

Logic and Metaphysics: Part I

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1 Opening Discussion

2 Recap: Systems of Modal Logic

Possible worlds semantics for MPL help us interpret sentences involving modal operators ($\Diamond$, $\Box$) by means of a relative accessibility relation on a set of possible worlds.

$\Diamond P$ is true at w iff there's some world w' such that $R(w, w')$ and P is true at w'

$\Box P$ is true at w iff for every world w' such that $R(w, w')$ and P is true at w'

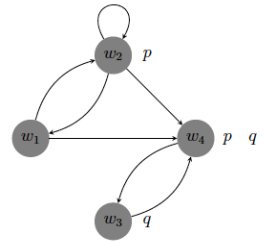
Reminder

A model M for the language of modal propositional logic is $\langle W, R, V \rangle$, where:

1. W is a non-empty set of worlds
2. R is a binary accessibility relation on W
3. V is a valuation function that assigns to each propositional variable p a set $V(p)$ of worlds.

When $R(w, v)$ we say that v is accessible from w , or sometimes that “ w sees v ”.

When $w \in V(p)$, we say that p is true at w .



One of the handiest features of Kripke models is their ability to reflect correspondences between structural constraints on *relative possibility* and a variety of modal axioms. The usual celebrities:

If R is ...	then all instances of ... are true at every world
Serial ($\forall u \exists w R u w$)	$\Box A \rightarrow \Diamond A$ (D)
Reflexive ($\forall w R w w$)	$\Box A \rightarrow A$ (T)
Symmetric ($\forall u \forall w (R u w \rightarrow R w u)$)	$A \rightarrow \Box \Diamond A$ (B)
Transitive ($\forall u \forall t \forall w (R u t \wedge R t w \rightarrow R u w)$)	$\Box A \rightarrow \Box \Box A$ (4)
Euclidean ($\forall u \forall w \forall t (R u w \wedge R u t \rightarrow R w t)$)	$\Diamond A \rightarrow \Box \Diamond A$ (5)

Quick reminder: although (eg.) T is true at every world in models where R is reflexive, there are models where T is true at every world even though R is *not* reflexive. To get at the correspondences between modal axioms and accessibility conditions, we look instead at the frames $\langle W, R \rangle$ that classes of models are based on.

Finally, (just for reference!) here are some **normal modal logics**, with their characteristic axioms and the corresponding conditions on frames. (Check out the SEP entry on Modal Logic for more.)

System Name	Axioms	Corresponding Frame Condition
D	K,D	R is <i>serial</i>
T	K,T	R is <i>reflexive</i>
B	K,T, B	R is <i>reflexive and symmetric</i>
S4	K, T, 4	R is <i>reflexive and transitive</i>
S5	KT5 or KTB4	R is <i>reflexive, symmetric, and transitive</i>

The R relation allows us to model the relation of *relative* accessibility between worlds (situations, states of affairs, total ways for things to be). But when it comes to *metaphysical possibility*, it is tempting to think that the ‘world-relativity’ falls out: every *total way for things to be* is possible relative to every possible world; to be a total way for things to be just is to be a (metaphysically) *possible* way for things to be.

3 The Simplest Quantified Modal Logic

The system that is sometimes called the “Simplest Quantified Modal Logic” is given by S5 together with *classical quantification theory* — plain vanilla first-order predicate logic you may have encountered before. Adding axioms for identity, we get the more-or-less orthodox logic of metaphysical modality:

Axioms of SQML	
→	Axioms of S5 MPL
→	Universal Instantiation: $\forall x A \rightarrow A[t/x]$, where t is substitutable for x in A
→	Id. $x = x$
→	LL. $x = t \rightarrow (A \rightarrow A[t/x])$
And the usual rules of inference:	
Modus Ponens:	$A, A \rightarrow B \vdash B$
Universal Generalization:	$A \rightarrow B \vdash A \rightarrow \forall x B$
Necessitation:	$A \vdash \Box A$

S5 alone gives us some very significant **modal reduction theorems**:

- $\Box A$ iff $\Box \Box A$
- $\Diamond A$ iff $\Diamond \Diamond A$
- $\Box A$ iff $\Diamond \Box A$
- $\Diamond A$ iff $\Box \Diamond A$

CQT has plenty to say on its own, as well:

Something. Something exists.

