PIKSI: Logic and Metaphysics

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For the rest of today, we're going to dive deeper into a specific challenge to SQML and the modal ontological debate—the debate concerned with whether or not it's contingent what exists. Logic has played an interesting and controversial role in the discussion.

0. Preliminaries

Question. Is it contingent what there is?

Necessitism: necessarily everything necessarily is something
$\Box \forall x \Box \exists y(y = x)$.[1]

Contingentism: the negation of necessitism
$\neg \Box \forall x \Box \exists y(y = x)$.

[1] $\Box$ is read as expressing a kind of metaphysical necessity and the quantifiers are read unrestrictedly.

1. Are Barcan, converse Barcan, and NNE valid in SQML?

Barcan schema: $\forall x \Box \phi \rightarrow \Box \forall x \phi$

Converse Barcan schema: $\Box \forall x \phi \rightarrow \forall x \Box \phi$

NNE: $\Box \forall x \Box \exists y(y = x)$[2]

[2] This one on handout.

2. Necessitism is valid in SQML (with =)

Assumptions

I= $a = a$, where a is an individual constant

$\exists$**G** $\phi \rightarrow \exists x \phi[x/a]$, where x is free for a in ϕ

RN If $\vdash \phi$, then $\vdash \Box \phi$

Gen if $\vdash \phi$, then $\vdash \forall x \phi[x/a]$

These principles entail necessitism

1. $a = a$ (by **I=**)
2. $\exists x(x = a)$(by $\exists$**G**)
3. $\Box \exists x(x = a)$ (by **RN**)

The above is the core argument; by **Gen** and **RN**, we get NNE.

Direct argument

NI $\Box \forall x \Box(x = x)$
IE $\Box \forall x \Box(x = x \rightarrow \exists y(y = x))$
Therefore, $\Box \forall x \Box \exists y(y = x)$[3]

[3] Goodman 2016.

One distinction between contingentists

Serious actualism: Having a property or standing in a relation entails existing.

The being constraint: **BC** $\Box\forall x_1 ... \Box\forall x_n \Box(Rx_1...x_n \to \exists y(y = x_i))$ for $1 \leq i \leq n$.

Some responses

Restrict $\exists$**G**: $\exists$**GR** $\forall y(\phi \to \exists x\phi[x/y])$, where x is free for y in ϕ.

Restrict **I=**: **=IR** $\forall x(x = x)$.

Deny **RN**.

3. Necessitist vs. contingentist logic?

Part of the ongoing necessitist challenge to contingentists is that contingentist logics are inferior to necessitist ones and that considerations of theory choice (esp. strength) favor necessitism.

"Without [$\exists$ G], the axiomatization of quantified modal logic becomes much harder; the required complications are greater than in the formal semantics, and completeness proofs are more convoluted. Such complications are a warning sign of philosophical error." (Williamson, 1998, p. 262)

4. Entrenched modal judgments

Are logical considerations or the strength of abstract principles relevant to a debate where one of the views flies in the face of our entrenched judgments?

Do our entrenched modal judgments favor contingentism?

Necessitists: contingently concrete, contingently non-concrete, chunky

5. Going higher-order

First-order contingentism: roughly, it is contingent what individuals there are

Higher-order contingentism: roughly, it is contingent what propositions and properties there are

Thorough-going contingentism: roughly, it is contingent what individuals, propositions, and properties there are

Hybrid contingentism: roughly, it is contingent what individuals there are, but it is necessary what propositions and properties there are

6. *Higher-order logic and lambda: basics*

First-order quantifiers allow us to quantify into the position that a singular term occurs. For example, in classical FOL:

from Fa we can infer $\exists xFx$

where x is a bound variable taking the place of the singular term a. We may like to pronounce the latter as, there is some*thing* that is F

In higher-order logic, we can quantify into the position of any expression whatsoever. For example, in standard discussions:

from Fa we may infer $\exists XXa$

where X is a bound variable taking the place of *concepts*, following Frege. We may like to pronounce the latter as, there is some property a has, or there is some*way* that a is.

We will also make use of λ expressions, a device for creating further operators, predicates, and other expressions using expressions we already have, and reduction and abstraction rules that govern these expressions.

For any formula ϕ, the expression $\ulcorner(\lambda x.\phi)\urcorner$ is a monadic predicate that takes expressions of the same syntactic type as x as arguments and in which all occurrences of x are bound. Some people read the expression as, is such that ϕ, but other pronunciations may sound more natural in specific cases.

