

Barriers to Entailment

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The Plan

Part 1:

- Introduction to Hume's Law
- Three Counterexamples
- More barriers to entailment
- Group work: can the three counterexamples be converted to work against Hume's Law's friends?

The Plan

Part 2:

- Results from the Group work
- Two ways to formulate the particular/universal barrier
- Introduction to Tense Logic
- Formulating the past/future barrier
- Group work: Prior's new counterexample

The Plan

Part 3:

- Results from the Group Work
- Introduction to Deontic Logic
- Formulating the is/ought barrier
- What about the counterexamples?
- The End!

Hume's Law

"In every system of morality, which I have hitherto met with, I have always remark'd, that the author proceeds for some time in the ordinary way of reasoning, and establishes the being of a God, or makes observations concerning human affairs; when of a sudden I am surpriz'd to find, that instead of the usual copulations of propositions, *is*, and *is not*, I meet with no proposition that is not connected with an *ought*, or an *ought not*. This change is imperceptible; but is, however, of the last consequence. For as this *ought*, or *ought not*, expresses some new relation or affirmation, 'tis necessary that it shou'd be observ'd and explain'd; and at the same time that a reason should be given, for what seems altogether inconceivable, how this new relation can be a deduction from others, which are entirely different from it." (Hume, 1739, 3.1.1)

Popper called this "perhaps the simplest and the most important point about ethics."

Hudson's 1969 anthology *The Is/Ought Question* is subtitled *A Collection of Papers on **the Central Problem in Moral Philosophy***.

Pigden suggests that the is/ought passage "is perhaps the most influential thing Hume ever wrote." (Pigden, 1989)

But Hume's Law is also controversial.

It has been endorsed or defended in some form by Hume (1739), Popper (1948), Prior (1949), Hare (1952), Nowell-Smith (1954), Anscombe (1958a), Shorter (1961), Thomson and Thomson (1964), Flew (1964), Jaggar (1974), von Kutschera (1977), Karmo (1988), Pigden (1989), Schurz (1991, 1997), Guevera (2008), Singer (2015), Brown (2015), and Fine (2021),

but rejected in some form by Rynin (1957), Foot (1958), Anscombe (1958b), Prior (1960), Black (1964), Searle (1964), Mavrodes (1964), Kurtzman (1970), Geach (1976), MacIntyre (1981), Nelson (1995, 2007), Maitzen (1998, 2010) and Brown (2015).

"It has often been said—in fact, I have said it quite emphatically myself—that it is impossible to deduce ethical conclusions from non-ethical premisses. This now seems to me a mistake." (Prior, 1960, 199)

Counterexample 1

Tea-drinking is common in England.

Tea-drinking is common in England or all New Zealanders ought to be shot.

Counterexample 1

Tea-drinking is common in England or all New Zealanders ought to be shot.

Tea-drinking is not-common in England.

All New Zealanders ought to be shot.

Counterexample 1

D	$D \vee N$
	$\neg D$
$D \vee N$	N

Counterexample 2

"Many ethical philosophers appear to accept the view that 'ought' implies 'can'. This view, which seems quite plausible, can perhaps be formulated more precisely as (1) Statements of the form 'N ought to do X' entail the corresponding statements of the form 'N can (is able to) do X.' But (1) is equivalent to (2) Statements of the form 'N cannot (is unable to) do X' entail the corresponding statements of the form 'It is not the case that N ought to do X'. And (2) appears to say that there is a non-normative statement which entails a normative one." (Mavrodes, 1964, 42)

Counterexample 2

The key idea:

Ought implies (entails) can

Contraposing preserves entailment:

if $\phi \models \psi$, then $\neg\psi \models \neg\phi$

So *cannot* entails not *ought*.

$$\mathcal{O}p \models \Diamond p$$

$$\neg\Diamond p \models \neg\mathcal{O}p$$

Counterexample 3

Bitá said, "one ought to help others."

What Bitá said is true.

One ought to help others.

Our 3 Counterexamples:

- 1 Prior's Dilemma: $D \models D \vee N$ $D \vee N, \neg D \models N$
- 2 Mavrodes' Argument from Ought implies Can: $\neg\Diamond p \models \neg\mathcal{O}p$
- 3 Quotation

Other barriers

The particular/universal barrier: No set containing only particular premises entails a universal conclusion

You can never arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premises. (Russell, 1918, 101)

The Past/Future Barrier

The past/future barrier: No set containing only premises about the past (or present) entails a conclusion about the future.

