

The Epistemology of Logic

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1 Lewis Carroll (1895) on Logic

Zeno's Racetrack Paradox

Carroll's Paradox

What's the problem?

What's the solution?

2 A Little Bit of History

- Plato, *The Meno*

The Problem of A Priori Knowledge

- Early Modern Empiricism

Rationalism versus Empiricism: knowledge, concepts

Hume: Relations of Ideas versus Matters of Fact

Kant: The Analytic/Synthetic Distinction

- Conventionalism (Carnap, Ayer, ...)

(Inspired in part by the early Wittgenstein)

The analytic theory of the a priori + the conventionalist theory of the analytic

"Truth by (linguistic) convention"

Semantics, metaphysics, and epistemology

- W. V. O. Quine (1936; 1951; 1960)

Not explicit conventions: A Carrollian regress, Conventions use logical vocabulary

Not implicit conventions: What are those?

Quine's positive view: It's empirical knowledge

The "web of belief" model

Later: "Anti-exceptionalism about logic"

- Gerhard Gentzen (1934)

Natural deduction – introduction and elimination rules

Inferentialism – the meaning of a term determined by rules of use (e.g., introduction rules)

- A. N. Prior (1960)

The tonk problem

Replies to Prior (Stevenson: truth-functions, Belnap: harmony)

3 Knowledge of Logic – Some Issues

A few different things:

Deductive rule of inference: (MP) $p, \text{ if } p \text{ then } q / q$

Instance of rule: It's raining, If it's raining then the sidewalks are wet / the sidewalks are wet

Validity claim: MP is a logically valid rule of inference

Logical truth: If both it's raining and if it's raining then the sidewalks are wet, then the sidewalks are wet

A few different questions:

What explains our knowledge of the validity claim?

What explains our knowledge of the logical truth?

What explains the epistemic status of the rule?

What explains the epistemic status of a particular instance of the rule?

Skeptical worries versus explanatory questions.

Two puzzles:

Truth/Reliability. How do we get it (by and large) right about logic?

Justification. How is it that our belief or acceptance of a rule is justified?

4 The Justificatory Challenge

(This section draws on Paul Boghossian's paper "Knowledge of Logic".)

Which comes first:

- (i) Having justification for believing the validity claim, or
- (ii) Having justification for using the rule?

Analogy: Which comes first:

- (i) Having justification for believing that one's perception is reliable, or
- (ii) Having justification to rely on one's perception?

One motivation for thinking that the belief comes first: measuring instruments and new rules

One motivation for thinking that the rule comes first: young children and non-human animals

View 1: Having justification for believing the claim comes first.

Then we are derivatively justified in using the rule. How does that work?

Idea: In general, thinkers are justified in using a rule in their reasoning when (and by virtue of the fact that) they are justified in believing the rule is a good one.

Presumably, they don't need to actually have a justified belief. (Children and animals)

Better idea: Thinkers are justified in using a rule in their reasoning when (and by virtue of the fact that) they have justification for believing the rule is a good one.

Big question: What justifies us in believing the validity claim?
We'll come back to this!

View 2: Justification for using the rule comes first
Then we are derivatively justified in believing the claim. How?

Idea: We infer to the belief, using MP (or a related rule).

Example 1:

If MP preserves truth on every row of the truth table, then MP is valid.
MP preserves truth on every row of the truth table.
So, MP is valid.

Example 2:

Suppose both p is true and if p then q is true.
So p is true.
So if p then q is true.
So p.
So if p then q.
So q.
So q is true.
So if both p is true and if p then q is true then q is true.
That is, MP preserves truth.
We've just proved that! So MP is valid.

Potential Worries:

1. Isn't this circular? We're relying on something close to what we're trying to prove.
Compare: The inductive justification of induction. (Induction has worked in the past, so ...)
2. Couldn't we "justify" everything this way, even things that aren't really justified?

Example: We can "justify" the belief that the tonk rules are valid in a corresponding way.

Suppose A is true.

So A.
So A tonk B.
So A tonk B is true.

So if A is true then A tonk B is true.

So tonk-introduction preserves truth.

We've just proved that! So tonk-introduction is valid.

[*View 3: They are justified simultaneously*]

Responding to the worries for View 2. (This is due to Boghossian)

- Problem: It's circular!

Response: Distinguish premise circularity and rule circularity.

Response: Not relying on what we're trying to prove. Crucial difference.

- Problem: We can justify anything!

Response: We can justify anything if we are allowed to use any rules we want. But we're not.

We're only justified in using certain rules (without a backing belief).

Big Question: What justifies us in employing MP?

5 Accounts of Justification

On either View 1 or View 2, we need an account of justification.

