}, a_{10} \to \frac{1}{118}, a_{11} \to \frac{10}{67}, a_{12} \to \frac{1}{31}, a_{13} \to \frac{2}{63}, a_{14} \to \frac{1}{320}, a_{15} \to \frac{1}{38}, a_{16} \to \frac{2}{65}\}\}$$

CSe and **CSc** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSe[E, C1] ≥ CSe[E, C2], CSc[E, C1] < CSc[E, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{6\,552\,877}{19\,684\,665}, a_2 \to \frac{2}{23}, a_3 \to \frac{2}{39}, a_4 \to \frac{1}{63}, a_5 \to \frac{26}{105}, a_6 \to \frac{3}{38}, a_7 \to \frac{3}{46}, a_8 \to \frac{4}{33}\}\}$$

CSe and **CSc** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CSe[E1, C] ≥ CSe[E2, C], CSc[E1, C] < CSc[E2, C]}, Probabilities → Regular]
```

$$\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{90\,764\,601\,181}{260\,065\,893\,780}, a_2 \to \frac{1}{820}, a_3 \to \frac{1}{882}, a_4 \to \frac{10}{57}, a_5 \to \frac{1}{693}, a_6 \to \frac{3}{31}, a_7 \to \frac{1}{999}, a_8 \to \frac{46}{123}\}\}$$

- **Eells & Lewis Ratio**

CSe and **CSlr** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSe[E1, C1] ≥ CSe[E2, C2], CSlr[E1, C1] < CSlr[E2, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \{a_1 \to \frac{8\,004\,879\,309\,751}{21\,360\,609\,408\,600}, a_2 \to \frac{2}{25}, a_3 \to \frac{3}{50}, a_4 \to \frac{1}{70}, a_5 \to \frac{6}{53}, a_6 \to \frac{2}{37}, a_7 \to \frac{1}{60}, a_8 \to \frac{1}{11}, a_9 \to \frac{5}{56}, a_{10} \to \frac{1}{469}, a_{11} \to \frac{1}{30}, a_{12} \to \frac{1}{999}, a_{13} \to \frac{1}{30}, a_{14} \to \frac{1}{170}, a_{15} \to \frac{1}{69}, a_{16} \to \frac{1}{60}\}\}$$

CSe and **CSlr** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSe[E, C1] ≥ CSe[E, C2], CSlr[E, C1] < CSlr[E, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{6\,156\,667}{17\,272\,710}, a_2 \to \frac{1}{999}, a_3 \to \frac{1}{20}, a_4 \to \frac{5}{52}, a_5 \to \frac{1}{999}, a_6 \to \frac{1}{798}, a_7 \to \frac{1}{18}, a_8 \to \frac{25}{57}\}\}$$

CSe and **CSlr** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CSe[E1, C] ≥ CSe[E2, C], CSlr[E1, C] < CSlr[E2, C]}, Probabilities → Regular]
```

$$\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{18\,661\,411\,667}{75\,416\,638\,440}, a_2 \to \frac{1}{920}, a_3 \to \frac{2}{11}, a_4 \to \frac{1}{339}, a_5 \to \frac{2}{5}, a_6 \to \frac{2}{19}, a_7 \to \frac{1}{267}, a_8 \to \frac{3}{52}\}\}$$

- **Eells & Good**

CSe and **CSij** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSⅇ[𝔼1, ℂ1] ≥ CSⅇ[𝔼2, ℂ2], CSⅈⅉ[𝔼1, ℂ1] < CSⅈⅉ[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\left\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \left\{a_1 \to \frac{846\,077\,855\,323}{21\,499\,268\,659\,440}, a_2 \to \frac{1}{20}, a_3 \to \frac{3}{46}, a_4 \to \frac{1}{16}, a_5 \to \frac{1}{26}, a_6 \to \frac{1}{19}, a_7 \to \frac{3}{44}, a_8 \to \frac{4}{53}, a_9 \to \frac{5}{52}, a_{10} \to \frac{1}{34}, a_{11} \to \frac{3}{17}, a_{12} \to \frac{2}{37}, a_{13} \to \frac{1}{19}, a_{14} \to \frac{2}{43}, a_{15} \to \frac{1}{24}, a_{16} \to \frac{2}{39}\right\}\right\}$$

CSⅇ and **CSⅈⅉ** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSⅇ[𝔼, ℂ1] ≥ CSⅇ[𝔼, ℂ2], CSⅈⅉ[𝔼, ℂ1] < CSⅈⅉ[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\left\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \left\{a_1 \to \frac{1\,410\,773\,791\,657}{4\,029\,684\,167\,040}, a_2 \to \frac{21}{128}, a_3 \to \frac{11}{112}, a_4 \to \frac{3}{155}, a_5 \to \frac{37}{131}, a_6 \to \frac{2}{101}, a_7 \to \frac{1}{85}, a_8 \to \frac{7}{129}\right\}\right\}$$

CSⅇ and **CSⅈⅉ** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CSⅇ[𝔼1, ℂ] ≥ CSⅇ[𝔼2, ℂ], CSⅈⅉ[𝔼1, ℂ] < CSⅈⅉ[𝔼2, ℂ]}, Probabilities → Regular]
```

$$\left\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \left\{a_1 \to \frac{3\,281\,145\,539}{12\,793\,606\,020}, a_2 \to \frac{3}{13}, a_3 \to \frac{1}{27}, a_4 \to \frac{6}{59}, a_5 \to \frac{1}{20}, a_6 \to \frac{2}{17}, a_7 \to \frac{6}{79}, a_8 \to \frac{3}{23}\right\}\right\}$$

- **Suppes & Galton**

CSⅆ and **CSℊ** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSⅆ[𝔼1, ℂ1] ≥ CSⅆ[𝔼2, ℂ2], CSℊ[𝔼1, ℂ1] < CSℊ[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\left\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \left\{a_1 \to \frac{6\,922\,763\,391\,622\,931}{2\,336\,883\,816\,890\,975\,760}, a_2 \to \frac{1}{36}, a_3 \to \frac{1}{93}, a_4 \to \frac{4}{43}, a_5 \to \frac{1}{136}, a_6 \to \frac{1}{16}, a_7 \to \frac{3}{58}, a_8 \to \frac{16}{37}, a_9 \to \frac{1}{12}, a_{10} \to \frac{1}{47}, a_{11} \to \frac{4}{47}, a_{12} \to \frac{6}{67}, a_{13} \to \frac{1}{171}, a_{14} \to \frac{1}{97}, a_{15} \to \frac{1}{115}, a_{16} \to \frac{1}{136}\right\}\right\}$$

CSⅆ and **CSℊ** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSⅆ[𝔼, ℂ1] ≥ CSⅆ[𝔼, ℂ2], CSℊ[𝔼, ℂ1] < CSℊ[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\left\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \left\{a_1 \to \frac{18\,490\,511}{53\,199\,630}, a_2 \to \frac{2}{29}, a_3 \to \frac{3}{20}, a_4 \to \frac{1}{654}, a_5 \to \frac{1}{34}, a_6 \to \frac{1}{396}, a_7 \to \frac{1}{15}, a_8 \to \frac{1}{3}\right\}\right\}$$

CSⅆ and **CSℊ** *are* ordinally equivalent in the class of cases with two effects and a single cause (they *are* II-E):

```
PrSAT[{CSⅆ[𝔼1, ℂ] ≥ CSⅆ[𝔼2, ℂ], CSℊ[𝔼1, ℂ] < CSℊ[𝔼2, ℂ]}, Probabilities → Regular]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Suppes & Cheng**

CSⅆ and **CSⅽ** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSs[E1, C1] ≥ CSs[E2, C2], CSc[E1, C1] < CSc[E2, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \{a_1 \to \frac{54598177821239}{122557892826600}, a_2 \to \frac{1}{29}, a_3 \to \frac{1}{290}, a_4 \to \frac{1}{275}, a_5 \to \frac{3}{25}, a_6 \to \frac{1}{126}, a_7 \to \frac{2}{53}, a_8 \to \frac{1}{28}, a_9 \to \frac{1}{999}, a_{10} \to \frac{4}{21}, a_{11} \to \frac{1}{999}, a_{12} \to \frac{2}{33}, a_{13} \to \frac{1}{73}, a_{14} \to \frac{1}{213}, a_{15} \to \frac{1}{45}, a_{16} \to \frac{1}{56}\}\}$$

CSs and **CSc** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSs[E, C1] ≥ CSs[E, C2], CSc[E, C1] < CSc[E, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{3538711217}{14154875280}, a_2 \to \frac{1}{92}, a_3 \to \frac{5}{27}, a_4 \to \frac{1}{35}, a_5 \to \frac{11}{78}, a_6 \to \frac{1}{303}, a_7 \to \frac{6}{31}, a_8 \to \frac{3}{16}\}\}$$