All inferences from experience suppose, as their foundation, that the future will resemble the past...if there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion. It is impossible, therefore, that any argument from experience can prove this resemblance of the past to the future; since all these arguments are founded on the supposition of that resemblance. (Hume, 1748, 4.21/37)

More Barriers in Philosophy

The *Is/Must* Barrier:

No set containing only premises about which say how things are **in the actual world** entails a conclusion about how things are **in all worlds**.

The Indexical Barrier:

No set containing only **non-indexical** premises entails a conclusion which is **indexical**.

Five Barriers

- you can't get **universal** conclusions from **particular** premises.
- you can't get conclusions about the **future** from premises about the **past or present**.
- you can't get conclusions about how things **must** be from premises about how they **are**.
- you can't get **indexical** conclusions from **non-indexical** premises.
- you can't get **normative** conclusions from **descriptive** premises. (Hume's Law)

Group Work Question:

Q. Which of the three counterexamples against the *is/ought* barrier can you reformulate to work against the other barriers?

- 1 Prior's Dilemma: $D \models D \vee N$ $D \vee N, \neg D \models N$
- 2 Mavrodes' Argument from Ought implies Can: $\neg \Diamond p \models \neg Op$
- 3 Quotation:

Bitá said, "one ought to help others."
 What Bitá said is true.

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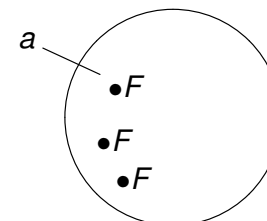
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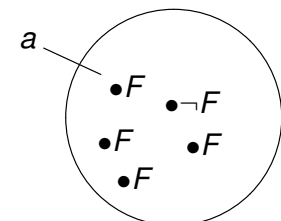
 One ought to help others.

Formulating the Universal Barrier

Particular sentences:



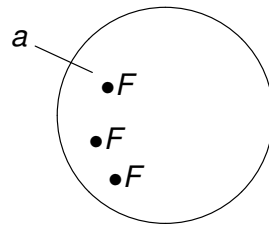
Fa is true



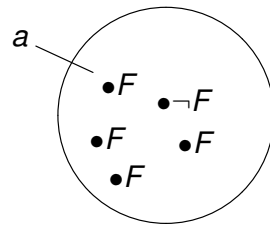
Fa is still true

Formulating the Universal Barrier

Universal sentences:



$\forall x Fx$ is true



$\forall x Fx$ is no longer true

There are (at least) two ways to use this idea in definitions of **Universal**.

Definition (Extension-**breakability**)

*Sentences are extension-breakable if it is **at least sometimes** possible to make them false by extending the model.*

Sentences that are extension-breakable: $\forall x Fx$, $\neg \exists x Fx$, $Fa \vee \forall x Gx$, $\neg Fa \rightarrow \forall x Gx$, $\forall x (Fx \rightarrow Gx)$, $\forall x (Fx \rightarrow \neg Gx)$.

Sentences that are not extension-breakable: Fa , $\neg Fa$, $Fa \vee \neg Fa$, $Fa \wedge Fa$, $\forall x (Fx \vee \neg Fx)$.

Definition (Extension-fragility)

*Sentences are extension-fragile if it is **always** possible to make them false by extending the model.*

Sentences that are extension-fragile: $\forall x Fx$, $\neg \exists x Fx$.

Some sentence are not extension-sensitive at all, i.e. extension never changes their truth-value.

Examples: Fa , $Fa \wedge Gb$, $\neg Fa$, $Fa \vee \neg Fa$, $Fa \wedge \neg Fa$, $\forall x (Fx \vee \neg Fx)$.

If we use extension-fragility as the definition of Universal sentence, and extension-insensitivity as the definition of Particular sentence, then **some sentences turn out to be neither Universal nor Particular**.

$$Fa \vee \forall xGx$$

$$\neg Fa \rightarrow \forall xGx$$

One other difference:

- Extension-fragility makes it very easy to prove a barrier.
- Extension-breakability does not.

Theorem (Universal Barrier)

If Γ is satisfiable and not extension-sensitive, and ϕ is extension-fragile, then $\Gamma \not\models \phi$.

Proof.

Suppose Γ is true in a model M . Either M makes ϕ true or it does not. If it does not, then M is a counterexample and so $\Gamma \not\models \phi$. Suppose instead M makes ϕ true. Then just extend M until ϕ goes false (**this will be possible since ϕ is extension-fragile**). The new model still makes Γ true since Γ is extension-insensitive. So again we have a counterexample, and $\Gamma \not\models \phi$. $\square$

But if we were to use extension-breakability, instead of extension-fragility, the proof won't go through. In fact, there are counterexamples! $Fa \models Fa \vee \forall xGx$.