Here are a few possible accounts: (I've stated the View 2 versions.)

A thinker is justified in employing a rule of inference as basic in thought if (and by virtue of the fact that):

Reliabilism. ... the rule is reliable in the sense that it tends to yield truths from truths.

Starting Point. ... the rule is one the thinker started out with.

Psychological Unavoidability. ... reasoning with the rule is psychologically unavoidable for the thinker.

Phenomenology. ... applications of the rule are accompanied by the appropriate phenomenology (e.g., feelings of obviousness).

Acquaintance. ... the thinker somehow is "in touch with" the validity of the rule.

Evolution. ... the thinker is a descendant of a population of creatures that were selected for using the rule.

Concepts. ... the rule is built into one of the thinker's concepts (or words).

Innocence. ... using the rule is "epistemically innocent" (e.g., is a conservative extension).

Importance. ... using the rule is "epistemically important" (e.g., is indispensable to an important cognitive end).

These all have pros and cons!

6 The Reliability Challenge

(This section draws from my paper “The Reliability Challenge and the Epistemology of Logic”)

How is it that we (by and large) get it right about logic?

Again, we can raise the question about both our logical beliefs and our deductive rules.

Natural view: We are reliable in our logical beliefs because we are reliable in our deductive reasoning. Our reliability about logic is a byproduct of our more basic deductive competence.

Question: What explains the reliability of our cognitive mechanism for deductive inference?

Two questions:

- The Operational Question: How does our cognitive mechanism for deductive inference work such that it is reliable?
- The Etiological Question: How is it that we have a cognitive mechanism for deductive inference that is reliable?

The latter question is the more difficult one!

Rational Insight. We have some faculty broadly analogous to perception by which we “see” that certain deductive rules are valid, or that certain logical claims ought to be accepted, or something similar.

Problems:

- There is no independent evidence that we have such a faculty.
- It is very mysterious how such a faculty could work.
- Appeal to such a faculty does not help to answer the etiological question, since explanation is then needed of how we have a reliability faculty of rational insight.

Concepts: There are rules of inference “built into” our logical concepts (or words). The semantic value of a concept is assigned to guarantee that its constitutive rules necessarily preserve truth.

Problem:

- Not every conceptual role (that is, package of rules) corresponds to a genuine concept with reliable constitutive rules. At best, the semantic thesis is true when restricted to “good” conceptual roles. An explanation is needed of how we came to have good conceptual roles.

Evolution: Our ancestors were selected for employing reliable deductive rules – a heritable trait – and this explains how we, their descendants, came to employ reliable deductive rules.

Problems

- An evolutionary account can only explain why a particular trait predominates in a population. It cannot explain why particular individuals have the traits they do.
- Even if an evolutionary account can explain why we employ useful rules, such an account cannot explain why we employ reliable rules.
- Since engaging in deductive reasoning does not yield new information about the world, there could not have been any evolutionary advantage in doing so.
- Even if an evolutionary account can explain how we employ rules that are actually truth-preserving concerning a narrow range of simple propositions, such an account cannot explain the full extent of our reliability.

Some Relevant Sources

Plato (380 BCE), *The Meno*

Lewis Carroll (1895) "What the Tortoise Said to Achilles"

W. V. O. Quine (1936) "Truth by Convention"

W. V. O. Quine (1951) "Two Dogmas of Empiricism"

W. V. O. Quine (1960) "Carnap and Logical Truth"

Arthur N. Prior (1960) "The Runabout Inference-Ticket"

Michael Dummett (1973) "The Justification of Deduction" (this one is hard!)

Susan Haack (1976) "The Justification of Deduction"

Paul Boghossian (2000) "Knowledge of Logic"

Paul Boghossian (2003) "Blind Reasoning"

Crispin Wright (2004) "Intuition, Entitlement and the Epistemology of Logical Laws"

Timothy Williamson (2007) *The Philosophy of Philosophy* (especially chapter 4)

Joshua Schechter (2010) "The Reliability Challenge and the Epistemology of Logic"

Ole Hjortland (2017) "Anti-Exceptionalism about Logic"

XII.—NOTES.

WHAT THE TORTOISE SAID TO ACHILLES.

BY LEWIS CARROLL.

ACHILLES had overtaken the Tortoise, and had seated himself comfortably on its back.

"So you've got to the end of our race-course?" said the Tortoise. "Even though it *does* consist of an infinite series of distances? I thought some wiseacre or other had proved that the thing couldn't be done?"

"It *can* be done," said Achilles. "It *has* been done! *Solvitur ambulando*. You see the distances were constantly *diminishing*; and so—"

"But if they had been constantly *increasing*?" the Tortoise interrupted. "How then?"