CSs and **CSc** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CSs[E1, C] ≥ CSs[E2, C], CSc[E1, C] < CSc[E2, C]}, Probabilities → Regular]
```

$$\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{10679500945811}{45156373654770}, a_2 \to \frac{2}{35}, a_3 \to \frac{1}{999}, a_4 \to \frac{7}{34}, a_5 \to \frac{1}{122}, a_6 \to \frac{7}{31}, a_7 \to \frac{1}{758}, a_8 \to \frac{14}{53}\}\}$$

- **Suppes & Lewis Ratio**

CSs and **CSlr** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSs[E1, C1] ≥ CSs[E2, C2], CSlr[E1, C1] < CSlr[E2, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \{a_1 \to \frac{21928990223}{647751861900}, a_2 \to \frac{4}{55}, a_3 \to \frac{1}{29}, a_4 \to \frac{2}{51}, a_5 \to \frac{2}{27}, a_6 \to \frac{1}{29}, a_7 \to \frac{1}{75}, a_8 \to \frac{8}{29}, a_9 \to \frac{4}{45}, a_{10} \to \frac{1}{83}, a_{11} \to \frac{9}{41}, a_{12} \to \frac{1}{204}, a_{13} \to \frac{1}{348}, a_{14} \to \frac{2}{55}, a_{15} \to \frac{2}{39}, a_{16} \to \frac{1}{164}\}\}$$

CSs and **CSlr** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSs[E, C1] ≥ CSs[E, C2], CSlr[E, C1] < CSlr[E, C2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{31048357}{56814408}, a_2 \to \frac{19}{85}, a_3 \to \frac{1}{24}, a_4 \to \frac{1}{10}, a_5 \to \frac{1}{19}, a_6 \to \frac{1}{38}, a_7 \to \frac{1}{126}, a_8 \to \frac{1}{698}\}\}$$

CSs and **CSlr** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CSs[E1, C] ≥ CSs[E2, C], CSlr[E1, C] < CSlr[E2, C]}, Probabilities → Regular]
```

$$\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{18735952583407}{74400225940586}, a_2 \to \frac{1}{988}, a_3 \to \frac{20}{59}, a_4 \to \frac{1}{116}, a_5 \to \frac{9}{34}, a_6 \to \frac{1}{223}, a_7 \to \frac{1}{749}, a_8 \to \frac{4}{31}\}\}$$

- **Suppes & Good**

CSs and **CSij** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSs[𝔼1, ℂ1] ≥ CSs[𝔼2, ℂ2], CSij[𝔼1, ℂ1] < CSij[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \{a_1 \to \frac{10\,538\,490\,604\,381}{40\,767\,510\,250\,630\,800}, a_2 \to \frac{5}{42}, a_3 \to \frac{1}{999}, a_4 \to \frac{5}{42}, a_5 \to \frac{1}{999}, a_6 \to \frac{26}{69}, a_7 \to \frac{3}{62}, a_8 \to \frac{7}{43}, a_9 \to \frac{1}{700}, a_{10} \to \frac{1}{999}, a_{11} \to \frac{5}{37}, a_{12} \to \frac{1}{34}, a_{13} \to \frac{1}{999}, a_{14} \to \frac{1}{956}, a_{15} \to \frac{1}{624}, a_{16} \to \frac{1}{972}\}\}$$

CSs and **CSij** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSs[𝔼, ℂ1] ≥ CSs[𝔼, ℂ2], CSij[𝔼, ℂ1] < CSij[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{183\,180\,441\,981}{766\,827\,368\,230}, a_2 \to \frac{1}{211}, a_3 \to \frac{1}{19}, a_4 \to \frac{1}{22}, a_5 \to \frac{3}{35}, a_6 \to \frac{1}{109}, a_7 \to \frac{8}{43}, a_8 \to \frac{20}{53}\}\}$$

CSs and **CSij** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CSs[𝔼1, ℂ] ≥ CSs[𝔼2, ℂ], CSij[𝔼1, ℂ] < CSij[𝔼2, ℂ]}, Probabilities → Regular]
```

$$\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{36\,642\,643\,319}{113\,226\,380\,280}, a_2 \to \frac{1}{65}, a_3 \to \frac{1}{999}, a_4 \to \frac{5}{54}, a_5 \to \frac{1}{158}, a_6 \to \frac{9}{89}, a_7 \to \frac{1}{120}, a_8 \to \frac{14}{31}\}\}$$

- **Galton & Cheng**

CSg and **CSc** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CSg[𝔼1, ℂ1] ≥ CSg[𝔼2, ℂ2], CSc[𝔼1, ℂ1] < CSc[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \{a_1 \to \frac{16\,352\,691\,536\,717}{56\,256\,292\,461\,600}, a_2 \to \frac{1}{84}, a_3 \to \frac{3}{74}, a_4 \to \frac{1}{59}, a_5 \to \frac{1}{28}, a_6 \to \frac{5}{32}, a_7 \to \frac{5}{84}, a_8 \to \frac{1}{84}, a_9 \to \frac{1}{400}, a_{10} \to \frac{2}{15}, a_{11} \to \frac{1}{423}, a_{12} \to \frac{3}{77}, a_{13} \to \frac{1}{23}, a_{14} \to \frac{1}{344}, a_{15} \to \frac{1}{77}, a_{16} \to \frac{7}{50}\}\}$$

CSg and **CSc** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CSg[𝔼, ℂ1] ≥ CSg[𝔼, ℂ2], CSc[𝔼, ℂ1] < CSc[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{106\,387}{345\,950}, a_2 \to \frac{1}{25}, a_3 \to \frac{1}{12}, a_4 \to \frac{14}{33}, a_5 \to \frac{1}{11}, a_6 \to \frac{1}{148}, a_7 \to \frac{1}{22}, a_8 \to \frac{1}{561}\}\}$$

CSg and **CSc** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CSg[𝔼1, ℂ] ≥ CSg[𝔼2, ℂ], CSc[𝔼1, ℂ] < CSc[𝔼2, ℂ]}, Probabilities → Regular]
```

$$\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \{a_1 \to \frac{73\,624}{209\,715}, a_2 \to \frac{1}{41}, a_3 \to \frac{1}{41}, a_4 \to \frac{14}{55}, a_5 \to \frac{1}{124}, a_6 \to \frac{5}{41}, a_7 \to \frac{1}{31}, a_8 \to \frac{11}{60}\}\}$$

- **Galton & Lewis Ratio**

CSg and **CSlr** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CS𝕘[𝔼1, ℂ1] ≥ CS𝕘[𝔼2, ℂ2], CS𝕝𝕣[𝔼1, ℂ1] < CS𝕝𝕣[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \Big\{a_1 \to \frac{1\,421\,801\,290\,792\,405\,981}{4\,737\,362\,510\,908\,510\,560}, a_2 \to \frac{1}{30}, a_3 \to \frac{1}{827}, a_4 \to \frac{7}{36}, a_5 \to \frac{1}{186}, a_6 \to \frac{1}{864}, a_7 \to \frac{13}{62}, a_8 \to \frac{1}{482}, a_9 \to \frac{1}{30}, a_{10} \to \frac{7}{54}, a_{11} \to \frac{1}{392}, a_{12} \to \frac{3}{55}, a_{13} \to \frac{1}{417}, a_{14} \to \frac{1}{368}, a_{15} \to \frac{1}{412}, a_{16} \to \frac{1}{40}\Big\}\Big\}$$

CS𝕘 and **CS𝕝𝕣** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CS𝕘[𝔼, ℂ1] ≥ CS𝕘[𝔼, ℂ2], CS𝕝𝕣[𝔼, ℂ1] < CS𝕝𝕣[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \Big\{a_1 \to \frac{101\,643\,217}{234\,554\,100}, a_2 \to \frac{5}{33}, a_3 \to \frac{1}{678}, a_4 \to \frac{2}{25}, a_5 \to \frac{1}{660}, a_6 \to \frac{7}{37}, a_7 \to \frac{7}{85}, a_8 \to \frac{2}{33}\Big\}\Big\}$$

CS𝕘 and **CS𝕝𝕣** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CS𝕘[𝔼1, ℂ] ≥ CS𝕘[𝔼2, ℂ], CS𝕝𝕣[𝔼1, ℂ] < CS𝕝𝕣[𝔼2, ℂ]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \Big\{a_1 \to \frac{1\,417\,850\,087}{4\,609\,269\,280}, a_2 \to \frac{1}{29}, a_3 \to \frac{1}{11}, a_4 \to \frac{1}{582}, a_5 \to \frac{11}{60}, a_6 \to \frac{3}{49}, a_7 \to \frac{1}{32}, a_8 \to \frac{11}{38}\Big\}\Big\}$$

- **Galton & Good**

CS𝕘 and **CS𝕚𝕛** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CS𝕘[𝔼1, ℂ1] ≥ CS𝕘[𝔼2, ℂ2], CS𝕚𝕛[𝔼1, ℂ1] < CS𝕚𝕛[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\}, \mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\}, \Big\{a_1 \to \frac{1\,489\,965\,617\,368\,492\,451\,701}{3\,639\,223\,726\,139\,996\,794\,800}, a_2 \to \frac{1}{590}, a_3 \to \frac{11}{164}, a_4 \to \frac{11}{164}, a_5 \to \frac{1}{432}, a_6 \to \frac{14}{151}, a_7 \to \frac{9}{154}, a_8 \to \frac{21}{200}, a_9 \to \frac{1}{52}, a_{10} \to \frac{3}{151}, a_{11} \to \frac{4}{109}, a_{12} \to \frac{3}{83}, a_{13} \to \frac{1}{22}, a_{14} \to \frac{1}{253}, a_{15} \to \frac{2}{103}, a_{16} \to \frac{2}{129}\Big\}\Big\}$$