- So if we characterise universality as extension-fragility, we get a weird taxonomy, but an easy and simple barrier thesis.
- If we characterise it as extension-breakability, we get a simple taxonomy, but the barrier thesis has some exceptions. (We can still prove a limited barrier thesis it turns out, but it has an "unless" clause on the end.)

There's more to be said about which is the best way to go, but for now I just want to leave you with the dilemma and go on to think about the past/future barrier...

The Past/Future Barrier

No set of premises just about the past (or present) entails a conclusion about the future.

Introduction to Tense Logic

The expressions of our language—which we'll call TL (for tense logic)—are as follows:

sentence letters:	p, q, r etc.
connectives:	$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
tense operators:	F, G, P, H
punctuation:	$(,)$

Sentences

Definition (Sentence (of TL))

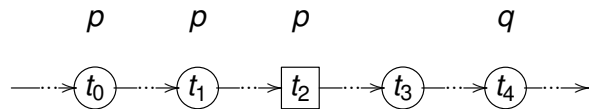
- 1 *Sentence letters are sentences.*
- 2 *If ϕ is a sentence, then $\neg\phi$ is a sentence.*
- 3 *If ϕ and ψ are sentences then $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$, $(\phi \leftrightarrow \psi)$ are sentences.*
- 4 *If ϕ is a sentence then $F\phi$, $G\phi$, $P\phi$, and $H\phi$ are sentences.*
- 5 *Nothing else is a sentence.*

Models, Truth, and Consequence

Definition (TL model)

A TL model is an ordered quadruple: $(T, <, n, I)$. T such that:

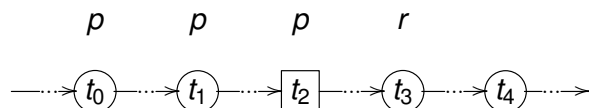
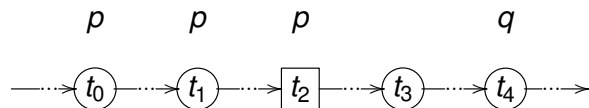
- 1 *T is a non-empty set of points (the set of times),*
- 2 *$<$ a binary relation on those points (the earlier than relation)*
- 3 *n an element of T (the model's 'now')*
- 4 *I an interpretation function mapping pairs of sentence letters and times into the set of values $\{1, 0\}$ (1 represents true, and 0 represents false.) Essentially $I(q, t_3) = 1$ means that the sentence letter q is true at time t_3 .*



Future-Switching

- An idea: a sentence is really about the future if you can change its truth-value by changing the future.
- A simple way to implement this: one model is a **future-switch** of another just in case the models are identical except that times than n , and any sentence letter ϕ , the values of $I(\phi, t)$ may differ.

Future-switching



Future Sentences

Again we could define sentences as being about the future if they are a) fragile with respect to future-switching, or b) breakable with respect to future-switching.

Definition (Future-switch breakable)

At least one model of the sentence has a future-switch that makes it false.

Definition (Future-switch fragile)

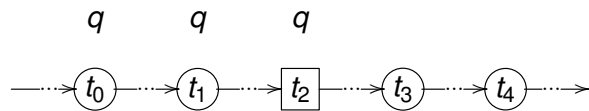
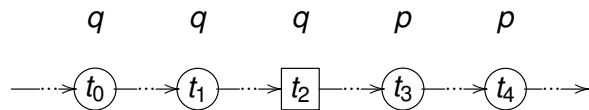
Every model of the sentence has a future-switch that makes it false.

Definition (Past)

Future-switching doesn't change the truth-value.

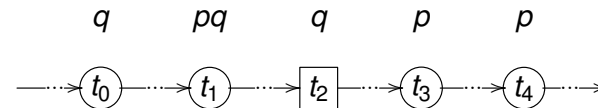
Examples

Fp and Gp are both Future-switch breakable AND future-switch fragile:



Examples

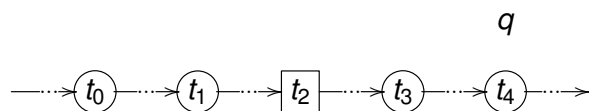
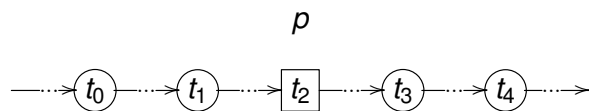
p, q, Pp, Hq are all Past (not sensitive to future-switching at all.)



Examples

$p \vee Fq$ is future-switch breakable but NOT future-switch fragile.