"Then I shouldn't be *here*," Achilles modestly replied; "and *you* would have got several times round the world, by this time!"

"You flatter me—*flatten*, I mean," said the Tortoise; "for you *are* a heavy weight, and *no* mistake! Well now, would you like to hear of a race-course, that most people fancy they can get to the end of in two or three steps, while it *really* consists of an infinite number of distances, each one longer than the previous one?"

"Very much indeed!" said the Grecian warrior, as he drew from his helmet (few Grecian warriors possessed *pockets* in those days) an enormous note-book and a pencil. "Proceed! And speak *slowly*, please! *Short-hand* isn't invented yet!"

"That beautiful First Proposition of Euclid!" the Tortoise murmured dreamily. "You admire Euclid?"

"Passionately! So far, at least, as one *can* admire a treatise that won't be published for some centuries to come!"

"Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:"

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?"

"Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*."

"And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?"

"No doubt such a reader might exist. He might say 'I accept as true the Hypothetical Proposition that, *if A and B be true, Z must be true*; but, I *don't* accept *A and B as true.*' Such a reader would do wisely in abandoning Euclid, and taking to football."

"And might there not *also* be some reader who would say 'I accept *A and B as true*, but I *don't* accept the Hypothetical'?"

"Certainly there might. *He*, also, had better take to football."

"And *neither* of these readers," the Tortoise continued, "is *as yet* under any logical necessity to accept *Z as true*?"

"Quite so," Achilles assented.

"Well, now, I want you to consider *me* as a reader of the *second* kind, and to force me, logically, to accept *Z as true.*"

"A tortoise playing football would be—" Achilles was beginning

"—an anomaly, of course," the Tortoise hastily interrupted. "Don't wander from the point. Let's have *Z* first, and football afterwards!"

"I'm to force you to accept *Z*, am I?" Achilles said musingly. "And your present position is that you accept *A and B*, but you *don't* accept the Hypothetical—"

"Let's call it *C*," said the Tortoise.

"—but you *don't* accept

(*C*) If *A and B* are true, *Z must be true.*"

"That is my present position," said the Tortoise.

"Then I must ask you to accept *C*."

"I'll do so," said the Tortoise, "as soon as you've entered it in that note-book of yours. What else have you got in it?"

"Only a few memoranda," said Achilles, nervously fluttering the leaves: "a few memoranda of—of the battles in which I have distinguished myself!"

"Plenty of blank leaves, I see!" the Tortoise cheerily remarked. "We shall need them *all*!" (Achilles shuddered.) "Now write as I dictate:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*C*) If *A and B* are true, *Z must be true.*

(*Z*) The two sides of this Triangle are equal to each other."

"You should call it *D*, not *Z*," said Achilles. "It comes *next* to the other three. If you accept *A and B* and *C*, you *must* accept *Z*."

"And why *must* I?"

"Because it follows *logically* from them. If *A and B* and *C* are true, *Z must be true*. You don't dispute *that*, I imagine?"

"If *A and B* and *C* are true, *Z must be true*," the Tortoise thoughtfully repeated. "That's *another* Hypothetical, isn't it? And, if I failed to see its truth, I might accept *A and B* and *C*, and *still* not accept *Z*, mightn't I?"

"You might," the candid hero admitted; "though such obtuseness would certainly be phenomenal. Still, the event is *possible*. So I must ask you to grant *one* more Hypothetical."

"Very good. I'm quite willing to grant it, as soon as you've written it down. We will call it

(*D*) If *A and B* and *C* are true, *Z must be true.*

Have you entered that in your note-book?"

"I *have*!" Achilles joyfully exclaimed, as he ran the pencil into its sheath. "And at last we've got to the end of this ideal race-course! Now that you accept *A and B* and *C* and *D*, of course you accept *Z*."

"Do I?" said the Tortoise innocently. "Let's make that quite clear. I accept *A and B* and *C* and *D*. Suppose I *still* refused to accept *Z*?"

"Then Logic would take you by the throat, and *force* you to do it!" Achilles triumphantly replied. "Logic would tell you 'You ca'n't help yourself. Now that you've accepted *A* and *B* and *C* and *D*, you *must* accept *Z*!' So you've no choice, you see."

"Whatever *Logic* is good enough to tell me is worth *writing down*," said the Tortoise. "So enter it in your book, please. We will call it

(*E*) If *A* and *B* and *C* and *D* are true, *Z* must be true. Until I've granted *that*, of course I needn't grant *Z*. So it's quite a *necessary* step, you see?"

"I see," said Achilles; and there was a touch of sadness in his tone.