CS𝕘 and **CS𝕚𝕛** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CS𝕘[𝔼, ℂ1] ≥ CS𝕘[𝔼, ℂ2], CS𝕚𝕛[𝔼, ℂ1] < CS𝕚𝕛[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \Big\{a_1 \to \frac{82\,557\,216\,031}{174\,544\,966\,524}, a_2 \to \frac{5}{24}, a_3 \to \frac{1}{129}, a_4 \to \frac{1}{632}, a_5 \to \frac{1}{233}, a_6 \to \frac{3}{17}, a_7 \to \frac{4}{47}, a_8 \to \frac{1}{23}\Big\}\Big\}$$

CS𝕘 and **CS𝕚𝕛** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CS𝕘[𝔼1, ℂ] ≥ CS𝕘[𝔼2, ℂ], CS𝕚𝕛[𝔼1, ℂ] < CS𝕚𝕛[𝔼2, ℂ]}, Probabilities → Regular]
```

$$\Big\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\}, \Big\{a_1 \to \frac{156\,979}{608\,304}, a_2 \to \frac{4}{19}, a_3 \to \frac{2}{23}, a_4 \to \frac{3}{29}, a_5 \to \frac{1}{16}, a_6 \to \frac{3}{38}, a_7 \to \frac{1}{24}, a_8 \to \frac{3}{19}\Big\}\Big\}$$

- **Cheng & Lewis Ratio**

CS𝕔 and **CS𝕝𝕣** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CS𝕔[𝔼1, ℂ1] ≥ CS𝕔[𝔼2, ℂ2], CS𝕝𝕣[𝔼1, ℂ1] < CS𝕝𝕣[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\},$$
$$\mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\},$$
$$\Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\},$$
$$\{a_1 \to \frac{71\,659\,914\,453\,187\,427}{730\,589\,028\,241\,010\,400}, a_2 \to \frac{1}{999}, a_3 \to \frac{1}{517}, a_4 \to \frac{3}{25}, a_5 \to \frac{1}{989}, a_6 \to \frac{1}{999}, a_7 \to \frac{3}{14},$$
$$a_8 \to \frac{1}{999}, a_9 \to \frac{10}{53}, a_{10} \to \frac{19}{94}, a_{11} \to \frac{1}{999}, a_{12} \to \frac{5}{61}, a_{13} \to \frac{1}{999}, a_{14} \to \frac{1}{96}, a_{15} \to \frac{1}{79}, a_{16} \to \frac{3}{47}\}\}$$

CS𝕔 and **CS𝕝𝕣** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CS𝕔[𝔼, ℂ1] ≥ CS𝕔[𝔼, ℂ2], CS𝕝𝕣[𝔼, ℂ1] < CS𝕝𝕣[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$$
$$\{a_1 \to \frac{3\,229\,633}{11\,888\,100}, a_2 \to \frac{1}{72}, a_3 \to \frac{11}{50}, a_4 \to \frac{1}{840}, a_5 \to \frac{1}{999}, a_6 \to \frac{1}{153}, a_7 \to \frac{1}{5}, a_8 \to \frac{2}{7}\}\}$$

CS𝕔 and **CS𝕝𝕣** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CS𝕔[𝔼1, ℂ] ≥ CS𝕔[𝔼2, ℂ], CS𝕝𝕣[𝔼1, ℂ] < CS𝕝𝕣[𝔼2, ℂ]}, Probabilities → Regular]
```

$$\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$$
$$\{a_1 \to \frac{1\,417\,850\,087}{4\,609\,269\,280}, a_2 \to \frac{1}{29}, a_3 \to \frac{1}{11}, a_4 \to \frac{1}{582}, a_5 \to \frac{11}{60}, a_6 \to \frac{3}{49}, a_7 \to \frac{1}{32}, a_8 \to \frac{11}{38}\}\}$$

- **Cheng & Good**

CS𝕔 and **CS𝕚𝕛** *are* ordinally equivalent *in general* (they *are* G-E):

```
PrSAT[{CS𝕔[𝔼1, ℂ1] ≥ CS𝕔[𝔼2, ℂ2], CS𝕚𝕛[𝔼1, ℂ1] < CS𝕚𝕛[𝔼2, ℂ2]}, Probabilities → Regular]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Lewis Ratio & Good**

CS𝕝𝕣 and **CS𝕚𝕛** are *not* ordinally equivalent *in general* (they are *not* G-E):

```
PrSAT[{CS𝕝𝕣[𝔼1, ℂ1] ≥ CS𝕝𝕣[𝔼2, ℂ2], CS𝕚𝕛[𝔼1, ℂ1] < CS𝕚𝕛[𝔼2, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_6, a_7, a_8, a_{12}, a_{13}, a_{14}, a_{16}\}, \mathbb{C}2 \to \{a_3, a_6, a_9, a_{10}, a_{12}, a_{13}, a_{15}, a_{16}\},$$
$$\mathbb{E}1 \to \{a_4, a_7, a_9, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}\}, \mathbb{E}2 \to \{a_5, a_8, a_{10}, a_{11}, a_{13}, a_{14}, a_{15}, a_{16}\},$$
$$\Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}\},$$
$$\{a_1 \to \frac{1\,927\,075\,715\,645\,624\,223\,873}{6\,545\,193\,215\,139\,594\,055\,040}, a_2 \to \frac{1}{68}, a_3 \to \frac{1}{986}, a_4 \to \frac{1}{640}, a_5 \to \frac{2}{33}, a_6 \to \frac{1}{139}, a_7 \to \frac{3}{35},$$
$$a_8 \to \frac{3}{56}, a_9 \to \frac{1}{985}, a_{10} \to \frac{1}{71}, a_{11} \to \frac{1}{998}, a_{12} \to \frac{1}{521}, a_{13} \to \frac{7}{52}, a_{14} \to \frac{5}{56}, a_{15} \to \frac{1}{132}, a_{16} \to \frac{19}{82}\}\}$$

CS𝕝𝕣 and **CS𝕚𝕛** are *not* ordinally equivalent in the class of cases with two causes and a single effect (they are *not* I-E):

```
PrSAT[{CS𝕝𝕣[𝔼, ℂ1] ≥ CS𝕝𝕣[𝔼, ℂ2], CS𝕚𝕛[𝔼, ℂ1] < CS𝕚𝕛[𝔼, ℂ2]}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$$
$$\{a_1 \to \frac{50\,912\,868\,509}{162\,069\,276\,936}, a_2 \to \frac{16}{97}, a_3 \to \frac{1}{24}, a_4 \to \frac{1}{86}, a_5 \to \frac{3}{47}, a_6 \to \frac{9}{38}, a_7 \to \frac{14}{111}, a_8 \to \frac{2}{49}\}\}$$

CS𝕝𝕣 and **CS𝕚𝕛** are *not* ordinally equivalent in the class of cases with two effects and a single cause (they are *not* II-E):

```
PrSAT[{CS𝕝𝕣[𝔼1, ℂ] ≥ CS𝕝𝕣[𝔼2, ℂ], CS𝕚𝕛[𝔼1, ℂ] < CS𝕚𝕛[𝔼2, ℂ]}, Probabilities → Regular]
```

$$\left\{\{\mathbb{C} \to \{a_2, a_5, a_6, a_8\}, \mathbb{E}1 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E}2 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},\ \left\{a_1 \to \frac{1\,862\,780\,176\,883}{6\,521\,744\,272\,680}, a_2 \to \frac{1}{680}, a_3 \to \frac{1}{82}, a_4 \to \frac{15}{47}, a_5 \to \frac{1}{91}, a_6 \to \frac{1}{5}, a_7 \to \frac{1}{927}, a_8 \to \frac{10}{59}\right\}\right\}$$

Continuity Property Verification (Table 5)

In this section, we verify the continuity properties of all the measures, as reported in Table 5 of the paper (again, using **PrSAT**, as above — so if a model is found, then it is a counterexample to the salient continuity property, and if no models are found, then the salient continuity property holds generally in that case).

Causation-Prevention Continuity (CPC)

Eells

```
PrSAT[{CS𝕖[Y, X] ≠ -CS𝕖[¬ Y , X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Suppes

```
PrSAT[{CS𝕤[Y, X] ≠ -CS𝕤[¬ Y , X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Galton

```
PrSAT[{CS𝕘[Y, X] ≠ -CS𝕘[¬ Y , X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Cheng

```
PrSAT[{CS𝕔[Y, X] ≠ -CS𝕔[¬ Y , X]}, Probabilities → Regular]
```

$$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\},\ \left\{a_1 \to \frac{7\,510\,271}{14\,910\,716}, a_2 \to \frac{1}{943}, a_3 \to \frac{1}{268}, a_4 \to \frac{29}{59}\right\}\right\}$$

Lewis Ratio (first rescaling)

```
PrSAT[{CS𝕝𝕣1[Y, X] ≠ -CS𝕝𝕣1[¬ Y , X]}, Probabilities → Regular]
```

$$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\},\ \left\{a_1 \to \frac{997}{1998}, a_2 \to \frac{1}{4}, a_3 \to \frac{1}{999}, a_4 \to \frac{1}{4}\right\}\right\}$$

Lewis Ratio (second rescaling)

```
PrSAT[{CS𝕝𝕣2[Y, X] ≠ -CS𝕝𝕣2[¬ Y , X]}, Probabilities → Regular]
```

$$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\},\ \left\{a_1 \to \frac{4031}{4\,091\,020}, a_2 \to \frac{108}{215}, a_3 \to \frac{35}{71}, a_4 \to \frac{1}{268}\right\}\right\}$$