Some (but not all) models of the sentence can be future-switched to make them false. Compare:



Taxonomy Summary

Future-switch Fragile: Fp, Gp

Future-switch breakable: $p \vee Fq, \neg p \rightarrow Gq$

Past: p, q, Pp, Hp .

Fragility-theorem

Theorem (Future Barrier)

If Γ is satisfiable and future-switch-insensitive, and ϕ is future-switch-fragile, then $\Gamma \not\models \phi$

Proof.

Suppose Γ is true in a model M . Either M makes ϕ true or it does not. If it does not, then M is a counterexample and $\Gamma \not\models \phi$. So suppose M makes ϕ true. Then just future-switch M until ϕ goes false (this will be possible since ϕ is future-switch-fragile). The new model still makes Γ true since it is future-switch-preserved. So again we have a counterexample, and $\Gamma \not\models \phi$. $\square$

Prior's Other Counterexample

"J.F. Bennett recently described Leibniz as having discovered, and Hume as having re-discovered, the principle that 'if Q is an immediate consequence of P then there cannot be a time-reference in Q later than the latest time-reference in P '. One thing that the development of tense-logic makes quite clear—if it was not clear before—is that this alleged 'discovery' is in fact a falsehood." (Prior, 1967, 57))

$$\frac{Pp}{FPp}$$

At some time in the past p .

At some time in the future, at some time in the past p .

Group Work (end of part 2)

Q. Where does Prior's conclusion FPp fit in our taxonomy? Is it Future-switch breakable, fragile, or insensitive? (Draw models to support your answer.)

Definition (Future-switch breakable)

At least one model of the sentence has a future-switch that makes it false.

Definition (Future-switch fragile)

Every model of the sentence has a future-switch that makes it false.

Definition (Past)

Future-switching doesn't change the truth-value.

The Plan

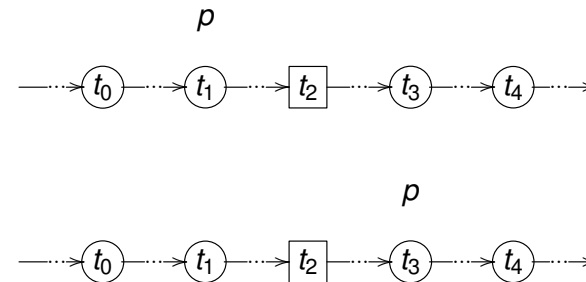
Part 3:

- Results from the Group Work
- Introduction to Deontic Logic
- Formulating the is/ought barrier
- Solution to the three counterexamples
- The End!

Results from the Group Work:

Q. Where does Prior's conclusion FPp fit in our taxonomy? Is it Future-switch breakable, fragile, or insensitive? (Draw models to support your answer.)

FPp is future-switch breakable, but not future-switch fragile.
Consider:



The *Is/Ought* Barrier

To apply all this to Hume's Law we will need:

- 1 A suitable logic
- 2 A taxonomy (normative vs descriptive sentences.)

Deontic Modal Logic

sentence letters:	$p, q, r, \dots$ etc.
connectives:	$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
modal operators:	$\Box, \Diamond$
deontic operators:	$\mathcal{O}, \mathcal{P}$
punctuation:	$(,)$

DML Models

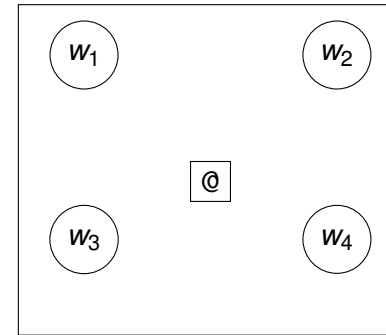
A DML model is an ordered quadruple

$$(W, S, @, I)$$

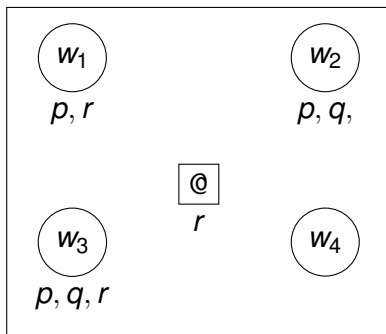
where:

- 1 W is a non-empty set of points (the set of *possible worlds*),
- 2 S is a non-empty subset of W (the set of *superb worlds*)
- 3 $@$ is an element of W (the ‘actual world’)
- 4 I is an interpretation function mapping pairs of sentence letters and elements of W into the set of values $\{1, 0\}$.