Here the narrator, having pressing business at the Bank, was obliged to leave the happy pair, and did not again pass the spot until some months afterwards. When he did so, Achilles was still seated on the back of the much-enduring Tortoise, and was writing in his note-book, which appeared to be nearly full. The Tortoise was saying "Have you got that last step written down? Unless I've lost count, that makes a thousand and one. There are several millions more to come. And *would* you mind, as a personal favour, considering what a lot of instruction this colloquy of ours will provide for the Logicians of the Nineteenth Century—*would* you mind adopting a pun that my cousin the Mock-Turtle will then make, and allowing yourself to be re-named *Taught-Us*?"

"As you please!" replied the weary warrior, in the hollow tones of despair, as he buried his face in his hands. "Provided that *you*, for *your* part, will adopt a pun the Mock-Turtle never made, and allow yourself to be re-named *A Kill-Ease*!"

THE RUNABOUT INFERENCE-TICKET

By A. N. PRIOR

IT is sometimes alleged that there are inferences whose validity arises solely from the meanings of certain expressions occurring in them. The precise technicalities employed are not important, but let us say that such inferences, if any such there be, are analytically valid.

One sort of inference which is sometimes said to be in this sense analytically valid is the passage from a conjunction to either of its conjuncts, e.g., the inference 'Grass is green and the sky is blue, therefore grass is green'. The validity of this inference is said to arise solely from the meaning of the word 'and'. For if we are asked what is the meaning of the word 'and', at least in the purely conjunctive sense (as opposed to, e.g., its colloquial use to mean 'and then'), the answer is said to be *completely* given by saying that (i) from any pair of statements P and Q we can infer the statement formed by joining P to Q by 'and' (which statement we hereafter describe as 'the statement P-and-Q'), that (ii) from any conjunctive statement P-and-Q we can infer P, and (iii) from P-and-Q we can always infer Q. Anyone who has learnt to perform these inferences knows the meaning of 'and', for there is simply nothing more *to* knowing the meaning of 'and' than being able to perform these inferences.

A doubt might be raised as to whether it is really the case that, for any pair of statements P and Q, there is always a statement R such that given P and given Q we can infer R, and given R we can infer P and can also infer Q. But on the view we are considering such a doubt is quite misplaced, once we have introduced a word, say the word 'and', precisely in order to form a statement R with these properties from any pair of statements P and Q. The doubt reflects the old superstitious view that an expression must have some independently determined meaning before we can discover whether inferences involving it are valid or invalid. With analytically valid inferences this just isn't so.

I hope the conception of an analytically valid inference is now at least as clear to my readers as it is to myself. If not, further illumination is obtainable from Professor Popper's paper on 'Logic without Assumptions' in *Proceedings of the Aristotelian Society* for 1946-7, and from Professor Kneale's contribution to *Contemporary British Philosophy*, Volume III. I have also been much helped in my understanding of the notion by some lectures of Mr. Strawson's and some notes of Mr. Hare's.

I want now to draw attention to a point not generally noticed, namely that in this sense of 'analytically valid' any statement whatever may be

inferred, in an analytically valid way, from any other. '2 and 2 are 5', for instance, from '2 and 2 are 4'. It is done in two steps, thus:

2 and 2 are 4.

Therefore, 2 and 2 are 4 tonk 2 and 2 are 5.

Therefore, 2 and 2 are 5.

There may well be readers who have not previously encountered this conjunction 'tonk', it being a comparatively recent addition to the language; but it is the simplest matter in the world to explain what it means. Its meaning is completely given by the rules that (i) from any statement P we can infer any statement formed by joining P to any statement Q by 'tonk' (which compound statement we hereafter describe as 'the statement P-tonk-Q'), and that (ii) from any 'contonk-tive' statement P-tonk-Q we can infer the contained statement Q.

A doubt might be raised as to whether it is really the case that, for any pair of statements P and Q, there is always a statement R such that given P we can infer R, and given R we can infer Q. But this doubt is of course quite misplaced, now that we have introduced the word 'tonk' precisely in order to form a statement R with these properties from any pair of statements P and Q.

As a matter of simple history, there have been logicians of some eminence who have seriously doubted whether sentences of the form 'P and Q' express single propositions (and so, e.g., have negations). Aristotle himself, in *De Soph. Elench.* 176 a 1 ff., denies that 'Are Callias and Themistocles musical?' is a single question; and J. S. Mill says of 'Caesar is dead and Brutus is alive' that 'we might as well call a street a complex house, as these two propositions a complex proposition' (*System of Logic* I, iv. 3). So it is not to be wondered at if the form 'P tonk Q' is greeted at first with similar scepticism. But more enlightened views will surely prevail at last, especially when men consider the extreme *convenience* of the new form, which promises to banish *falsche Spitzfindigkeit* from Logic for ever.

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