Good (first rescaling)

```
PrSAT[{CS𝕚𝕛1[Y, X] ≠ -CS𝕚𝕛1[¬ Y , X]}, Probabilities → Regular]
```

$$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\},\ \left\{a_1 \to \frac{505\,481}{1\,010\,988}, a_2 \to \frac{63}{253}, a_3 \to \frac{1}{999}, a_4 \to \frac{1}{4}\right\}\right\}$$

Good (second rescaling)

```
PrSAT[{CSij2[Y, X] ≠ -CSij2[¬ Y , X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{119\,333}{244\,790}, a_2 \to \frac{1}{910}, a_3 \to \frac{1}{269}, a_4 \to \frac{33}{65}\right\}\right\}$

- **Causation-Omission Continuity (COC)**

Eells

```
PrSAT[{CSe[Y, X] ≠ -CSe[Y , ¬ X]}, Probabilities → Regular]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

{}

Suppes

```
PrSAT[{CSs[Y, X] ≠ -CSs[Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{9896}{46\,953}, a_2 \to \frac{1}{999}, a_3 \to \frac{1}{999}, a_4 \to \frac{37}{47}\right\}\right\}$

Galton

```
PrSAT[{CSg[Y, X] ≠ -CSg[Y , ¬ X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Cheng

```
PrSAT[{CSc[Y, X] ≠ -CSc[Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{1}{300}, a_2 \to \frac{29}{60}, a_3 \to \frac{12}{25}, a_4 \to \frac{1}{30}\right\}\right\}$

Lewis Ratio (first rescaling)

```
PrSAT[{CSlr1[Y, X] ≠ -CSlr1[Y , ¬ X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Lewis Ratio (second rescaling)

```
PrSAT[{CSlr2[Y, X] ≠ -CSlr2[Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{22\,266\,503}{44\,695\,430}, a_2 \to \frac{1}{998}, a_3 \to \frac{1}{265}, a_4 \to \frac{84}{169}\right\}\right\}$

Good (first rescaling)

```
PrSAT[{CSij1[Y, X] ≠ -CSij1[Y , ¬ X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Good (second rescaling)

```
PrSAT[{CSij2[Y, X] ≠ -CSij2[Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{3304061}{6618375}, a_2 \to \frac{1}{265}, a_3 \to \frac{1}{999}, a_4 \to \frac{62}{125}\right\}\right\}$

- **Causation = Prevention By Omission Continuity (CPO)**

Eells

```
PrSAT[{CSe[Y, X] ≠ CSe[¬ Y , ¬ X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Suppes

```
PrSAT[{CSs[Y, X] ≠ CSs[¬ Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{1105}{892107}, a_2 \to \frac{37}{47}, a_3 \to \frac{4}{19}, a_4 \to \frac{1}{999}\right\}\right\}$

Galton

```
PrSAT[{CSg[Y, X] ≠ CSg[¬ Y , ¬ X]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Cheng

```
PrSAT[{CSc[Y, X] ≠ CSc[¬ Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{995}{3996}, a_2 \to \frac{1}{999}, a_3 \to \frac{1}{4}, a_4 \to \frac{1}{2}\right\}\right\}$

Lewis Ratio (first rescaling)

```
PrSAT[{CSlr1[Y, X] ≠ CSlr1[¬ Y , ¬ X]}]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{1}{4}, a_2 \to 0, a_3 \to \frac{1}{4}, a_4 \to \frac{1}{2}\right\}\right\}$

Lewis Ratio (second rescaling)

```
PrSAT[{CSlr2[Y, X] ≠ CSlr2[¬ Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{505481}{1010988}, a_2 \to \frac{1}{4}, a_3 \to \frac{1}{999}, a_4 \to \frac{63}{253}\right\}\right\}$

Good (first rescaling)

```
PrSAT[{CSij1[Y, X] ≠ CSij1[¬ Y , ¬ X]}]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{1}{4}, a_2 \to \frac{1}{2}, a_3 \to \frac{1}{4}, a_4 \to 0\right\}\right\}$

Good (second rescaling)

```
PrSAT[{CSij2[Y, X] ≠ CSij2[¬ Y , ¬ X]}, Probabilities → Regular]
```

$\left\{\{X \to \{a_2, a_4\}, Y \to \{a_3, a_4\}, \Omega \to \{a_1, a_2, a_3, a_4\}\}, \left\{a_1 \to \frac{995}{3996}, a_2 \to \frac{1}{2}, a_3 \to \frac{1}{4}, a_4 \to \frac{1}{999}\right\}\right\}$

Causal Independence (definitions, and two fundamental properties)

First, we define the various causal independence relations, for the various measures of causal strength:

```
ICSe[E_, C1_, C2_] := CSe[E, C1, C2] == CSe[E, C1, ¬ C2];
ICSs[E_, C1_, C2_] := CSs[E, C1, C2] == CSs[E, C1, ¬ C2];
ICSg[E_, C1_, C2_] := CSg[E, C1, C2] == CSg[E, C1, ¬ C2];
ICSc[E_, C1_, C2_] := CSc[E, C1, C2] == CSc[E, C1, ¬ C2];
ICSlr[E_, C1_, C2_] := CSlr[E, C1, C2] == CSlr[E, C1, ¬ C2];
ICSij[E_, C1_, C2_] := CSij[E, C1, C2] == CSij[E, C1, ¬ C2];
```

Then, we set-up our *background conditions* (**BACK**), which include the following: (*i*) that **C1** and **C2** are unconditionally probabilistically independent, (*ii*) that **C1** and **C2** are both positively causally relevant to **E** [*i.e.*, that **Pr[E|C1]>Pr[E]** and **Pr[E|C2]>Pr[E]**]. Finally, to simplify the searches, we will also assume (as part of **BACK**) — without loss of generality in this context — (*iii*) that **Pr[E|C1]=1/2** and **Pr[E]=1/4** and that **Pr[E|C2]=1/2** and **Pr[E]=1/4**. This last assumption [which is just a more precise way of asserting (*ii*)] could be relaxed, but the searches would take much longer to complete.

```
BACK := {Pr[C1 | C2] == Pr[C1], Pr[E | C1] > Pr[E], Pr[E | C2] > Pr[E],
   Pr[E | C1] == 1 / 2, Pr[E] == 1 / 4, Pr[E | C2] == 1 / 2, Pr[E] == 1 / 4};
```

The following two fundamental properties involving causal Independence judgments are satisfied by all of our measures, *given* **BACK**:

- **ICS(E,C1,C2)** iff **ICS(E,C2,C1)** [Symmetry of **ICS** in **C1,C2**]
- **ICS(E,C1,C2)** iff **ICS(E,C1,C2)=ICS(E,C1)** [Equivalence of conditional/unconditional definitions of **ICS**]

Here are **PrSAT**-verifications of these fundamental properties (given **BACK**), for each of our measures of causal strength. First, we define a *non-equivalence relation* (≢), to make it easier to assert that a logical equivalence fails to hold:

```
p_ ≡ q_ := (p ⇒ q) && (q ⇒ p);
p_ ≢ q_ := ¬ p ≡ q;
```

Eells

Symmetry of **ICSe** in **C1,C2**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSe[E, C1, C2] ≢ ICSe[E, C2, C1]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Equivalence of conditional/unconditional definitions of **ICSe**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSe[E, C1, C2] ≢ (CSe[E, C1, C2] == CSe[E, C1])}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Suppes

Symmetry of **ICSs** in **C1,C2**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSs[E, C1, C2] ≢ ICSs[E, C2, C1]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Equivalence of conditional/unconditional definitions of **ICSs**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSs[E, C1, C2] ≢ (CSs[E, C1, C2] == CSs[E, C1])}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Galton

Symmetry of **ICSg** in **C1,C2**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSg[E, C1, C2] ≢ ICSg[E, C2, C1]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Equivalence of conditional/unconditional definitions of **ICSg**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSg[E, C1, C2] ≢ (CSg[E, C1, C2] == CSg[E, C1])}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Cheng**

Symmetry of **ICSc** in **C1,C2**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSc[E, C1, C2] ≢ ICSc[E, C2, C1]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Equivalence of conditional/unconditional definitions of **ICSg**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSc[E, C1, C2] ≢ (CSc[E, C1, C2] == CSc[E, C1])}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Lewis Ratio**

Symmetry of **ICSlr** in **C1,C2**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSlr[E, C1, C2] ≢ ICSlr[E, C2, C1]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Equivalence of conditional/unconditional definitions of **ICSlr**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSlr[E, C1, C2] ≢ (CSlr[E, C1, C2] == CSlr[E, C1])}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Good**

Symmetry of **ICSij** in **C1,C2**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSij[E, C1, C2] ≢ ICSij[E, C2, C1]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Equivalence of conditional/unconditional definitions of **ICSij**, *given* **BACK**:

```
PrSAT[BACK ⋃ {ICSij[𝔼, ℂ1, ℂ2] ≢ (CSij[𝔼, ℂ1, ℂ2] == CSij[𝔼, ℂ1])}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Agreement on Causal Independence Judgments (Table 6)

In this section, we verify the claims about agreement on independence judgments reported in Table 6 of the paper (using **PrSAT**, as above).

Eells & Suppes

CSe and **CSs** agree on *all* independence judgments (assuming **BACK**). First, we show that **ICSe[𝔼,ℂ1,ℂ2] ⇒ ICSs[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {ICSe[𝔼, ℂ1, ℂ2], ¬ ICSs[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Then, we show that **ICSs[𝔼,ℂ1,ℂ2] ⇒ ICSe[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {¬ ICSe[𝔼, ℂ1, ℂ2], ICSs[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Finally, we show that there *are* some cases (satisfying **BACK**) in which **CSe** and **CSs** agree that **ℂ1** and **ℂ2** are independent causes of **𝔼** (*non-triviality*):