The $\lambda-$calculus[4] allows us to form functions from formulas and vice versa using expressions we already have. Consider a simple example:

Ted is sitting and eating.

How would you translate this into standard first-order symbolization you're used to? The standard treatment of λ expressions allows us to take the closed formula above and abstract the complex predicate $\lambda x(Sx\&Ex)$, pronounced 'is sitting and eating', such that when we apply it to 'Ted' we get something that—on a standard treatment of reduction and abstraction—entails and is entailed by the original formula

REDUCTION: we may substitute a term of the form $(\lambda x.\phi)a$ for $\phi[a/x]$, where a is free for x in ϕ[5]
ABSTRACTION: we may substitute a term of the form $\phi[a/x]$ for $(\lambda x.\phi)a$, where a is free for x in ϕ[6]

How would you translate the following into both the standard first-order symbolization you're used to and in the new λ symbolization?

1. (if time for quantifiers) Something is sitting and eating
2. (if time for quantifiers) There is someone who respects everyone.

[4] The $\lambda-$calculus is a formal system in mathematical logic which is based on function application (applying functions to arguments) and abstraction (forming functions by abstracting on formulas). It is a non-extensional theory of functions. One might think of it in contrast to an extensional view of functions as sets of ordered pairs.

[5] That is, x is everywhere substitutable for a in ϕ if for any variable y that appears in x it is not the case that a appears in the scope of a quantifier of the form $\forall y$ or $\exists y$ within ϕ.

[6] Another important rule is the η rule, where if I apply a predicate ϕ to a variable (i.e. ϕx) and lambda out the variable (i.e. $\lambda x\phi x$) I get something formally equivalent to what I started with.

How would you express the following: the complex predicate *being necessarily F* instantiates the higher-order concept of *applying to a*?[7]

$\beta - equivalence : (\lambda x \phi)a \leftrightarrow \phi[a/x]$, where a is free for x in ϕ.[8]

Prove that $(\lambda X Xa)\lambda x \Box Fx$ is equivalent to $\Box Fa$.

[7] Example from Bacon 2018.

[8] For any formula ϕ, $(\lambda x_1, ..., x_n.\phi)(a_1, ..., a_n)$ and $\phi[a_i/x_i]$ are one-step β−equivalent, where the $\phi[a_i/x_i]$ means that we replace all occurrences of x_i in ϕ with a_i, and the a's are free for the x's in ϕ. $x_1, ..., x_n$ are distinct variables, but $a_1, ..., a_n$ are not necessarily distinct terms.

7. The higher-order argument

The pre-theoretically favored strengthening of the typical non-modal comprehension principle is

Comp$_M$: $\exists X^{\langle t_1,...,t_n\rangle} \Box \forall x_1^{t_1}...x_n^{t_n}(Xx_1...x_n \leftrightarrow A)$, where X does not occur free in A.[9]

[9] Here, A is any formula, x is an individual variable that may occur free in A, and X is a monadic predicate variable that applies to individuals. From now on, I will omit the prefixing of multiple quantifiers and variables and n−placed predicates.

This comprehension principle (with RN, Gen, and the assumption the properties are identical iff they're coextensive) implies property necessitism:

NNE$_M$: $\Box \forall X \Box \exists Y \Box \forall x(Xx \leftrightarrow Yx)$

Note that Comp$_M$ can be derived from (primarily) the following two principles:

$\exists$**G2** $\phi \rightarrow \exists X\phi[X/P]$, where X is free for P in ϕ.
$\beta - equivalence : (\lambda x\phi)a \leftrightarrow \phi[a/x]$, where a is free for x in ϕ.

As follows:

1. $\Box \forall x((\lambda y Ay)x \leftrightarrow A)$ (β−equivalence, RN)
2. $\exists X \Box \forall x(Xx \leftrightarrow A)$ (1, $\exists$**G2**)

Some responses

Option 1: reject 1 by restricting β−equivalence:

Rβ−equivalence: $\exists y(y = a) \rightarrow ((\lambda x\phi)a \leftrightarrow \phi[x/a])$, where a is free for x in ϕ

Option 2: reject 2 by restricting $\exists$**G2**

R$\exists$**G2**: $\forall X(\phi(A) \rightarrow \exists X\phi[X/A])$, where X is free for A in ϕ

8. What is the role of logic in metaphysics

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