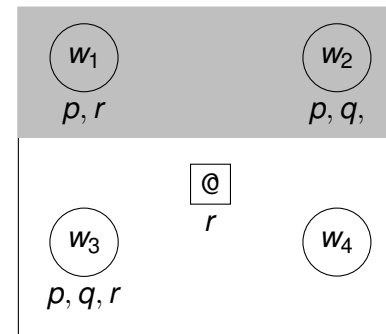
A DML Model



A DML Model



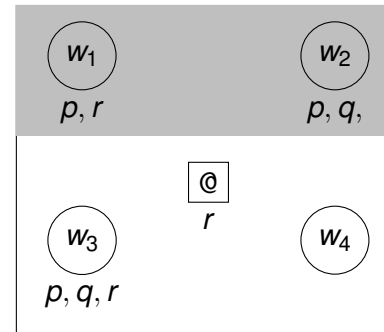
A DML Model



Truth in a model

- $V_M(\mathcal{P}\phi, w) = 1$ iff there is $u \in S$ such that $V_M(\phi, u) = 1$
- $V_M(\mathcal{O}\phi, w) = 1$ iff for all $u \in S$, $V_M(\phi, u) = 1$

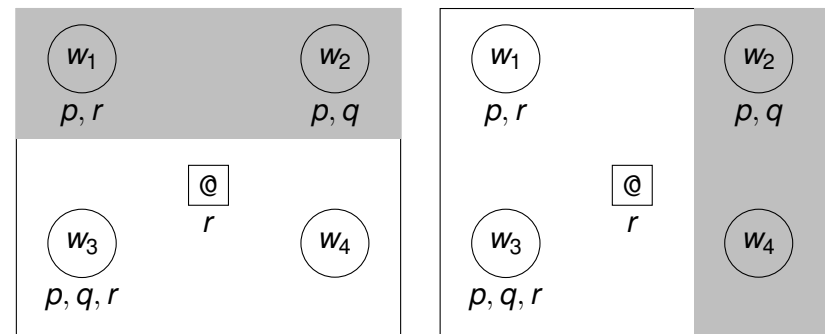
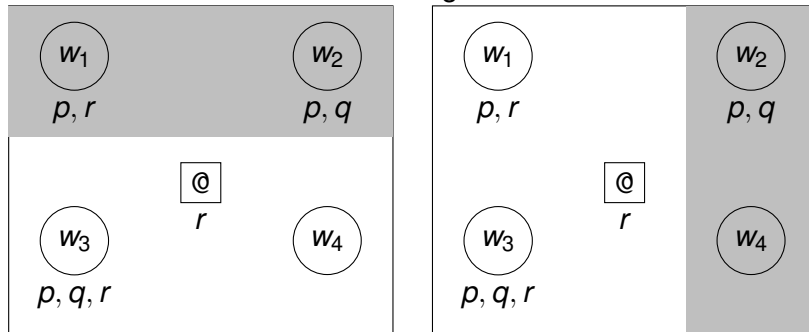
A DML Model



In our model $\mathcal{O}p$ is true and so is $\mathcal{P}r$.

Taxonomy

We need a relation on models with respect to which normative sentences are sensitive: S-shifting!



S-shift fragile sentences: ???

S-shift breakable sentences: $\mathcal{O}p$, $\mathcal{P}r$, $\neg \mathcal{O}p$, $r \vee \mathcal{O}p$, $r \rightarrow \mathcal{O}q$

S-shift insensitive sentences: p , q , r , $\Box p$, $\Diamond p$, $\mathcal{O}p \vee \neg \mathcal{O}p$, $\mathcal{O}p \wedge \neg \mathcal{O}p$, $\mathcal{O}p \rightarrow \mathcal{P}p$.

Reasons to use R-breakability

- better taxonomy:
 - partition the set of sentences,
 - classify $p \rightarrow Oq$ as normative,
 - classify Op as normative even in contexts where $\Box p \models Op$ (without thereby classifying $\Box p$ as normative.)
- remaining worry? We can't prove the barrier with breakability.
- Solution: we can still prove a limited barrier.

The Limited Barrier

No descriptive set Γ entails a normative sentence ϕ , **unless C** (where C is some condition on Γ and ϕ .)

But what should C say?

No descriptive set Γ entails a normative sentence ϕ unless for any model M of Γ , every c-shift N of M , is a model of ϕ .

Back to our counterexamples

- 1 Prior's Dilemma: $D \models D \vee N$ $D \vee N, \neg D \models N$
- 2 Mavrodes' Argument from Ought implies Can: $\neg \Diamond p \models \neg Op$
- 3 Quotation

The End.

Thank you!

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