```
PrSAT[BACK ⋃ {ICSe[𝔼, ℂ1, ℂ2], ICSs[𝔼, ℂ1, ℂ2], Pr[ℂ1] == 1/3}, Probabilities → Regular]
```

$$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$$
$$\{a_1 \to \frac{391}{876}, a_2 \to \frac{61}{438}, a_3 \to \frac{10}{73}, a_4 \to \frac{1}{876}, a_5 \to \frac{2}{73}, a_6 \to \frac{37}{438}, a_7 \to \frac{6}{73}, a_8 \to \frac{6}{73}\}\}$$

Eells & Galton

CSe and **CSg** agree on *all* independence judgments (assuming **BACK**). First, we show that **ICSe[𝔼,ℂ1,ℂ2] ⇒ ICSg[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {ICSe[𝔼, ℂ1, ℂ2], ¬ ICSg[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Then, we show that **ICSg[𝔼,ℂ1,ℂ2] ⇒ ICSe[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {¬ ICSe[𝔼, ℂ1, ℂ2], ICSg[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Finally, we show that there *are* some cases (satisfying **BACK**) in which **CSe** and **CSg** agree that **ℂ1** and **ℂ2** are independent causes of **𝔼** (*non-triviality*):

```
PrSAT[BACK ⋃ {ICSe[𝔼, ℂ1, ℂ2], ICSg[𝔼, ℂ1, ℂ2], Pr[ℂ1] == 1/3}, Probabilities → Regular]
```

$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$
$\{a_1 \to \frac{949}{2124}, a_2 \to \frac{74}{531}, a_3 \to \frac{145}{1062}, a_4 \to \frac{1}{708}, a_5 \to \frac{29}{1062}, a_6 \to \frac{5}{59}, a_7 \to \frac{29}{354}, a_8 \to \frac{29}{354}\}\}$

- **Eells & Cheng**

CSe and **CSc** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSe[𝔼, ℂ1, ℂ2], ICSc[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Eells & Lewis Ratio**

CSe and **CSlr** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSe[𝔼, ℂ1, ℂ2], ICSlr[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Eells & Good**

CSe and **CSij** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSe[𝔼, ℂ1, ℂ2], ICSij[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Suppes & Galton**

CSs and **CSg** agree on *all* independence judgments (assuming **BACK**). First, we show that **ICSs[𝔼,ℂ1,ℂ2] ⇒ ICSg[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {ICSs[𝔼, ℂ1, ℂ2], ¬ ICSg[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Then, we show that **ICSg[𝔼,ℂ1,ℂ2] ⇒ ICSs[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {¬ ICSs[𝔼, ℂ1, ℂ2], ICSg[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Finally, we show that there *are* some cases (satisfying **BACK**) in which **CSs** and **CSg** agree that **ℂ1** and **ℂ2** are independent causes of **𝔼** (*non-triviality*):

```
PrSAT[BACK ⋃ {ICSs[E, C1, C2], ICSg[E, C1, C2], Pr[C1] == 1/3}, Probabilities → Regular]
```

$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$
$\{a_1 \to \frac{949}{2124}, a_2 \to \frac{74}{531}, a_3 \to \frac{145}{1062}, a_4 \to \frac{1}{708}, a_5 \to \frac{29}{1062}, a_6 \to \frac{5}{59}, a_7 \to \frac{29}{354}, a_8 \to \frac{29}{354}\}\}$

- **Suppes & Cheng**

CSs and **CSc** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSs[E, C1, C2], ICSc[E, C1, C2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Suppes & Lewis Ratio**

CSs and **CSlr** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSs[E, C1, C2], ICSlr[E, C1, C2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Suppes & Good**

CSs and **CSij** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[Union[BACK, {ICSs[E, C1, C2], ICSij[E, C1, C2]}]]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Galton & Cheng**

CSg and **CSc** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSg[E, C1, C2], ICSc[E, C1, C2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Galton & Lewis Ratio**

CSg and **CSlr** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSg[E, C1, C2], ICSlr[E, C1, C2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Galton & Good**

CSg and **CSij** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSg[𝔼, ℂ1, ℂ2], ICSij[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Cheng & Lewis Ratio**

CSc and **CSlr** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSc[𝔼, ℂ1, ℂ2], ICSlr[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Cheng & Good**

CSc and **CSij** agree on *all* independence judgments (assuming **BACK**). First, we show that **ICSc[𝔼,ℂ1,ℂ2] ⇒ ICSij[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {ICSc[𝔼, ℂ1, ℂ2], ¬ ICSij[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Then, we show that **ICSij[𝔼,ℂ1,ℂ2] ⇒ ICSc[𝔼,ℂ1,ℂ2]**, given **BACK**:

```
PrSAT[BACK ⋃ {¬ ICSc[𝔼, ℂ1, ℂ2], ICSij[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Finally, we show that there *are* some cases (satisfying **BACK**) in which **CSc** and **CSij** agree that **ℂ1** and **ℂ2** are independent causes of **𝔼** (*non-triviality*):

$$\text{PrSAT}\left[\text{BACK} \bigcup \left\{\text{ICSc}[\mathbb{E}, \mathbb{C}1, \mathbb{C}2], \text{ICSij}[\mathbb{E}, \mathbb{C}1, \mathbb{C}2], \Pr[\mathbb{C}1] == \frac{1}{3}\right\}, \text{Probabilities} \to \text{Regular}\right]$$

$$\left\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},\right.$$
$$\left.\left\{a_1 \to \frac{343}{732}, a_2 \to \frac{49}{366}, a_3 \to \frac{7}{61}, a_4 \to \frac{1}{732}, a_5 \to \frac{2}{61}, a_6 \to \frac{37}{366}, a_7 \to \frac{5}{61}, a_8 \to \frac{4}{61}\right\}\right\}$$

- **Lewis Ratio & Good**

CSlr and **CSij** agree on *no* independence judgments (assuming **BACK**).

```
PrSAT[BACK ⋃ {ICSlr[𝔼, ℂ1, ℂ2], ICSij[𝔼, ℂ1, ℂ2]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Constraint (†) on the values of independent causal strengths

In this section, we will show that some of our measures m are such that:

(†) **ICSm[𝔼, ℂ1, ℂ2] ⇒ CSm[𝔼, ℂ1] + CSm[𝔼, ℂ2] ≤ 1.**

That is, for some of our measures m, if ℂ1 and ℂ2 are independent causes of 𝔼 according to m, then the individual m-causal-strengths of ℂ1 and ℂ2 cannot sum to more than 1. We will also show that some measures do *not* imply any such constraint (†) on independent individual causal strengths.

Eells

CSe *does* entail (†):

```
PrSAT[{ICSe[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1],
  CSe[𝔼, ℂ1] + CSe[𝔼, ℂ2] > 1, CSe[𝔼, ℂ1] > 0, CSe[𝔼, ℂ2] > 0}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Suppes

CSs *does* entail (†):

```
PrSAT[{ICSs[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1], CSs[𝔼, ℂ1] + CSs[𝔼, ℂ2] > 1, CSs[𝔼, ℂ1] > 0, CSs[𝔼, ℂ2] > 0}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Galton

CSg *does* entail (†):

```
PrSAT[{ICSg[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1], CSg[𝔼, ℂ1] + CSg[𝔼, ℂ2] > 1, CSg[𝔼, ℂ1] > 0, CSg[𝔼, ℂ2] > 0}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Cheng

CSc does *not* entail (†):

```
PrSAT[{ICSc[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1], CSc[𝔼, ℂ1] + CSc[𝔼, ℂ2] > 1, CSc[𝔼, ℂ1] > 0,
  CSc[𝔼, ℂ2] > 0, Pr[ℂ1] == 1 / 2, Pr[ℂ2] == 1 / 2}, Probabilities → Regular, BypassSearch → True]
```

$\{\{ℂ1 \to \{a_2, a_5, a_6, a_8\}, ℂ2 \to \{a_3, a_5, a_7, a_8\}, 𝔼 \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$
$\{a_1 \to \frac{1}{6}, a_2 \to \frac{1}{16}, a_3 \to \frac{1}{16}, a_4 \to \frac{1}{12}, a_5 \to \frac{3}{128}, a_6 \to \frac{3}{16}, a_7 \to \frac{3}{16}, a_8 \to \frac{29}{128}\}\}$

Lewis Ratio (first rescaling)