1. $x = x$ *Id*
2. $\exists y y = x$ *EG*

And then things *really* get going:

NI. $x = y \rightarrow \Box x = y$

ND. $x \neq y \rightarrow \Box x \neq y$

(More to come this afternoon!)

Today's Theme: Is SQML the logic of metaphysical modality?

You'll sometimes hear debates over SQML described as debates about whether *SQML is the true modal logic*. We're going to be talking today about "*challenges to SQML*," but there's a particular spirit in which we want to approach those challenges. *(Discussion)*

4 Challenges to S5

We're going to be looking first at challenges that target the characteristic axiom of **S4** — that is, challenges that target the *transitivity* of metaphysical possibility.

4.1 Essentialism

Essentialism: (At least some) objects have (at least some) essential properties.

Essential properties are properties that constrain how an object can be. We can mostly get by with the *modal characterization of essence*:

F is an essential property of x iff necessarily, if x exists, x is F.

Some varieties of essentialism, roughly sketched:

Origin Essentialism. Objects have their origins essentially

(eg. *Socrates couldn't have had different parents.*)

Overlap Origin Essentialism. If a collection C of atoms originally composed x, then necessarily, x is not originally composed by any collection of atoms with no members in common with C.

(eg. *this table couldn't have originally been made from a hunk of wood that had no parts in common with the wood it was actually made from.*)

Sortal Essentialism. Objects have their sortal properties essentially.

(eg. *Socrates is essentially human.*)

4.2 Salmón and the Tables

The Chandler-Salmón argument targets *the transitivity of possibility*, and so threatens both S4 and S5. Here's the usual case, from Salmón (1989):

“A particular wooden table which we may call "Woody" could have originated from matter slightly different from its actual original matter m^* (...) but not from entirely different matter. Wherever one may choose to draw the line between what matter Woody might have originated from and what matter Woody could not have originated from, it would seem that, by stretching things to the limit, we may select some (presumably scattered) matter m such that, although Woody could not have originated from m , m is close enough to being a possibility for Woody that if Woody *had* originated from certain matter m' that *is in fact* possible for Woody — matter differing in as many molecules from the original matter m^* as is possible and sharing as many molecules with m as possible, while remaining a possibility for Woody — then it would have been possible for Woody to have originated from m , even though it is not actually possible.”

It'll be helpful for our initial discussion to isolate a few commitments playing an important role here. (We'll revisit generalizations of these shortly.)

Some Limits.	It is not possible for Woody to have originated from radically different matter <i>m</i> .
Minimal Tolerance.	It is possible for Woody to have originated from slightly different matter <i>m</i> '.
Counterfactual Tolerance.	If Woody <i>had</i> originated from matter <i>m</i> ', Woody <i>could have</i> originated from matter <i>m</i> .

The official target here, though, is **4** (or, shortly, **Iteration**): whatever is possibly possible is possible. From Salmón (1989):

“Recall that it is (just barely) impossible for Woody the table to have originated from certain matter *m*. Woody cannot be in the state of originating from *m*. That is, originating from *m* is a state metaphysically unavailable to Woody; it is a way that Woody cannot be. **But it is still a way for an individual to be.**”

“Since Woody originates from *m* according to *W*, and Woody metaphysically cannot do so, *W* is a total way things cannot be. (...) **Total ways things cannot be are thus also “worlds”, or maximal ways for things to be. They are impossible worlds.** (...) Although *W* is impossible relative to the actual world, it is possible relative to *W*', which is itself possible relative to the actual world. Thus, *W* is a possibly possible world. (...) The binary relation between (possible or impossible) worlds of relative possibility — the modal relation of accessibility — is not transitive.”