CSlr1 does *not* entail (†):

```
PrSAT[{ICS𝕃𝕣[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1], CS𝕃𝕣1[𝔼, ℂ1] + CS𝕃𝕣1[𝔼, ℂ2] > 1, CS𝕃𝕣1[𝔼, ℂ1] > 0,
  CS𝕃𝕣1[𝔼, ℂ2] > 0, Pr[ℂ1] == 1 / 2, Pr[ℂ2] == 1 / 2}, Probabilities → Regular, BypassSearch → True]
```

```
{{ℂ1 → {𝕒2, 𝕒5, 𝕒6, 𝕒8}, ℂ2 → {𝕒3, 𝕒5, 𝕒7, 𝕒8}, 𝔼 → {𝕒4, 𝕒6, 𝕒7, 𝕒8}, Ω → {𝕒1, 𝕒2, 𝕒3, 𝕒4, 𝕒5, 𝕒6, 𝕒7, 𝕒8}},
 {𝕒1 → 31/128, 𝕒2 → 15/64, 𝕒3 → 3/16, 𝕒4 → 1/128, 𝕒5 → 1/8, 𝕒6 → 1/64, 𝕒7 → 1/16, 𝕒8 → 1/8}}
```

- **Lewis Ratio (second rescaling)**

CS𝕃𝕣2 does *not* entail (†):

```
PrSAT[{ICS𝕃𝕣[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1], CS𝕃𝕣2[𝔼, ℂ1] + CS𝕃𝕣2[𝔼, ℂ2] > 1, CS𝕃𝕣2[𝔼, ℂ1] > 0,
  CS𝕃𝕣2[𝔼, ℂ2] > 0, Pr[ℂ1] == 1 / 2, Pr[ℂ2] == 1 / 2}, Probabilities → Regular, BypassSearch → True]
```

```
{{ℂ1 → {𝕒2, 𝕒5, 𝕒6, 𝕒8}, ℂ2 → {𝕒3, 𝕒5, 𝕒7, 𝕒8}, 𝔼 → {𝕒4, 𝕒6, 𝕒7, 𝕒8}, Ω → {𝕒1, 𝕒2, 𝕒3, 𝕒4, 𝕒5, 𝕒6, 𝕒7, 𝕒8}},
 {𝕒1 → 21/92, 𝕒2 → 3/16, 𝕒3 → 3/16, 𝕒4 → 1/46, 𝕒5 → 9/128, 𝕒6 → 1/16, 𝕒7 → 1/16, 𝕒8 → 23/128}}
```

- **Good (first rescaling)**

CS𝕚𝕛1 does *not* entail (†):

```
PrSAT[{ICS𝕚𝕛[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1], CS𝕚𝕛1[𝔼, ℂ1] + CS𝕚𝕛1[𝔼, ℂ2] > 1, CS𝕚𝕛1[𝔼, ℂ1] > 0,
  CS𝕚𝕛1[𝔼, ℂ2] > 0, Pr[ℂ1] == 1 / 2, Pr[ℂ2] == 1 / 2}, Probabilities → Regular, BypassSearch → True]
```

```
{{ℂ1 → {𝕒2, 𝕒5, 𝕒6, 𝕒8}, ℂ2 → {𝕒3, 𝕒5, 𝕒7, 𝕒8}, 𝔼 → {𝕒4, 𝕒6, 𝕒7, 𝕒8}, Ω → {𝕒1, 𝕒2, 𝕒3, 𝕒4, 𝕒5, 𝕒6, 𝕒7, 𝕒8}},
 {𝕒1 → 1/6, 𝕒2 → 1/16, 𝕒3 → 1/24, 𝕒4 → 1/12, 𝕒5 → 1/64, 𝕒6 → 3/16, 𝕒7 → 5/24, 𝕒8 → 15/64}}
```

- **Good (second rescaling)**

CS𝕚𝕛2 does *not* entail (†):

```
PrSAT[{ICS𝕚𝕛[𝔼, ℂ1, ℂ2], Pr[ℂ1 | ℂ2] == Pr[ℂ1], CS𝕚𝕛2[𝔼, ℂ1] + CS𝕚𝕛2[𝔼, ℂ2] > 1, CS𝕚𝕛2[𝔼, ℂ1] > 0,
  CS𝕚𝕛2[𝔼, ℂ2] > 0, Pr[ℂ1] == 1 / 2, Pr[ℂ2] == 1 / 2}, Probabilities → Regular, BypassSearch → True]
```

```
{{ℂ1 → {𝕒2, 𝕒5, 𝕒6, 𝕒8}, ℂ2 → {𝕒3, 𝕒5, 𝕒7, 𝕒8}, 𝔼 → {𝕒4, 𝕒6, 𝕒7, 𝕒8}, Ω → {𝕒1, 𝕒2, 𝕒3, 𝕒4, 𝕒5, 𝕒6, 𝕒7, 𝕒8}},
 {𝕒1 → 1/6, 𝕒2 → 1/16, 𝕒3 → 1/16, 𝕒4 → 1/12, 𝕒5 → 3/128, 𝕒6 → 3/16, 𝕒7 → 3/16, 𝕒8 → 29/128}}
```

Causal Independence and The Causal Strength of Conjunctive Factors

In this section, we show that some of our measures 𝕞 *appear* to violate the following "independence synergy property":

(S) **ICS𝕞[𝔼, ℂ1, ℂ2] ⇒ (CS𝕞[𝔼, ℂ1 ∧ ℂ2] > CS𝕞[𝔼, ℂ1] & CS𝕞[𝔼, ℂ1 ∧ ℂ2] > CS𝕞[𝔼, ℂ2])**

But, that the appearance of the failure of (S) for (all but one of) these measures 𝕞 depends on an incorrect way of calculating "**CS𝕞[𝔼, ℂ1 ∧ ℂ2]**". Once this is corrected, we see that — on a proper understanding of "**CS𝕞[𝔼, ℂ1 ∧ ℂ2]**", all but one of our measures 𝕞 *do* satisfy (S). There is but one "recalcitrant" measure — the Galton measure **CS𝕘**.

- **Eells**

CS𝕖 *appears* to violate (S), as the existence of the following model indicates:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSe[𝔼, ℂ1, ℂ2] == CSe[𝔼, ℂ1, ¬ ℂ2],
  CSe[𝔼, ℂ1 ⋀ ℂ2] < CSe[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

$$\left\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},\right.$$
$$\left\{a_1 \to \frac{10\,736\,022\,587}{122\,536\,789\,540}, a_2 \to \frac{36\,278\,969}{452\,458\,083}, a_3 \to \frac{2}{5},\right.$$
$$\left. a_4 \to \frac{146\,564\,197}{122\,536\,789\,540}, a_5 \to \frac{15}{41}, a_6 \to \frac{1}{101}, a_7 \to \frac{1}{131}, a_8 \to \frac{1}{21}\right\}\}$$

But, on the following proper reformulation of **CSe[𝔼, ℂ1 ∧ ℂ2]**

CSe[𝔼, ℂ1 ∧ ℂ2] = Pr[𝔼 | ℂ1 ∧ ℂ2] - Pr[𝔼 | ¬ℂ1 ∧ ¬ℂ2],

which compares **Pr[𝔼 | ℂ1 ∧ ℂ2]** and **Pr[𝔼 | ¬ℂ1 ∧ ¬ℂ2]** rather than **Pr[𝔼 | ℂ1 ∧ ℂ2]** and **Pr[𝔼 | ¬(ℂ1 ∧ ℂ2)]**, such examples do not exist. So (S) is satisfied by **CSe** — once it is properly understood.

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSe[𝔼, ℂ1, ℂ2] == CSe[𝔼, ℂ1, ¬ ℂ2],

  Pr[𝔼 | ℂ1 ⋀ ℂ2] - Pr[𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2] ≤ CSe[𝔼, ℂ1]
 }
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

This is to be expected, in light of the fact that **CSe** (assuming a proper reformulation of **CSe[𝔼, ℂ1 ∧ ℂ2]**) admits of the following (additive) "decomposition" of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing *E*:

ICSe[𝔼, ℂ1, ℂ2] ⇒ CSe[𝔼, ℂ1 ∧ ℂ2] = Pr[𝔼 | ℂ1 ∧ ℂ2] - Pr[𝔼 | ¬ℂ1 ∧ ¬ℂ2] = CSe[𝔼, ℂ1] + CSe[𝔼, ℂ1]

This can be verified using **PrSAT**, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSe[𝔼, ℂ1, ℂ2] == CSe[𝔼, ℂ1, ¬ ℂ2],

  Pr[𝔼 | ℂ1 ⋀ ℂ2] - Pr[𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2] ≠ CSe[𝔼, ℂ1] + CSe[𝔼, ℂ2]
 }
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Suppes**

CSs does *not* even *appear* to violate (S):

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕤[𝔼, ℂ1, ℂ2] == CS𝕤[𝔼, ℂ1, ¬ ℂ2],
  CS𝕤[𝔼, ℂ1 ⋀ ℂ2] < CS𝕤[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Thus, **CS𝕤** satisfies (S) — even on *naive* application. This is to be expected, in light of the following (*formal*) *additivity* property of **CS𝕤**:

```
ICS𝕤[𝔼, ℂ1, ℂ2] ⇒ CS𝕤[𝔼, ℂ1 ∧ ℂ2] = CS𝕤[𝔼, ℂ1] + CS𝕤[𝔼, ℂ2]
```

which can be verified using **PrSAT**, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕤[𝔼, ℂ1, ℂ2] == CS𝕤[𝔼, ℂ1, ¬ ℂ2],
  CS𝕤[𝔼, ℂ1 ⋀ ℂ2] ≠ CS𝕤[𝔼, ℂ1] + CS𝕤[𝔼, ℂ2]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Since the Suppes measure does not involve conditioning on ~*C*, there is no need to consider reformulations of **CS𝕤[𝔼, ℂ1 ∧ ℂ2]**.

- **Galton**

CS𝕘 *appears* to violate (S), as the existence of the following model indicates:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕘[𝔼, ℂ1, ℂ2] == CS𝕘[𝔼, ℂ1, ¬ ℂ2],
  CS𝕘[𝔼, ℂ1 ⋀ ℂ2] < CS𝕘[𝔼, ℂ1],
  Pr[ℂ1] == 1 / 3
 },
 Probabilities → Regular
]
```

$$\Big\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$$
$$\Big\{a_1 \to \frac{668\,143\,271}{1\,860\,173\,964}, a_2 \to \frac{1096}{10\,989}, a_3 \to \frac{151\,739}{1\,109\,889}, a_4 \to \frac{506\,617}{206\,685\,996}, a_5 \to \frac{1}{999}, a_6 \to \frac{3}{37}, a_7 \to \frac{17}{101}, a_8 \to \frac{5}{33}\Big\}\Big\}$$

Surprisingly, even on a proper reformulation of **CS𝕘[𝔼, ℂ1 ∧ ℂ2]**, which (presumably) would be given by the following:

```
CS𝕘[𝔼, ℂ1 ∧ ℂ2] = 4 (Pr[𝔼 ∧ (ℂ1 ∧ ℂ2)] - Pr[𝔼]Pr[ℂ1 ∧ ℂ2])
```

such examples *still* exist. So (S) seems to be violated by **CS𝕘** — even once **CS𝕘[𝔼, ℂ1 ∧ ℂ2]** is properly reformulated. Here‘s a “recalcitrant“ model:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕘[𝔼, ℂ1, ℂ2] == CS𝕘[𝔼, ℂ1, ¬ ℂ2],
  CS𝕘[𝔼, ℂ1 ⋀ ℂ2] < CS𝕘[𝔼, ℂ1],

  4 (Pr[𝔼 ⋀ (ℂ1 ⋀ ℂ2)] - Pr[𝔼] Pr[ℂ1 ⋀ ℂ2]) < CS𝕘[𝔼, ℂ1],

  Pr[ℂ1] == 1 / 4,
  Pr[ℂ2] == 1 / 4,
  Pr[𝔼] == 1 / 2
 },
 Probabilities → Regular,
 BypassSearch → True
]
```

$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$

$\{a_1 \to \frac{45}{128}, a_2 \to \frac{3}{64}, a_3 \to \frac{3}{32}, a_4 \to \frac{27}{128}, a_5 \to \frac{1}{128}, a_6 \to \frac{9}{64}, a_7 \to \frac{3}{32}, a_8 \to \frac{7}{128}\}\}$

- **Cheng**

CS𝕔 *appears* to violate (S), as the existence of the following model indicates:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕔[𝔼, ℂ1, ℂ2] == CS𝕔[𝔼, ℂ1, ¬ ℂ2] == 1 / 2,
  CS𝕔[𝔼, ℂ1 ⋀ ℂ2] < CS𝕔[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$

$\{a_1 \to \frac{1312788}{33167407}, a_2 \to \frac{2}{19}, a_3 \to \frac{656394}{6234475},$

$a_4 \to \frac{4922955}{1956877013}, a_5 \to \frac{7}{25}, a_6 \to \frac{7}{59}, a_7 \to \frac{35117079}{3229458050}, a_8 \to \frac{25}{74}\}\}$

But, on a proper reformulation of `CS𝕔[𝔼, ℂ1 ∧ ℂ2]`, *i.e.*:

$$\text{CS}\mathbb{c}[\mathbb{E}, \mathbb{C}1 \wedge \mathbb{C}2] = \frac{\Pr[\mathbb{E} \mid \mathbb{C}1 \wedge \mathbb{C}2] - \Pr[\mathbb{E} \mid \neg\mathbb{C}1 \wedge \neg\mathbb{C}2]}{1 - \Pr[\mathbb{E} \mid \neg\mathbb{C}1 \wedge \neg\mathbb{C}2]}$$

which compares `Pr[𝔼 | ℂ1 ∧ ℂ2]` and `Pr[𝔼 | ¬ℂ1 ∧ ¬ℂ2]` rather than `Pr[𝔼 | ℂ1 ∧ ℂ2]` and `Pr[𝔼 | ¬(ℂ1 ∧ ℂ2)]`, such examples do not exist. So (S) really is satisfied by `CS𝕔` — once it is properly understood.

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSc[𝔼, ℂ1, ℂ2] == CSc[𝔼, ℂ1, ¬ ℂ2],
```

$$\frac{\Pr[\mathbb{E} \mid \mathbb{C}1 \wedge \mathbb{C}2] - \Pr[\mathbb{E} \mid \neg \mathbb{C}1 \wedge \neg \mathbb{C}2]}{1 - \Pr[\mathbb{E} \mid \neg \mathbb{C}1 \wedge \neg \mathbb{C}2]} \leq \mathrm{CSc}[\mathbb{E}, \mathbb{C}1]$$

```
 }
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

This is to be expected, in light of the fact that **CSc** (assuming a proper reformulation of **CSc[𝔼, ℂ1 ∧ ℂ2]**) admits of the following (multiplicative) "decomposition" of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing *E*:

$$\mathrm{ICSc}[\mathbb{E}, \mathbb{C}1, \mathbb{C}2] \Rightarrow \mathrm{CSc}[\mathbb{E}, \mathbb{C}1 \wedge \mathbb{C}2] = \frac{\Pr[\mathbb{E}|\mathbb{C}1\wedge\mathbb{C}2] - \Pr[\mathbb{E}|\neg\mathbb{C}1\wedge\neg\mathbb{C}2]}{1 - \Pr[\mathbb{E}|\neg\mathbb{C}1\wedge\neg\mathbb{C}2]} = 1-(1-\mathrm{CSc}[\mathbb{E},\mathbb{C}1])(1-\mathrm{CSc}[\mathbb{E}, \mathbb{C}2])$$

This can be verified using **PrSAT**, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSc[𝔼, ℂ1, ℂ2] == CSc[𝔼, ℂ1, ¬ ℂ2],
```

$$\frac{\Pr[\mathbb{E} \mid \mathbb{C}1 \wedge \mathbb{C}2] - \Pr[\mathbb{E} \mid \neg \mathbb{C}1 \wedge \neg \mathbb{C}2]}{1 - \Pr[\mathbb{E} \mid \neg \mathbb{C}1 \wedge \neg \mathbb{C}2]} \neq 1 - (1 - \mathrm{CSc}[\mathbb{E}, \mathbb{C}1]) (1 - \mathrm{CSc}[\mathbb{E}, \mathbb{C}2])$$

```
 }
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

- **Lewis Ratio (non-rescaled)**

CSlr does *not* even *appear* to violate (S):

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSlr[𝔼, ℂ1, ℂ2] == CSlr[𝔼, ℂ1, ¬ ℂ2],
  CSlr[𝔼, ℂ1 ⋀ ℂ2] ≤ CSlr[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Thus, **CSlr** satisfies (S) — even on *naive* application. This is *despite* the fact that **CSlr[E, C1 ∧ C2]** is not properly formulated (on *naive* application). What's more important here is that **CSlr** satisfies (S), once **CSlr[E, C1 ∧ C2]** *is* properly reformulated, as follows:

$$\text{CSlr}[\mathbb{E}, \mathbb{C}1 \wedge \mathbb{C}2] = \frac{\Pr[\mathbb{E} \mid \mathbb{C}1 \wedge \mathbb{C}2]}{\Pr[\mathbb{E} \mid \neg\mathbb{C}1 \wedge \neg\mathbb{C}2]}$$

This can be verified using **PrSAT**, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSlr[𝔼, ℂ1, ℂ2] == CSlr[𝔼, ℂ1, ¬ ℂ2],
  Pr[𝔼 | ℂ1 ⋀ ℂ2] / Pr[𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2] ≤ CSlr[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

This is to be expected, in light of the fact that **CSlr** (assuming a proper reformulation of **CSlr[E, C1 ∧ C2]**) admits of the following (multiplicative) "decomposition" of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing *E*:

$$\text{CSlr}[\mathbb{E}, \mathbb{C}1, \mathbb{C}2] \Rightarrow \text{CSlr}[\mathbb{E}, \mathbb{C}1 \wedge \mathbb{C}2] = \frac{\Pr[\mathbb{E} \mid \mathbb{C}1 \wedge \mathbb{C}2]}{\Pr[\mathbb{E} \mid \neg\mathbb{C}1 \wedge \neg\mathbb{C}2]} = \text{CSlr}[\mathbb{E}, \mathbb{C}1] * \text{CSlr}[\mathbb{E}, \mathbb{C}2]$$

This can be verified using **PrSAT**, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSlr[𝔼, ℂ1, ℂ2] == CSlr[𝔼, ℂ1, ¬ ℂ2],
  Pr[𝔼 | ℂ1 ⋀ ℂ2] / Pr[𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2] ≠ CSlr[𝔼, ℂ1] * CSlr[𝔼, ℂ2]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

- **Lewis Ratio (second rescaling)**

CSlr2 *appears* to violate (S), as the existence of the following model indicates:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕝𝕣2[𝔼, ℂ1, ℂ2] == CS𝕝𝕣2[𝔼, ℂ1, ¬ ℂ2],
  CS𝕝𝕣2[𝔼, ℂ1 ⋀ ℂ2] < CS𝕝𝕣2[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