“**If worlds include ways things metaphysically cannot be in addition to ways things metaphysically might have been, then the idea that metaphysical necessity corresponds to truth in every world whatsoever is flatly mistaken.** (...) Do worlds, *qua* ways for things to be, include ways things cannot be in addition to ways things might have been? I know of no plausible grounds for denying that they do.”

Some counterpoints:

“Say I: This is no defense, this is capitulation. (...) By what right do we ignore worlds that are deemed inaccessible? Accessible or not, they're still worlds. We still believe in them. Why don't they count?” (Lewis 1986: 246)

That is: the “inaccessible worlds” iteration-denying strategy is no defense of **Some Limits**; it is ultimately just capitulation to radical *anti*-essentialism — we're still granting *ways for the world to be where Woody is made of m*, while merely denying that they're ‘relatively possible’.

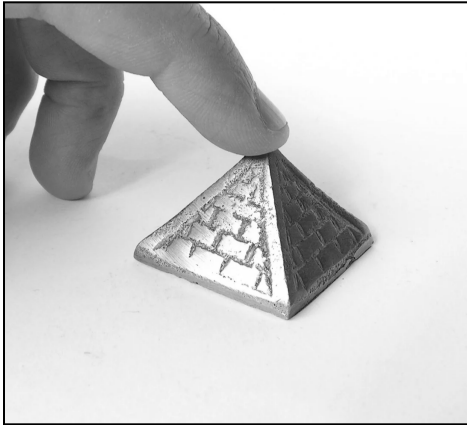
“We look in vain...for an account of what it means to deny that some world is ‘relatively possible’. I think it is like saying: there are things such that, ignoring them, there are no such

things. Ignoring all of the worlds where such and such obnoxious things happen, it is impossible that such things happen. Yes. Small comfort.” (*ibid.* 248)¹

(Discussion)

5 Tolerance Puzzles

Let’s look instead at the formulation in *Bounds of Possibility* (Dorr, Hawthorne, and Yli-Vakkuri 2021):



The Great Pyramid could have been a little bit smaller. But it couldn’t have been as small as a thimble.

The *Mona Lisa* could have been slightly different as regards its spatial distribution of colors. But it could not have had the spatial distribution of colors that we find in Edvard Munch’s *The Scream*.

The Vienna Circle could have had somewhat different members. But it couldn’t have had Sigmund Freud, Arnold Schoenberg, and Franz Kafka as its only members.

“These arguments purport to show that if the objects in question are *tolerant* — capable of being somewhat different in the relevant respects from the way they in fact are — they are also *hypertolerant* — capable of being vastly different in these respects from the way they in fact are. The motivating thought in these arguments is that the fact that the relevant objects are tolerant — assuming it is a fact — doesn’t seem to be a mere accident.” (*ibid.* 2)

Every *tolerance argument* starts from the thought that something could have had some properties that are (in some respect to be filled in each time) *close to* the ones it actually has. We’ll use a couple of schematic definitions:

x is tolerant: for any properties *F* and *G* such that *x* instantiates *F* and *G* is **close to** *F*, it is possible for *x* to instantiate *G*.

x is hypertolerant: for any *F* and *G* such that *x* instantiates *F* and *G* is **ancestrally close to** *F*, it is possible for *x* to instantiate *G*.

Quick Details Note: ‘ancestral closeness’ is just the transitive closure of ‘closeness’. *G* is ancestrally close to *F* iff there’s a finite sequence from *F* to *G* where every property in the sequence is close to its predecessor. Today we’ll just treat **Persistent Closeness** as a background assumption: when properties are close, they are necessarily close.

¹ Lewis does ultimately go for something similar to Iteration-denial, but *via* counterpart theory.

5.1 A Schema For Tolerance Arguments (*lightly adapted*)

Tolerance	a is tolerant
Non-Contingency	If a is tolerant, then it is necessary that a is tolerant.
Iteration	Whatever is possible is possibly possible.
Hypertolerance	a is hypertolerant.