$\{\{\mathbb{C}1 \to \{a_2, a_5, a_6, a_8\}, \mathbb{C}2 \to \{a_3, a_5, a_7, a_8\}, \mathbb{E} \to \{a_4, a_6, a_7, a_8\}, \Omega \to \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}\},$

$\{a_1 \to \frac{426\,722\,843}{7\,206\,311\,529}, a_2 \to \frac{632\,350\,171}{52\,668\,350\,928}, a_3 \to \frac{2}{9}, a_4 \to \frac{64}{63\,423}, a_5 \to \frac{1}{999}, a_6 \to \frac{4}{29}, a_7 \to \frac{1}{243}, a_8 \to \frac{9}{16}\}\}$

But, on a proper reformulation of **CS𝕝𝕣2[E, C1 ∧ C2]**, *i.e.*:

$$\text{CS}\mathbb{lr}2[\mathbb{E}, \mathbb{C}1 \wedge \mathbb{C}2] = 1 - \frac{\Pr[\mathbb{E} \mid \neg\mathbb{C}1 \wedge \neg\mathbb{C}2]}{\Pr[\mathbb{E} \mid \mathbb{C}1 \wedge \mathbb{C}2]}$$

which involves **Pr[𝔼 | ℂ1 ∧ ℂ2]** and **Pr[𝔼 | ¬ℂ1 ∧ ¬ℂ2]** rather than **Pr[𝔼 | ℂ1 ∧ ℂ2]** and **Pr[𝔼 | ¬(ℂ1 ∧ ℂ2)]**, such examples do not exist. So (S) really is satisfied by **CS𝕝𝕣2** — once it is properly understood.

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕝𝕣2[𝔼, ℂ1, ℂ2] == CS𝕝𝕣2[𝔼, ℂ1, ¬ ℂ2],
  1 - Pr[𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2] / Pr[𝔼 | ℂ1 ⋀ ℂ2] ≤ CS𝕝𝕣2[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

This is to be expected, in light of the fact that **CS𝕝𝕣2** (assuming a proper reformulation of **CS𝕝𝕣2[𝔼, ℂ1 ∧ ℂ2]**) admits of the following (multiplicative) “decomposition“ of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing *E*:

$$\text{CS}\mathbb{lr}2[\mathbb{E}, \mathbb{C}1, \mathbb{C}2] \Rightarrow \text{CS}\mathbb{lr}2[\mathbb{E}, \mathbb{C}1 \wedge \mathbb{C}2] = 1 - \frac{\Pr[\mathbb{E} \mid \neg\mathbb{C}1 \wedge \neg\mathbb{C}2]}{\Pr[\mathbb{E} \mid \mathbb{C}1 \wedge \mathbb{C}2]} = 1 - (1 - \text{CS}\mathbb{lr}2[\mathbb{E}, \mathbb{C}1])\,(1 - \text{CS}\mathbb{lr}2[\mathbb{E}, \mathbb{C}2])$$

This can be verified using **PrSAT**, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕝𝕣2[𝔼, ℂ1, ℂ2] == CS𝕝𝕣2[𝔼, ℂ1, ¬ ℂ2],
      Pr[𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2]
  1 - ──────────────────── ≠ 1 - (1 - CS𝕝𝕣2[𝔼, ℂ1]) (1 - CS𝕝𝕣2[𝔼, ℂ2])
        Pr[𝔼 | ℂ1 ⋀ ℂ2]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

- **Good**

`CS𝕚𝕛` does *not* even *appear* to violate (S):

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕚𝕛[𝔼, ℂ1, ℂ2] == CS𝕚𝕛[𝔼, ℂ1, ¬ ℂ2] == 1 / 2,
  CS𝕚𝕛[𝔼, ℂ1 ⋀ ℂ2] < CS𝕚𝕛[𝔼, ℂ1]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

Thus, `CS𝕚𝕛` satisfies (S) — even on *naive* application. This is *despite* the fact that `CS𝕚𝕛[𝔼, ℂ1 ∧ ℂ2]` is not properly formulated (on *naive* application). What‘s more important here is that `CS𝕚𝕛` satisfies (S), once `CS𝕚𝕛[𝔼, ℂ1 ∧ ℂ2]` *is* properly reformulated, as follows:

```
                      Pr[¬𝔼|¬ℂ1⋀¬ℂ2]
CS𝕚𝕛[𝔼, ℂ1 ∧ ℂ2] = ──────────────
                       Pr[¬𝔼|ℂ1⋀ℂ2]
```

This can be verified using `PrSAT`, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CS𝕚𝕛[𝔼, ℂ1, ℂ2] == CS𝕚𝕛[𝔼, ℂ1, ¬ ℂ2] == 1 / 2,
   Pr[¬ 𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2]
  ───────────────────── < CS𝕚𝕛[𝔼, ℂ1]
    Pr[¬ 𝔼 | ℂ1 ⋀ ℂ2]
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

This is to be expected, in light of the fact that **CSij** (assuming a proper reformulation of **CSij[E, C1 ∧ C2]**) admits of the following (multiplicative) "decomposition" of the causal strength of a conjunctive causal factor, in cases where it judges the conjuncts to be independent in causing *E*:

$$\text{CSij}[\mathbb{E}, \mathbb{C}1, \mathbb{C}2] \Rightarrow \text{CSij}[\mathbb{E}, \mathbb{C}1 \wedge \mathbb{C}2] = \frac{\Pr[\neg\mathbb{E}|\neg\mathbb{C}1\bigwedge\neg\mathbb{C}2]}{\Pr[\neg\mathbb{E}|\mathbb{C}1\bigwedge\mathbb{C}2]} = 1-((1-\text{CSij}[\mathbb{E},\mathbb{C}1])(1-\text{CSij}[\mathbb{E},\mathbb{C}2]))$$

This can be verified using **PrSAT**, as follows:

```
PrSAT[
 {
  Pr[ℂ1 ⋀ ℂ2] == Pr[ℂ1] Pr[ℂ2],
  Pr[𝔼 | ℂ2] > Pr[𝔼],
  Pr[𝔼 | ℂ1] > Pr[𝔼],
  CSij[𝔼, ℂ1, ℂ2] == CSij[𝔼, ℂ1, ¬ ℂ2] == 1 / 2,
  Pr[¬ 𝔼 | ¬ ℂ1 ⋀ ¬ ℂ2] / Pr[¬ 𝔼 | ℂ1 ⋀ ℂ2] ≠ 1 - (1 - CSij[𝔼, ℂ1]) (1 - CSij[𝔼, ℂ2])
 },
 Probabilities → Regular
]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Boolean Representations of Cheng, Eells, Suppes, and Lewis Ratio (second rescaling)

Cheng

Assume that (1) 𝔸, ℚ, and ℂ are pairwise independent and jointly independent, and (2) 𝔼 = 𝔸 ∨ (ℚ ∧ ℂ). Then, we have the following Boolean representation of **CSc**:

```
CSc[𝔼, ℂ] = Pr[ℚ]
```

Here is a verification:

```
ASSc = {Pr[𝔸 ⋀ ℚ] == Pr[𝔸] Pr[ℚ], Pr[𝔸 ∧ ℂ] == Pr[𝔸] Pr[ℂ],
   Pr[ℚ ⋀ ℂ] == Pr[ℚ] Pr[ℂ], Pr[𝔸 ⋀ (ℚ ⋀ ℂ)] == Pr[𝔸] Pr[ℚ ⋀ ℂ]};
𝔼 =
  𝔸 ⋁
   (ℚ ⋀ ℂ);

PrSAT[ASSc ⋃ {CSc[𝔼, ℂ] ≠ Pr[ℚ]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

{}

Eells

Assume that (1) 𝔸 and ℚ are mutually exclusive, (2) 𝔸 and ℂ are independent, (3) ℚ and ℂ are independent, and (4) 𝔼 = 𝔸 ∨ (ℚ ∧ ℂ). Then, we have the following Boolean representation of **CSe**:

```
CSe[𝔼, ℂ] = Pr[ℚ]
```

Here is a verification:

```
ASSe =  {Pr[𝔸 ⋀ ℚ] == 0, Pr[𝔸 ∧ ℂ] == Pr[𝔸] Pr[ℂ], Pr[ℚ ⋀ ℂ] == Pr[ℚ] Pr[ℂ]};
```

```
PrSAT[ASSe ⋃ {CSe[𝔼, ℂ] ≠ Pr[ℚ]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

- **Suppes**

The same assumptions used for the Eells measure above yield the following Boolean representation of **CSs**:

```
CSs[𝔼, ℂ] = Pr[ℚ ∧ ¬ℂ]
```

Here is a verification:

```
PrSAT[ASSe ⋃ {CSs[𝔼, ℂ] ≠ Pr[ℚ ⋀ ¬ ℂ]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```

- **Lewis Ratio (second rescaling)**

The same assumptions used for the Eells and Suppes measures above yield the following Boolean representation of **CSlr2**:

```
CSlr2[𝔼, ℂ] = Pr[ℚ | ℂ ∧ 𝔼]
```

Here is a verification:

```
PrSAT[ASSe ⋃ {CSlr2[𝔼, ℂ] ≠ Pr[ℚ | ℂ ∧ 𝔼]}]
```

PrSAT::srchfail : Search phase failed; attempting FindInstance

```
{}
```