The gears turn roughly as follows. (See 2021:57) **Tolerance** and **Non-Contingency** tell us that:

Necessitated Tolerance. It is necessary that a is tolerant.

Given **Persistent Closeness**, this means that for any close properties F and G , necessarily, if a is F , then it is possible for a to be G . And so, in particular, if it is possible for a to be F , then it is possibly possible for a to be G . And so by **Iteration**,

Possibility Transfer. For any close properties F and G , if it is possible for a to be F , then it is possible for a to be G .

Every property close to a property a possibly has is itself a property a possibly has. And so if a is F , then every property that can be reached by a finite series of closeness-hops from F is a property a could have. That is: a is **Hypertolerant**.

5.2 Discussion: what are the options?

In the *hard* cases, should we...

Deny Tolerance?
Deny Non-Contingency?
Deny Iteration?
Accept Hypertolerance?

Selected References and Further Reading

Some selections from our discussion, to get you started. For other recent work on topics we discussed (eg. essence and essentialism, plenitude, higher-order metaphysics, possible worlds) check out the relevant entries in the SEP, search Philosophy Compass, or email us!

Chandler, Hugh S. (1976). Plantinga and the Contingently Possible. *Analysis* 36 (2):106 - 109.

“One of the interesting things in Alvin Plantinga's latest book, *The Nature of Necessity*, is his contention that there are no states of affairs which are only *contingently* possible, i.e. possible in some possible worlds and not possible in others. (...) Nevertheless I want to suggest that what is possible varies from world to world. (...) We intuit that a particular bicycle (...) could not have come into existence made up of totally different parts. On the other hand, this bicycle *could* have come into existence with *one* of its parts different from the one it actually had. (...)”

Dorr, Cian; Hawthorne, John & Yli-Vakkuri, Juhani (2021). *The Bounds of Possibility: Puzzles of Modal Variation*. Oxford: Oxford University Press.

Leslie, Sara-Jane (2011). Essence, Plenitude, and Paradox. *Philosophical Perspectives* 25 (1):277-296.

Lewis, David (1986). *On the Plurality of Worlds*. Wiley-Blackwell.

Miller, Kristie (forthcoming). Contingentism in Metaphysics. In Rikki Bliss & James Miller (eds.), *The Routledge Handbook of Metametaphysics*. Routledge.

“While entity contingentism and entity necessitarianism are views about the modal status of entities, *metaphysical* contingentism and necessitarianism are views about the modal status of metaphysical principles. *Metaphysical contingentism* about some metaphysical principle, P, is the view that P is contingent: it is true in some worlds, and false in others. *Metaphysical necessitarianism* about P is the view that P is necessary: either P is true in every world, or false in every world. (...) This chapter principally focuses on metaphysical rather than entity contingentism, though §2 briefly discusses the latter.”

Nolan, D. (2011). The Extent of Metaphysical Necessity. *Philosophical Perspectives*, 25(1):313–39.

“Rather than chart all the different ways people have used [“metaphysical necessity”], I thought it better to try to chart what seem to be the best options for subject matters in the area that people seem to be aiming at by using the expression. So after some preliminary remarks about what it is that people want from “metaphysical necessity”, I will discuss a range of options for answering what seem to me to be the interesting questions.”

Paul, L. A. (2012). Metaphysics as Modeling: The Handmaiden's Tale. *Philosophical Studies* 160 (1):1-29.

Rosen, G. (2006). The Limits of Contingency. In MacBride, F., editor, *Identity and Modality*, pages 13–40. Oxford University Press.

Salmon, Nathan (1989). The Logic of What Might Have Been. *Philosophical Review* 98 (1):3-34.

Yablo, Stephen (forthcoming). Everything Exists: Plenitude, Kinds, and Paradox. In Cumpa, J., editor, *The Question of Ontology: The Contemporary Debate*. (OUP)